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Transmission lines with distributed electric coefficients. Metamaterials

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Metamaterials are complex structures consisting of in-built microscopic functional devices exhibiting, basically, a non-linear behaviour. They are expected to possess a complex response to various inputs, like electric, mechanical, thermal, optical, etc loads.

Let C and L be the distributed capacitance and, respectively, inductance along an electric transmission line of coordinate z. The density of electric current is given by

$$i = \frac{\partial}{\partial t}[q(z) - q(z + \Delta z)]/\Delta z = -\partial^2 q/\partial t \partial z \ \ , \tag{1}$$

where q is the local electric charge. Similarly, Similarly, the voltage drop is given by

$$-u(z + \Delta z) = aL\frac{\partial}{\partial t}[i(z + \Delta z) - i(z)] \ \ , \tag{2}$$

or

$$u = -a^2 L \partial^2 i/\partial t \partial z \ \ , \tag{3}$$

where a is a characteristic length of the microscopic distribution of electric coefficients. By (1) and (3) we get[1]

$$a^2 L \partial^4 q/\partial t^2 \partial z^2 = u \ . \tag{4}$$

Let $q = Cu$, for a linear regime, so that (4) becomes

$$\partial^4 u/\partial t^2 \partial z^2 - (\omega_0/a)^2 u = 0 \ \ , \tag{5}$$

where $\omega_0 = 1/\sqrt{LC}$ is the line frequency. The plane wave solution of this equation has the interesting dispersion law

$$\omega = \omega_0/ak \tag{6}$$

Equation (6) gives a negative group velocity,

$$v = \partial\omega/\partial k \sim -\omega^2 \ \ , \tag{7}$$

which makes these metamaterials be called left-handed (alternatively, they may also be viewed as negative-refractive index materials, $n = ck/\omega < 0$ by taking $\omega = -\omega_0/ak$ in (6); waves go one way in these materials, while de energy goes the opposite way) and exhibits also an anomalous dispersion, which may be indicative of solitons. In particular, high frequencies move faster, which makes, for instance, the fore-wing of a pulse to move away, while the trail-wing to be compressed, indicating thus a new mechanism of pulse compression.

The non-linearities are included through

$$q = Cu(1 - \alpha u) \tag{8}$$

in (4), leading to

$$\partial^4 u/\partial t^2 \partial z^2 - (\omega_0/a)^2 u = \alpha \partial^4 u^2/\partial t^2 \partial z^2 \ \ , \tag{9}$$

where α may be viewed as a formally small perturbation parameter. A cursory analysis of this equation seems to indicate higher harmonics, whose superposition, at least to lowest orders, may give $\delta(z - vt)$, *i.e.* a singular pulse. It might be relevant for ultra-wide band radar and communications.[2]

References

[1] C. Caloz et al, Microwave and Opt. Techn. Lett. **40** 471 (2004)

[2] T. W. Barrett, Progress in Electromagnetics Symposium 2000, Cambrige, MA, July, 2000