## The Antiphysical Review

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PART I : The MESSAGE

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Fields Today

1) Gauge\&Symmetries --Imitation of QElectrodyn
2) Geometrization

> The Third Way

Principle of Equivalence, Non-Inertial Motion

A "New Deal" in Fields

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## PART II : MOTION DEPENDS ON OBSERVER

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## Curved Space

Newton's Law

$$
m \frac{d v_{\alpha}}{d t}=f_{\alpha}
$$

Equivalent with a free motion in a curved space

$$
D u^{i} / d s=\frac{d u^{i}}{d s}+\Gamma_{j k}^{i} u^{j} u^{k}=0
$$

Metric tensor

$$
d s^{2}=(1+h) c^{2} d t^{2}+2 c d t g_{\alpha} d x^{\alpha}-d x^{\alpha} d x^{\alpha}
$$

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$$
g_{i j}=\left(\begin{array}{cccc}
1+h & g_{1} & g_{2} & g_{3} \\
g_{1} & -1 & 0 & 0 \\
g_{2} & 0 & -1 & 0 \\
g_{3} & 0 & 0 & -1
\end{array}\right)
$$

Basic equation

$$
\begin{gathered}
\frac{\partial g_{\alpha}}{c \partial t}-\frac{1}{2} \cdot \frac{\partial h}{\partial x^{\alpha}}=f_{\alpha} / m c^{2} \\
\mathbf{g}(t), h(\mathbf{r}) \\
f_{\alpha}=-\partial \varphi / \partial x^{\alpha}, h=2 \varphi / m c^{2}
\end{gathered}
$$

(gravitational potential)
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For the first time Einstein "suspected the time" (1905)
Curved Spaces: Gauss ( $\sim 1830$ )
Riemann: Uber die Hypothesen welche der Geometrie zugrunde liegen, 1854
Grassman, Christoffel, Ricci, Levi-Civita
Klein: Programm zum Eintritt in die philosophische Fakultat in Erlangen, 1872
Einstein, Poincare, Minkowski, Sommerfeld, Kottler, Weyl, Hilbert
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Motion depends on subjectivity, though a "universal subjectivity" ("inter-subjectivity"?)
Pauli 1921
Covariance; Dirac
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Non-Uniform Translation

$$
\begin{aligned}
& \mathbf{r}=\mathbf{r}^{\prime}+\mathbf{R}\left(t^{\prime}\right), t=t^{\prime} \\
& m \frac{d \mathbf{v}^{\prime}}{d t^{\prime}}=\mathbf{f}^{\prime}-m d \mathbf{V} / d t^{\prime}
\end{aligned}
$$

Inertial force

$$
\mathbf{g}=-\mathbf{V} / c
$$

Equivalent with a curved space (Principle of equivalence)

## Coordinate Transformations

From a flat space to a curved space

$$
\begin{gathered}
d x^{i}=a_{j}^{i} d x^{\prime j}, d x^{\prime i}=b_{j}^{i} d x^{j}, d x_{i}=b_{i}^{j} d x_{j}^{\prime} a_{k}^{i} b_{j}^{k}=b_{k}^{i} a_{j}^{k}=\delta_{j}^{i} \\
d s^{2}=\eta_{i j} d x^{i} d x^{j}=\eta_{i j} a_{k}^{i} a_{l}^{j} d x^{\prime k} d x^{\prime l} \\
g_{i j}=\eta_{l m} a_{i}^{l} a_{j}^{m}, g^{i j}=\eta^{l m} b_{l}^{i} b_{m}^{j}, \text { where } \eta^{l m}=\eta_{l m}(+,---) \\
d t=\frac{(1+h) d t^{\prime}+(g+\beta \Delta) d x^{\prime} / c}{\sqrt{(1+h)\left(\left(1-\beta^{2}\right)\right.}}, d x=\frac{c \beta(1+h) d t^{\prime}+(\beta g+\Delta) d x^{\prime}}{\sqrt{(1+h)\left(\left(1-\beta^{2}\right)\right.}}
\end{gathered}
$$

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$\beta=d x / c d t, \Delta=\sqrt{1+h+g^{2}}$
Two frames, relative velociy $\beta$, one flat, the other curved
-For $h, \mathbf{g}=0$ - Lorentz transformations
-We put $\beta=V / c=-\mathbf{g}$ as before (to give a sense to our curved space)
-We put $h(\mathbf{r})$ and $\mathbf{g}(t)$
-We use $h, \mathbf{g} \ll 1$, to get corrections to the relativistic motion
-We get

$$
d t=(1+h / 2) d t^{\prime}, d x=d x^{\prime}-c g d t^{\prime}=d x^{\prime}+V d t^{\prime}
$$

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## Life in a curved space

Proper time $d \tau=\sqrt{1+h} d t$
Distance $d s^{2}=c^{2}(1+h)[d t+\mathbf{g} d \mathbf{r} / c(1+h)]^{2}-\left[d \mathbf{r}^{2}+(\mathbf{g} d \mathbf{r})^{2} /(1+h)\right]$
or

$$
d s^{2}=c^{2} d t^{\prime 2}-d \mathbf{l}^{2}
$$

Light propagates along curved geodesics $(d s=0)$ with velocity $c$ Time is indefinite, $d \tau$ and $d t^{\prime}$, depends on path

## A GENERALIZED HAMILTON-JACOBI EQUATION

-Free motion $t \rightarrow x$; Motion under forces $t \rightarrow x^{\prime}$
-Had we know $x \rightarrow x^{\prime}$, i.e. a global coordinate transf, we solve the motion
-Einstein's line of thought
-We have not that global coordinate transformation
(cannot get the $10 g_{i j}$ with four functions; local flat spaces, but axes are different from point to point)

- Our transformations are Local! $\left(x=(c t, \mathbf{r}), x^{\prime}=\left(c t^{\prime}, \mathbf{r}^{\prime}\right), d x \longleftrightarrow d x^{\prime}\right)$

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## One Exception: Special Theory of Relativity

From rest to motion (principle of inertia, $d s^{2}=$ const)

$$
x=c \beta \tau / \sqrt{1-\beta^{2}}, t=\tau / \sqrt{1-\beta^{2}}
$$

A vector: momentum $\mathbf{p}=\partial S / \partial \mathbf{r}$, energy $p_{0}=E / c=-\partial S / c \partial t$
Apply these transf to this vector

$$
\mathbf{p}=\mathbf{v} E / c^{2}, E=E_{0} / \sqrt{1-\beta^{2}}, E_{0}=m c^{2}
$$

Eqs of motion

$$
d \mathbf{p} / d t=\mathbf{f}
$$

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Additional "relativistic" forces $\left(\sim v^{2}\right)$
Hamilton-Jacobi equation $E^{2}-c^{2} p^{2}=m^{2} c^{4}$

$$
(\partial S / \partial t)^{2}-c^{2}(\partial S / \partial \mathbf{r})^{2}=m^{2} c^{4}
$$

This is the entire theory of special relativity
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## Hamilton-Jacobi Equation in Curved Space

## Motion in curved space

Let $\left(P_{0}=E_{0} / c,-\mathbf{P}\right)$ be the (cov) momentum of a free motion in the flat space, constant, $P_{0}^{2}-\mathbf{P}^{2}=$ $m^{2} c^{2}$

Apply the coord transf for our curved space

$$
\begin{gathered}
p_{0}=(1+h) p^{0}+g p^{1}=\sqrt{1+h} \cdot \frac{P_{0}-\beta P_{1}}{\sqrt{1-\beta^{2}}} \\
p_{1}=g p^{0}-p^{1}=\frac{(g+\beta \Delta) P_{0}-(g \beta+\Delta) P_{1}}{\sqrt{(1+h)\left(1-\beta^{2}\right)}}
\end{gathered}
$$

An integral of motion, already (by using $p_{i}=m c d u_{i} / d s$ )? NO! Different $x$ and $x^{\prime}$ ! 12
Use $P_{0}^{2}-\mathbf{P}^{2}=m^{2} c^{2}$ for $g=-\beta$
Hamilton-Jacobi Equation in curved space

$$
(E-c \mathbf{g} \mathbf{p})^{2}-c^{2}\left(1+h+g^{2}\right)\left(p^{2}+m^{2} c^{2}\right)=0
$$

or

$$
(\partial S / \partial t+c \mathbf{g} \partial S / \partial \mathbf{r})^{2}-c^{2}\left(1+h+g^{2}\right)\left[(\partial S / \partial \mathbf{r})^{2}+m^{2} c^{2}\right]=0
$$

## Euler-Lagrange Motion

The action

$$
\begin{gathered}
S=-m c \int d s=-m c^{2} \int d t \cdot\left(1+h+2 \mathbf{g v} / c-v^{2} / c^{2}\right)^{1 / 2}=\int d t \cdot L \\
m d \mathbf{p} / d t=\mathbf{F}
\end{gathered}
$$

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$$
\begin{gathered}
\mathbf{F}=\partial L / \partial \mathbf{r}=-\left(m c^{2} / 2\right) \cdot \frac{\partial h / \partial \mathbf{r}}{\left(1+h+2 \mathbf{g v} / c-v^{2} / c^{2}\right)^{1 / 2}} \\
E=\mathbf{p} \mathbf{v}-L=\frac{m c^{2}(1+h)+m c \mathbf{v} \mathbf{g}}{\left(1+h+2 \mathbf{g v} / c-v^{2} / c^{2}\right)^{1 / 2}}
\end{gathered}
$$

We get again the Hamilton-Jacobi equation given before

$$
(E-c \mathbf{g} \mathbf{p})^{2}-c^{2}\left(1+h+g^{2}\right)\left(p^{2}+m^{2} c^{2}\right)=0
$$

## Lagrange Motion

$$
S=-m c \int d s
$$

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$$
\begin{gathered}
\delta d s^{2}=2 d s \delta d s=\delta\left(g_{i j} d x^{i} d x^{j}\right)=2 g_{i j} d x^{i} d \delta x^{j}+d x^{i} d x^{j}\left(\partial g_{i j} / \partial x^{k}\right) \delta x^{k} \\
p_{i}=-\partial S / \partial x^{i}=m c u_{i}, p_{i}=\left(p_{0},-\mathbf{p}\right)
\end{gathered}
$$

Hamilton: $E=-\partial S / \partial t$; it follows $p_{0}=-E / c$
Since $p_{i} p^{i}=m^{2} c^{2}$, i.e. $g^{i j} p_{i} p_{j}=m^{2} c^{2}$ we get again the Hamilton-Jacobi equation in curved space

$$
(\partial S / \partial t+c \mathbf{g} \partial S / \partial \mathbf{r})^{2}-c^{2}\left(1+h+g^{2}\right)\left[(\partial S / \partial \mathbf{r})^{2}+m^{2} c^{2}\right]=0
$$

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Contravariant metric

$$
g^{i j}=\frac{1}{\Delta^{2}}\left(\begin{array}{cccc}
1 & g_{1} & g_{2} & g_{3} \\
g_{1} & -\Delta^{2}+g_{1}^{2} & g_{1} g_{2} & g_{1} g_{3} \\
g_{2} & g_{1} g_{2} & -\Delta^{2}+g_{2}^{2} & g_{2} g_{3} \\
g_{3} & g_{3} g_{1} & g_{3} g_{2} & -\Delta^{2}+g_{3}^{2}
\end{array}\right)
$$

## Eikonal Equation

-Waves go by $k_{i} d x^{i}=-d \Phi$, the eikonal (phase); flat space $k_{i}=\left(k_{0}=\omega / c,-\mathbf{k}\right)$, frequency and wavevector, $k_{i} k^{i}=(\omega / c)^{2}-\mathbf{k}^{2}=0$; straight line, $\Phi=-\omega t+\mathbf{k r}$; geometric optics
-Since $k_{i} k^{i}=g^{i j} k_{i} k_{j}=0$ we get the eikonal equation in curved space 16

$$
(\partial \Phi / \partial t+c \mathbf{g} \partial \Phi / \partial \mathbf{r})^{2}-c^{2}\left(1+h+g^{2}\right)(\partial \Phi / \partial \mathbf{r})^{2}=0
$$

-Solve it!
-neglect $g^{2}$
-first term does not depend on the time $t$ (the second doesnt!)

$$
\partial \Phi / c \partial t+\mathbf{g} \partial \Phi / \partial \mathbf{r}=-\omega_{0} / c
$$

where $\omega_{0}$ the frequency in the flat space; in addition

$$
(\partial \Phi / \partial \mathbf{r})^{2}=k^{2}=\frac{1}{1+h} \cdot\left(\omega_{0} / c\right)^{2}=\frac{1}{1+h} \cdot k_{0}^{2}
$$

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-It follows

$$
\partial \Phi / c \partial t=-\omega_{0} / c-\mathbf{g k}_{0}
$$

-What we measure? We measure the local, proper-time frequency

$$
\omega / c=-\partial \Phi / c \partial \tau=-\frac{1}{\sqrt{1+h}} \cdot \partial \Phi / c \partial t=\frac{1}{\sqrt{1+h}} \cdot \omega_{0} / c+\mathbf{g k}_{0}
$$

-Therefore a shift in frequency

$$
\Delta \omega / \omega_{0}=-h / 2+c \mathbf{g k}_{0} / \omega_{0}
$$

First term - the red shift; second term - Doppler effect (long)
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-Time-dependent part of the eikonal: $\Phi_{t}(t)=-\omega_{0} t+\mathbf{k}_{\mathbf{0}} \mathbf{R}(t)$ : a translation, as expected
-The path? $(\partial \Phi / \partial \mathbf{r})^{2}=(1-h) k_{0}^{2}$
-Write it in spherical coordinates; separate variables by $\Phi=\Phi_{r}(r)+M \varphi, M$ a constant; $\partial \Phi / \partial M=$ const gives the equation of the trajectory ( $M$ is a generalized coordinate, its momentum is constant)

The deflection angle (distance $M / k_{0}$ )

$$
\Delta \varphi=-\left(k_{0}^{2} / 2\right) \int_{\infty}^{r} d r \cdot \frac{h \cdot M / r^{2}}{\left(k_{0}^{2}-M^{2} / r^{2}\right)^{3 / 2}}
$$

(4 times smaller than in grav field of a point mass; our metric is not that metric!)

## PART IV : QUANTIZATION

## BASIC CHANGES

## Quantization

## What are we doing?

## Nothing Good (though NEW), even WORSE than before

Because it is bad to solve for a non-inertial motion; we just solve for an inertial frame and do the translation (for instance, the Ham-Jac eq is solved with $h$ for Mercury's perihelia precession; then apply the translation, etc)
This is perfectly true for classical motion with trajectory

## Things Change Fundamentally for the Quantal Motion

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## Quantization

$S=-i \hbar \ln \psi ; E \rightarrow i \hbar \partial / \partial t, \mathbf{p} \rightarrow-i \hbar \partial / \partial \mathbf{r}$
No trajectory, wavefunction $\psi$
No determined physical quantities ( $E, \mathbf{p}$ ) (operators)
Means and deviations: statistical meaning
$|\psi|^{2}$ density of probability (conservation)
Apply this procedure to the Ham-Jac eq $E^{2}-c^{2} p^{2}=m^{2} c^{2}$
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## Get the Klein-Gordon equation

$$
\partial^{2} \psi / \partial t^{2}-c^{2} \partial^{2} \psi / \partial \mathbf{r}^{2}+\left(m^{2} c^{4} / \hbar^{2}\right) \psi=0
$$

Troubles: the conserved quantity is $\psi^{*}(\partial \psi / \partial t)-\left(\partial \psi^{*} / \partial t\right) \psi$, both positive and negative (due to negative energies $\left.E=-\sqrt{p^{2} c^{2}+m^{2} c^{4}}\right)$ - nonsense

Dirac: $i \hbar \partial \psi / \partial t=\left(\alpha c \mathbf{p}+\beta m c^{2}\right) \psi$, matrices $\alpha$ and $\beta$; get a probability, but $\psi$ is a spinor; so, the question remains for the Klein-Gordon eq

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## Approximate Klein-Gordon equation

Apply the quantization to the Ham-Jac eq in curved space

$$
(E-c \mathbf{g} \mathbf{p})^{2}-c^{2}(1+h)\left(\mathbf{p}^{2}+m^{2} c^{2}\right)=0
$$

(neglecting $g^{2}$ )
Troubles: $1+h$ does not commute with $\mathbf{p}^{2}+m^{2} c^{2}$, ambiguities
We may transfer it to the lhs as $1 /(1+h)$, and neglect the $g h$-commutator
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Get then an approx Klein-Gordon eq in curved space

$$
(i \hbar \partial / \partial t-c \mathbf{g} \mathbf{p})^{2} \psi-c^{2}(1+h)\left(p^{2}+m^{2} c^{2}\right) \psi=0
$$

Still troubles, since we do not know where to put $1+h$ with respect to $p^{2}+m^{2} c^{2}$
However, in the non-relativistic limit this ambiguity does not matter, and we get the Schrodinger equation (recall $h=2 \varphi / m c^{2}$ )

$$
i \hbar \partial \psi / \partial t=H \psi=\left(m c^{2}+p^{2} / 2 m+\varphi\right) \psi+c \mathbf{g} \mathbf{p} \psi
$$

## The Fundamental Fact

The eigenstates are no more conserved due to the non-uniform translation

## We get quantal transition

An observer in a non-uniform translation sees quantal transitions

Esssential thing: do not conserve the momentum; the presence of the external potential $\varphi($ i.e. $h)$ is essential

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The approximate Klein-Gordon eq can be solved by pert theory

$$
(i \hbar \partial / \partial t-c \mathbf{g p})^{2} \psi-c^{2}(1+h)\left(p^{2}+m^{2} c^{2}\right) \psi=0
$$

Define $H^{2}=c^{2}(1+h)\left(p^{2}+m^{2} c^{2}\right)$, solve in the first order, get $E^{2}=(1+\bar{h})\left(p^{2} c^{2}+m^{2} c^{4}\right)$, the wavefunctions $\varphi(\mathbf{p})$-plane waves plus a weak admixture of plane waves (due to $h$ ); then we have $(i \hbar \partial / \partial t-c \mathbf{g p}) \psi=E \psi$

Get the transition amplitude

$$
-(i / \hbar) \int d t \cdot e^{-i\left[E(p)-E\left(p^{\prime}\right)\right] t / \hbar} \operatorname{cg}_{\mathbf{p}^{\prime} \mathbf{p}}
$$

## Conclusion: we do have quantal transitions!

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Restricting to the first-order of the perturbation theory we get also a Dirac equation

$$
(i \hbar \partial / \partial t-c \mathbf{g} \mathbf{p}) \psi=\left(\alpha c \mathbf{p}+\beta m c^{2}\right) \psi
$$

with leads to the same conclusion

## A "profound" argument

Let our eq be

$$
(\partial / \partial t+c \mathbf{g} \partial / \partial \mathbf{r})^{2} \psi-c^{2}(1+h)\left[\partial^{2} \psi / \partial \mathbf{r}^{2}-\left(m^{2} c^{2} / \hbar^{2}\right) \psi\right]=0
$$

like above
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Fourier transform; a homogeneous matricial equations in labels $(\omega, \mathbf{k})$; solve it by zeroing the determinant; get the eigenvalues; they are labelled by points ( $\omega, \mathbf{k}$ ) conveniently ordered; consequently, the eigenvalues are useless, they do not provide an algebraic relationship between $\omega$ and k

That means that for an $\omega$ we have many $\mathbf{k}$ and for a $\mathbf{k}$ we have many $\omega$
That means that the plane waves scatter both in $\omega$ and in $\mathbf{k}$
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That means that the quantization with plane waves is the only way to understand such solutions of the 2nd order diff eqs, and more, we have for them a statistical meaning; this is The Quantal Fields Theory!

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PART V : FIELDS; HOW THEY ARE and What THEY DO in a CURVED SPACE
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## Fields

Cannot forget that the above Klein-Gordon or Dirac equations in curved space are only approximate

## Way out: The Fields!

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Real Scalar Field (general note: covariant derivative)

$$
S=\int d x^{0} d \mathbf{r} \sqrt{-g} \cdot\left[\left(\partial_{i} \psi\right)\left(\partial^{i} \psi\right)+\left(m^{2} c^{2} / \hbar^{2}\right) \psi^{2}\right]
$$

Eqs of motion

$$
(i \hbar \partial / \partial t-c \mathbf{g} \mathbf{p})^{2} \psi-c^{2}(1+h)\left(p^{2}+m^{2} c^{2}\right) \psi+\left(i \hbar c^{2} / 2\right)(\partial h / \partial \mathbf{r}) \mathbf{p} \psi=0
$$

This is the real Klein-Gordon equation in curved space
Note the additional interacting term $(\partial h / \partial \mathbf{r}) \mathbf{p}$
Supports a similar treatment with the perturbation theory; same conclusion: quantal transitions 30
(Note: compare it with the KG eq in an electromagnetic field

$$
(i \hbar \partial / \partial t-e \varphi)^{2} \psi-c^{2}\left[(i \hbar \partial / \partial \mathbf{r}+e \mathbf{A} / c)^{2}+m^{2} c^{2}\right]=0
$$

Quite different! (Gauge fields!))

## The Hamiltonian of the Real Scalar Field

Quantization by $\Pi=\partial L / \partial(\partial \psi / \partial t)$, Hamiltonian by $\Pi(\partial \psi / \partial t)-L$, the Lagrangian in $S=\int d t \cdot L$

$$
H=H_{0}+H_{1 h}+H_{1 g}
$$

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$$
\begin{gathered}
H_{0}=\int d \mathbf{r} \cdot\left[c^{2} \Pi^{2} / 4+(\partial \psi / \partial \mathbf{r})^{2}+\left(m^{2} c^{2} / \hbar^{2}\right) \psi^{2}\right]= \\
=\sum_{\mathbf{p}}(\varepsilon / 2)\left(a_{\mathbf{p}} a_{\mathbf{p}}^{+}+a_{\mathbf{p}}^{+} a_{\mathbf{p}}\right) \\
H_{1 h}=\int d \mathbf{r} \cdot(\sqrt{1+h}-1)\left[c^{2} \Pi^{2} / 4+(\partial \psi / \partial \mathbf{r})^{2}+\left(m^{2} c^{2} / \hbar^{2}\right) \psi^{2}\right] \\
H_{1 g}=-(c / 2) \int d \mathbf{r} \cdot[\Pi(\mathbf{g} \partial \psi / \partial \mathbf{r})+(\mathbf{g} \partial \psi / \partial \mathbf{r}) \Pi]= \\
=-(c / 2) \sum_{\mathbf{p}}(\mathbf{g} \mathbf{p})\left(a_{\mathbf{p}} a_{\mathbf{p}}^{+}+a_{\mathbf{p}}^{+} a_{\mathbf{p}}\right)
\end{gathered}
$$

Systematic perturbation theory; scattering in the $h g$-order $\left(\varepsilon=\sqrt{p^{2} c^{2}+m^{2} c^{4}}\right)$ 32

Electromagnetic Field. Photons

$$
S=-(1 / 16 \pi c) \int d x^{0} d \mathbf{r} \cdot \sqrt{-g} F_{i j} F^{i j}
$$

$F_{i j}=\partial_{i} A_{j}-\partial_{j} A_{i} ; \partial_{i} F_{j k}+\partial_{j} F_{k i}+\partial_{k} F_{i j}=0$ (free Maxwell eqs)
Interacting Maxwell eqs $\partial_{j}\left(\sqrt{-g} F^{i j}\right)=0$

$$
\begin{gathered}
\operatorname{div}[(\mathbf{E}+g \times \mathbf{B}) / \Delta]=\operatorname{div} \mathbf{D}=0 \\
\frac{\partial}{c \partial t}[(\mathbf{E}+\mathbf{g} \times \mathbf{B}) / \Delta]=\operatorname{curl}[\Delta \mathbf{B}+\mathbf{g} \times \mathbf{E} / \Delta]
\end{gathered}
$$

Perturbation theory; scattering, both in $\mathbf{k}$ and $\omega$
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## The Hamiltonian of the Photons

$$
\begin{gathered}
S=(1 / 8 \pi) \int d t d \mathbf{r} \cdot \Delta\left(\mathbf{D}^{2}-\mathbf{B}^{2}\right)= \\
=(1 / 8 \pi) \int d t d \mathbf{r} \cdot(1 / \Delta)\left(\mathbf{E}^{2}+2 \mathbf{E}(\mathbf{g} \times \mathbf{B})-\Delta^{2} \mathbf{B}^{2}\right)
\end{gathered}
$$

$H=H_{0}+H_{1 h}+H_{1 g}$

$$
\begin{gathered}
H_{0}=\int d \mathbf{r} \cdot\left(c^{2} \Pi^{2} / 4+B^{2}\right)=\sum_{\alpha \mathbf{p}}(\varepsilon / 2)\left(a_{\alpha \mathbf{p}}^{+} a_{\alpha \mathbf{p}}+a_{\alpha \mathbf{p}} a_{\alpha \mathbf{p}}^{+}\right) \\
H_{1 h}=\int d \mathbf{r} \cdot(\sqrt{1+h}-1)\left(c^{2} \Pi^{2} / 4+\mathbf{B}^{2}\right)
\end{gathered}
$$

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$$
H_{1 g}=-\sum_{\alpha \mathbf{p}}(\mathbf{g p} / 2)\left(a_{\alpha \mathbf{p}}^{+} a_{\alpha \mathbf{p}}+a_{\alpha \mathbf{p}} a_{\alpha \mathbf{p}}^{+}\right)
$$

Systematic theory of perturbations ( $\varepsilon=c p=c \hbar k$ )
Photons are scatterred in frequency, as a consequence of a non-uniform translation, when in an external field (like a static grav field)
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## Other Fields. Quantum Gravity

Similar for other fields (spin-1/2 Dirac field) (technically more cumbersome; vierbeins)
Gravitons; quantized (with troubles); moving in a curved space $S=\int d x^{0} d \mathbf{r} \cdot \sqrt{-g} R ; g=g_{0}+\delta g$, background and gravitons; scattering of gravitons, i.e. of the space-time, on space-time, i.e. on matter or on the non-inertial motion

## Rotations

$$
\begin{gathered}
d \mathbf{r}^{\prime}=d \mathbf{r}+(\Omega \times \mathbf{r}) d t \\
d \mathbf{v}=d \mathbf{v}+(\dot{\Omega} \times \mathbf{r}) d t+2(\Omega \times \mathbf{v}) d t+[\Omega \times(\Omega \times \mathbf{r})] d t
\end{gathered}
$$

Non-uniform rotation, Coriolis, centrifugal

$$
\begin{gathered}
H=m v^{2} / 2-m(\Omega \times \mathbf{r})^{2} / 2+\varphi=p^{2} / 2 m-\Omega(\mathbf{r} \times \mathbf{p})+\varphi= \\
=p^{2} / 2 m-\Omega \mathbf{L}+\varphi
\end{gathered}
$$

No Coriolis, no centrifugal; just $\mathbf{L}$ we may neglect $\Omega^{2}$
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The above coordinate transformation gives the metric

$$
g_{i j}=\left(\begin{array}{cccc}
1+h & g_{1} & g_{2} & g_{3} \\
g_{1} & -1 & 0 & 0 \\
g_{2} & 0 & -1 & 0 \\
g_{3} & 0 & 0 & -1
\end{array}\right)
$$

with

$$
\mathbf{g}=-\Omega \times \mathbf{r} / c
$$

as before
Two distinctions: $g(t, \mathbf{r})$
$\Omega r / c \ll 1$
Coupling through the angular momentum $\mathbf{L}$
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## Conclusions

Non-inertial motion (for instance of the observer) produces quantal transitions in the presence of an external field
The coupling is through momentum $\mathbf{p}$ for translations or through the angular momentum $\mathbf{L}$ for rotations; so, the external field must not conserve these quantities
For instance, photons in a static gravitational field are scattered toward the blue (the blue shift) while seen from a non-uniform translation (or rotation)
Relation to the Unruh effect - quite distinct (the observer in the U effect sees its own motion as a bath of photons)
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Another more practical Conclusion:
The quantization in curved spaces has no meaning or it has the meaning given here

