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Curved space, Covariance, Motion and Quantization

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PART I : The MESSAGE

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Fields Today

- 1) Gauge&Symmetries —-Imitation of QElectrodyn
- 2) Geometrization

The Third Way

Principle of Equivalence, Non-Inertial Motion

A "New Deal" in Fields

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PART II : MOTION DEPENDS ON OBSERVER

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Curved Space

Newton's Law

$$m\frac{dv_{\alpha}}{dt} = f_{\alpha}$$

Equivalent with a free motion in a curved space

$$Du^i/ds = \frac{du^i}{ds} + \Gamma^i_{jk} u^j u^k = 0$$

151 (2008)

Metric tensor

$$ds^{2} = (1+h)c^{2}dt^{2} + 2cdtg_{\alpha}dx^{\alpha} - dx^{\alpha}dx^{\alpha}$$

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$$g_{ij} = \left(\begin{array}{cccc} 1+h & g_1 & g_2 & g_3 \\ g_1 & -1 & 0 & 0 \\ g_2 & 0 & -1 & 0 \\ g_3 & 0 & 0 & -1 \end{array} \right)$$

Basic equation

$$\frac{\partial g_{\alpha}}{c\partial t} - \frac{1}{2} \cdot \frac{\partial h}{\partial x^{\alpha}} = f_{\alpha}/mc^2$$

$${\bf g}(t), \ h({\bf r})$$

$$f_{\alpha}=-\partial\varphi/\partial x^{\alpha} \ , \ h=2\varphi/mc^2$$

(gravitational potential)

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For the first time Einstein "suspected the time" (1905)

Curved Spaces: Gauss (~ 1830)

Riemann: Uber die Hypothesen welche der Geometrie zugrunde liegen, 1854

Grassman, Christoffel, Ricci, Levi-Civita

Klein: Programm zum Eintritt in die philosophische Fakultat in Erlangen, 1872

Einstein, Poincare, Minkowski, Sommerfeld, Kottler, Weyl, Hilbert

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Motion depends on subjectivity, though a "universal subjectivity" ("inter-subjectivity"?) Pauli 1921

Covariance; Dirac

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Non-Uniform Translation

$$\mathbf{r} = \mathbf{r}' + \mathbf{R}(t') \ , \ t = t'$$

$$m\frac{d\mathbf{v}'}{dt'} = \mathbf{f}' - md\mathbf{V}/dt'$$

Inertial force

$$\mathbf{g} = -\mathbf{V}/c$$

Equivalent with a curved space (Principle of equivalence)

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Coordinate Transformations

From a flat space to a curved space

$$\begin{split} dx^{i} &= a_{j}^{i} dx'^{j}, \ dx'^{i} = b_{j}^{i} dx^{j}, \ dx_{i} = b_{i}^{j} dx'_{j} \ a_{k}^{i} b_{j}^{k} = b_{k}^{i} a_{j}^{k} = \delta_{j}^{i} \\ ds^{2} &= \eta_{ij} dx^{i} dx^{j} = \eta_{ij} a_{k}^{i} a_{l}^{j} dx'^{k} dx'^{l} \\ g_{ij} &= \eta_{lm} a_{i}^{l} a_{j}^{m}, \ g^{ij} = \eta^{lm} b_{l}^{i} b_{m}^{j}, \ \text{where} \ \eta^{lm} = \eta_{lm}(+, --) \end{split}$$

$$dt = \frac{(1+h)dt' + (g+\beta\Delta)dx'/c}{\sqrt{(1+h)((1-\beta^2))}}, \ dx = \frac{c\beta(1+h)dt' + (\beta g+\Delta)dx'}{\sqrt{(1+h)((1-\beta^2))}}$$

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 $\beta = dx/cdt, \, \Delta = \sqrt{1+h+g^2}$

Two frames, relative velociy β , one flat, the other curved

-For $h, \mathbf{g} = 0$ - Lorentz transformations

-We put $\beta = V/c = -\mathbf{g}$ as before (to give a sense to our curved space) -We put $h(\mathbf{r})$ and $\mathbf{g}(t)$

-We use $h, \mathbf{g} \ll 1$, to get corrections to the relativistic motion -We get

$$dt = (1 + h/2)dt'$$
, $dx = dx' - cgdt' = dx' + Vdt'$

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Life in a curved space

Proper time $d\tau = \sqrt{1+h}dt$ Distance $ds^2 = c^2(1+h)[dt + \mathbf{g}d\mathbf{r}/c(1+h)]^2 - [d\mathbf{r}^2 + (\mathbf{g}d\mathbf{r})^2/(1+h)]$ or $ds^2 = c^2\mu'^2 - \mathbf{n}^2$

$$ds^2 = c^2 dt'^2 - d\mathbf{l}^2$$

Light propagates along curved geodesics (ds = 0) with velocity cTime is indefinite, $d\tau$ and dt', depends on path 8

PART III : MOTION as a COORDINATE TRANSFORM; EINSTEIN'S VIEW

A GENERALIZED HAMILTON-JACOBI EQUATION

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General Theory of Motion

-Free motion $t \to x$; Motion under forces $t \to x'$

-Had we know $x \to x'$, *i.e.* a global coordinate transf, we solve the motion

-Einstein's line of thought

-We have not that global coordinate transformation

(cannot get the 10 g_{ij} with four functions; local flat spaces, but axes are different from point to point)

- Our transformations are Local! $(x = (ct, \mathbf{r}), x' = (ct', \mathbf{r}'), dx \leftrightarrow dx')$

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One Exception: Special Theory of Relativity

From rest to motion (principle of inertia, $ds^2 = const$)

$$x=c\beta\tau/\sqrt{1-\beta^2}$$
 , $t=\tau/\sqrt{1-\beta^2}$

A vector: momentum $\mathbf{p} = \partial S / \partial \mathbf{r}$, energy $p_0 = E/c = -\partial S / c \partial t$ Apply these transf to this vector

$$\mathbf{p} = \mathbf{v}E/c^2$$
, $E = E_0/\sqrt{1-\beta^2}$, $E_0 = mc^2$

Eqs of motion

$$d\mathbf{p}/dt = \mathbf{f}$$

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Additional "relativistic" forces (~ $v^2)$ Hamilton-Jacobi equation $E^2-c^2p^2=m^2c^4$

$$(\partial S/\partial t)^2 - c^2 (\partial S/\partial \mathbf{r})^2 = m^2 c^4$$

This is the entire theory of special relativity 11

Hamilton-Jacobi Equation in Curved Space

Motion in curved space

Let $(P_0 = E_0/c, -\mathbf{P})$ be the (cov) momentum of a free motion in the flat space, constant, $P_0^2 - \mathbf{P}^2 = m^2 c^2$

Apply the coord transf for our curved space

$$p_0 = (1+h)p^0 + gp^1 = \sqrt{1+h} \cdot \frac{P_0 - \beta P_1}{\sqrt{1-\beta^2}}$$
$$p_1 = gp^0 - p^1 = \frac{(g+\beta\Delta)P_0 - (g\beta+\Delta)P_1}{\sqrt{(1+h)(1-\beta^2)}}$$

An integral of motion, already (by using $p_i = mcdu_i/ds$)? NO! Different x and x'! 12

Use $P_0^2 - \mathbf{P}^2 = m^2 c^2$ for $g = -\beta$

Hamilton-Jacobi Equation in curved space

$$(E - c\mathbf{g}\mathbf{p})^2 - c^2(1 + h + g^2)(p^2 + m^2c^2) = 0$$

or

$$(\partial S/\partial t + c\mathbf{g}\partial S/\partial \mathbf{r})^2 - c^2(1+h+g^2)[(\partial S/\partial \mathbf{r})^2 + m^2c^2] = 0$$

Euler-Lagrange Motion

The action

$$S = -mc \int ds = -mc^2 \int dt \cdot (1 + h + 2\mathbf{g}\mathbf{v}/c - v^2/c^2)^{1/2} = \int dt \cdot L$$

$$md\mathbf{p}/dt = \mathbf{F}$$

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$$\mathbf{F} = \partial L / \partial \mathbf{r} = -(mc^2/2) \cdot \frac{\partial h / \partial \mathbf{r}}{(1+h+2\mathbf{g}\mathbf{v}/c-v^2/c^2)^{1/2}}$$

$$E = \mathbf{pv} - L = \frac{mc^2(1+h) + mc\mathbf{vg}}{(1+h+2\mathbf{gv}/c - v^2/c^2)^{1/2}}$$

We get again the Hamilton-Jacobi equation given before

$$(E - c\mathbf{g}\mathbf{p})^2 - c^2(1 + h + g^2)(p^2 + m^2c^2) = 0$$

Lagrange Motion

$$S = -mc \int ds$$

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$$\delta ds^2 = 2ds\delta ds = \delta(g_{ij}dx^i dx^j) = 2g_{ij}dx^i d\delta x^j + dx^i dx^j (\partial g_{ij}/\partial x^k) \delta x^k$$

$$p_i = -\partial S / \partial x^i = mcu_i , \ p_i = (p_0, -\mathbf{p})$$

Hamilton: $E = -\partial S/\partial t$; it follows $p_0 = -E/c$ Since $p_i p^i = m^2 c^2$, *i.e.* $g^{ij} p_i p_j = m^2 c^2$ we get again the Hamilton-Jacobi equation in curved space

$$(\partial S/\partial t + c\mathbf{g}\partial S/\partial \mathbf{r})^2 - c^2(1+h+g^2)[(\partial S/\partial \mathbf{r})^2 + m^2c^2] = 0$$

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Contravariant metric

$$g^{ij} = \frac{1}{\Delta^2} \begin{pmatrix} 1 & g_1 & g_2 & g_3 \\ g_1 & -\Delta^2 + g_1^2 & g_1g_2 & g_1g_3 \\ g_2 & g_1g_2 & -\Delta^2 + g_2^2 & g_2g_3 \\ g_3 & g_3g_1 & g_3g_2 & -\Delta^2 + g_3^2 \end{pmatrix}$$

Eikonal Equation

-Waves go by $k_i dx^i = -d\Phi$, the eikonal (phase); flat space $k_i = (k_0 = \omega/c, -\mathbf{k})$, frequency and wavevector, $k_i k^i = (\omega/c)^2 - \mathbf{k}^2 = 0$; straight line, $\Phi = -\omega t + \mathbf{kr}$; geometric optics -Since $k_i k^i = g^{ij} k_i k_j = 0$ we get the eikonal equation in curved space 16

$$(\partial \Phi / \partial t + c \mathbf{g} \partial \Phi / \partial \mathbf{r})^2 - c^2 (1 + h + g^2) (\partial \Phi / \partial \mathbf{r})^2 = 0$$

-Solve it!

-neglect g^2

-first term does not depend on the time t (the second doesnt!)

$$\partial \Phi / c \partial t + \mathbf{g} \partial \Phi / \partial \mathbf{r} = -\omega_0 / c$$

where ω_0 the frequency in the flat space; in addition

$$(\partial \Phi / \partial \mathbf{r})^2 = k^2 = \frac{1}{1+h} \cdot (\omega_0 / c)^2 = \frac{1}{1+h} \cdot k_0^2$$

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-It follows

$$\partial \Phi/c\partial t = -\omega_0/c - \mathbf{g}\mathbf{k}_0$$

-What we measure? We measure the local, proper-time frequency

$$\omega/c = -\partial\Phi/c\partial\tau = -\frac{1}{\sqrt{1+h}} \cdot \partial\Phi/c\partial t = \frac{1}{\sqrt{1+h}} \cdot \omega_0/c + \mathbf{g}\mathbf{k}_0$$

-Therefore a shift in frequency

$$\Delta\omega/\omega_0 = -h/2 + c\mathbf{g}\mathbf{k}_0/\omega_0$$

First term - the **red shift**; second term - **Doppler effect** (long) 18

-Time-dependent part of the eikonal: $\Phi_t(t) = -\omega_0 t + \mathbf{k_0} \mathbf{R}(t)$: a translation, as expected -The path? $(\partial \Phi / \partial \mathbf{r})^2 = (1 - h)k_0^2$

-Write it in spherical coordinates; separate variables by $\Phi = \Phi_r(r) + M\varphi$, M a constant; $\partial \Phi / \partial M = const$ gives the equation of the trajectory (M is a generalized coordinate, its momentum is constant)

The deflection angle (distance M/k_0)

$$\Delta \varphi = -(k_0^2/2) \int_{\infty}^r dr \cdot \frac{h \cdot M/r^2}{(k_0^2 - M^2/r^2)^{3/2}}$$

(4 times smaller than in grav field of a point mass; our metric is not that metric!) 19

PART IV : QUANTIZATION

BASIC CHANGES

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Quantization

What are we doing?

Nothing Good (though NEW), even WORSE than before

Because it is bad to solve for a non-inertial motion; we just solve for an inertial frame and do the translation (for instance, the Ham-Jac eq is solved with h for Mercury's perihelia precession; then apply the translation, etc)

This is perfectly true for classical motion with trajectory

Things Change Fundamentally for the Quantal Motion

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Quantization

 $S = -i\hbar \ln \psi; E \to i\hbar \partial / \partial t, \mathbf{p} \to -i\hbar \partial / \partial \mathbf{r}$

No trajectory, wavefunction ψ

No determined physical quantities (E, \mathbf{p}) (operators)

Means and deviations: statistical meaning

 $|\psi|^2$ density of probability (conservation)

Apply this procedure to the Ham-Jac eq $E^2 - c^2 p^2 = m^2 c^2$

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Get the Klein-Gordon equation

$$\partial^2 \psi / \partial t^2 - c^2 \partial^2 \psi / \partial \mathbf{r}^2 + (m^2 c^4 / \hbar^2) \psi = 0$$

Troubles: the conserved quantity is $\psi^*(\partial \psi/\partial t) - (\partial \psi^*/\partial t)\psi$, both positive and negative (due to negative energies $E = -\sqrt{p^2c^2 + m^2c^4}$)- nonsense

Dirac: $i\hbar\partial\psi/\partial t = (\alpha c\mathbf{p} + \beta mc^2)\psi$, matrices α and β ; get a probability, but ψ is a spinor; so, the question remains for the Klein-Gordon eq

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Approximate Klein-Gordon equation

Apply the quantization to the Ham-Jac eq in curved space

$$(E - c\mathbf{g}\mathbf{p})^2 - c^2(1+h)(\mathbf{p}^2 + m^2c^2) = 0$$

(neglecting g^2)

Troubles: 1 + h does not commute with $\mathbf{p}^2 + m^2 c^2$, ambiguities

We may transfer it to the lhs as 1/(1+h), and neglect the gh-commutator

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Get then an approx Klein-Gordon eq in curved space

$$(i\hbar\partial/\partial t - c\mathbf{g}\mathbf{p})^2\psi - c^2(1+h)(p^2 + m^2c^2)\psi = 0$$

Still troubles, since we do not know where to put 1 + h with respect to $p^2 + m^2 c^2$

However, in the non-relativistic limit this ambiguity does not matter, and we get the Schrödinger equation (recall $h = 2\varphi/mc^2$)

$$i\hbar\partial\psi/\partial t = H\psi = (mc^2 + p^2/2m + \varphi)\psi + c\mathbf{gp}\psi$$

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The Fundamental Fact

The eigenstates are no more conserved due to the non-uniform translation

We get quantal transition

An observer in a non-uniform translation sees quantal transitions

Esssential thing: do not conserve the momentum; the presence of the external potential $\varphi(i.e. h)$ is essential

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The approximate Klein-Gordon eq can be solved by pert theory

$$(i\hbar\partial/\partial t - c\mathbf{g}\mathbf{p})^2\psi - c^2(1+h)(p^2 + m^2c^2)\psi = 0$$

Define $H^2 = c^2(1+h)(p^2+m^2c^2)$, solve in the first order, get $E^2 = (1+\bar{h})(p^2c^2+m^2c^4)$, the wavefunctions $\varphi(\mathbf{p})$ -plane waves plus a weak admixture of plane waves (due to h); then we have $(i\hbar\partial/\partial t - c\mathbf{g}\mathbf{p})\psi = E\psi$

Get the transition amplitude

$$-(i/\hbar)\int dt\cdot e^{-i[E(p)-E(p')]t/\hbar}c\mathbf{g}\mathbf{p}_{\mathbf{p'p}}$$

Conclusion: we do have quantal transitions!

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Restricting to the first-order of the perturbation theory we get also a Dirac equation

$$(i\hbar\partial/\partial t - c\mathbf{g}\mathbf{p})\psi = (\alpha c\mathbf{p} + \beta mc^2)\psi$$

with leads to the same conclusion

A "profound" argument

Let our eq be

$$\left(\frac{\partial}{\partial t} + c\mathbf{g}\partial/\partial\mathbf{r}\right)^2\psi - c^2(1+h)\left[\frac{\partial^2\psi}{\partial\mathbf{r}^2} - (m^2c^2/\hbar^2)\psi\right] = 0$$

like above

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Fourier transform; a homogeneous matricial equations in labels (ω, \mathbf{k}) ; solve it by zeroing the determinant; get the eigenvalues; they are labelled by points (ω, \mathbf{k}) conveniently ordered; consequently, the eigenvalues are useless, they do not provide an algebraic relationship between ω and \mathbf{k}

That means that for an ω we have many ${\bf k}$ and for a ${\bf k}$ we have many ω

That means that the plane waves scatter both in ω and in ${\bf k}$

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That means that the quantization with plane waves is the only way to understand such solutions of the 2nd order diff eqs, and more, we have for them a statistical meaning; this is **The Quantal Fields Theory**!

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PART V : FIELDS; HOW THEY ARE and What THEY DO in a CURVED SPACE

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Fields

Cannot forget that the above Klein-Gordon or Dirac equations in curved space are only approximate

Way out: The Fields!

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Real Scalar Field (general note: covariant derivative)

$$S = \int dx^0 d\mathbf{r} \sqrt{-g} \cdot \left[(\partial_i \psi) (\partial^i \psi) + (m^2 c^2 / \hbar^2) \psi^2 \right]$$

Eqs of motion

$$(i\hbar\partial/\partial t - c\mathbf{g}\mathbf{p})^2\psi - c^2(1+h)(p^2 + m^2c^2)\psi + (i\hbar c^2/2)(\partial h/\partial \mathbf{r})\mathbf{p}\psi = 0$$

This is the real Klein-Gordon equation in curved space

Note the additional interacting term $(\partial h/\partial \mathbf{r})\mathbf{p}$

Supports a similar treatment with the perturbation theory; same conclusion: quantal transitions 30

(Note: compare it with the KG eq in an electromagnetic field

$$(i\hbar\partial/\partial t - e\varphi)^2\psi - c^2[(i\hbar\partial/\partial \mathbf{r} + e\mathbf{A}/c)^2 + m^2c^2] = 0$$

Quite different! (Gauge fields!))

The Hamiltonian of the Real Scalar Field

Quantization by $\Pi = \partial L / \partial (\partial \psi / \partial t)$, Hamiltonian by $\Pi (\partial \psi / \partial t) - L$, the Lagrangian in $S = \int dt \cdot L$

$$H = H_0 + H_{1h} + H_{1g}$$

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$$H_0 = \int d\mathbf{r} \cdot \left[c^2 \Pi^2 / 4 + (\partial \psi / \partial \mathbf{r})^2 + (m^2 c^2 / \hbar^2) \psi^2\right] =$$
$$= \sum_{\mathbf{p}} (\varepsilon/2) (a_{\mathbf{p}} a_{\mathbf{p}}^+ + a_{\mathbf{p}}^+ a_{\mathbf{p}})$$

$$H_{1h} = \int d\mathbf{r} \cdot (\sqrt{1+h} - 1) \left[c^2 \Pi^2 / 4 + (\partial \psi / \partial \mathbf{r})^2 + (m^2 c^2 / \hbar^2) \psi^2 \right]$$

$$H_{1g} = -(c/2) \int d\mathbf{r} \cdot \left[\Pi(\mathbf{g}\partial\psi/\partial\mathbf{r}) + (\mathbf{g}\partial\psi/\partial\mathbf{r})\Pi \right] =$$
$$= -(c/2) \sum_{\mathbf{p}} (\mathbf{g}\mathbf{p}) (a_{\mathbf{p}}a_{\mathbf{p}}^{+} + a_{\mathbf{p}}^{+}a_{\mathbf{p}})$$

Systematic perturbation theory; scattering in the hg-order ($\varepsilon = \sqrt{p^2 c^2 + m^2 c^4}$) 32

Electromagnetic Field. Photons

$$S = -(1/16\pi c) \int dx^0 d\mathbf{r} \cdot \sqrt{-g} F_{ij} F^{ij}$$

 $F_{ij} = \partial_i A_j - \partial_j A_i; \ \partial_i F_{jk} + \partial_j F_{ki} + \partial_k F_{ij} = 0$ (free Maxwell eqs) Interacting Maxwell eqs $\partial_j (\sqrt{-g} F^{ij}) = 0$

$$div[(\mathbf{E} + g \times \mathbf{B})/\Delta] = div\mathbf{D} = 0$$

$$\frac{\partial}{c\partial t}[(\mathbf{E} + \mathbf{g} \times \mathbf{B})/\Delta] = curl[\Delta \mathbf{B} + \mathbf{g} \times \mathbf{E}/\Delta]$$

Perturbation theory; scattering, both in ${\bf k}$ and ω

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The Hamiltonian of the Photons

$$S = (1/8\pi) \int dt d\mathbf{r} \cdot \Delta (\mathbf{D}^2 - \mathbf{B}^2) =$$
$$= (1/8\pi) \int dt d\mathbf{r} \cdot (1/\Delta) (\mathbf{E}^2 + 2\mathbf{E}(\mathbf{g} \times \mathbf{B}) - \Delta^2 \mathbf{B}^2)$$

 $H = H_0 + H_{1h} + H_{1g}$

$$H_0 = \int d\mathbf{r} \cdot (c^2 \Pi^2 / 4 + B^2) = \sum_{\alpha \mathbf{p}} (\varepsilon/2) (a^+_{\alpha \mathbf{p}} a_{\alpha \mathbf{p}} + a_{\alpha \mathbf{p}} a^+_{\alpha \mathbf{p}})$$

$$H_{1h} = \int d\mathbf{r} \cdot (\sqrt{1+h} - 1)(c^2 \Pi^2 / 4 + \mathbf{B}^2)$$

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$$H_{1g} = -\sum_{\alpha \mathbf{p}} (\mathbf{g}\mathbf{p}/2)(a^+_{\alpha \mathbf{p}}a_{\alpha \mathbf{p}} + a_{\alpha \mathbf{p}}a^+_{\alpha \mathbf{p}})$$

Systematic theory of perturbations $(\varepsilon = cp = c\hbar k)$

Photons are scatterred in frequency, as a consequence of a non-uniform translation, when in an external field (like a static grav field)

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Other Fields. Quantum Gravity

Similar for other fields (spin-1/2 Dirac field) (technically more cumbersome; vierbeins)

Gravitons; quantized (with troubles); moving in a curved space $S = \int dx^0 d\mathbf{r} \cdot \sqrt{-g}R$; $g = g_0 + \delta g$, background and gravitons; scattering of gravitons, *i.e.* of the space-time, on space-time, *i.e.* on matter or on the non-inertial motion

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PART VI : OTHER non-INERTIAL MOTIONS and MISCELLANEA; CONCLUSIONS

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Rotations

$$d\mathbf{r}' = d\mathbf{r} + (\Omega \times \mathbf{r}) dt$$
$$d\mathbf{v} = d\mathbf{v} + \left(\dot{\Omega} \times \mathbf{r}\right) dt + 2 \left(\Omega \times \mathbf{v}\right) dt + \left[\Omega \times (\Omega \times \mathbf{r})\right] dt$$

Non-uniform rotation, Coriolis, centrifugal

$$\begin{split} H &= mv^2/2 - m(\Omega\times\mathbf{r})^2/2 + \varphi = p^2/2m - \Omega(\mathbf{r}\times\mathbf{p}) + \varphi = \\ &= p^2/2m - \Omega\mathbf{L} + \varphi \end{split}$$

No Coriolis, no centrifugal; just L we may neglect
$$\Omega^2$$

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The above coordinate transformation gives the metric

$$g_{ij} = \begin{pmatrix} 1+h & g_1 & g_2 & g_3 \\ g_1 & -1 & 0 & 0 \\ g_2 & 0 & -1 & 0 \\ g_3 & 0 & 0 & -1 \end{pmatrix}$$

with

 $\mathbf{g} = -\Omega \times \mathbf{r}/c$

as before

Two distinctions: $g(t, \mathbf{r})$ $\Omega r/c \ll 1$

Coupling through the angular momentum L

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Conclusions

Non-inertial motion (for instance of the observer) produces quantal transitions in the presence of an external field

The coupling is through momentum \mathbf{p} for translations or through the angular momentum \mathbf{L} for rotations; so, the external field must not conserve these quantities

For instance, photons in a static gravitational field are scattered toward the blue (the blue shift) while seen from a non-uniform translation (or rotation)

Relation to the Unruh effect - quite distinct (the observer in the U effect sees its own motion as a bath of photons)

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Another more practical **Conclusion**:

The quantization in curved spaces has no meaning or it has the meaning given here

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