The Antiphysical Review

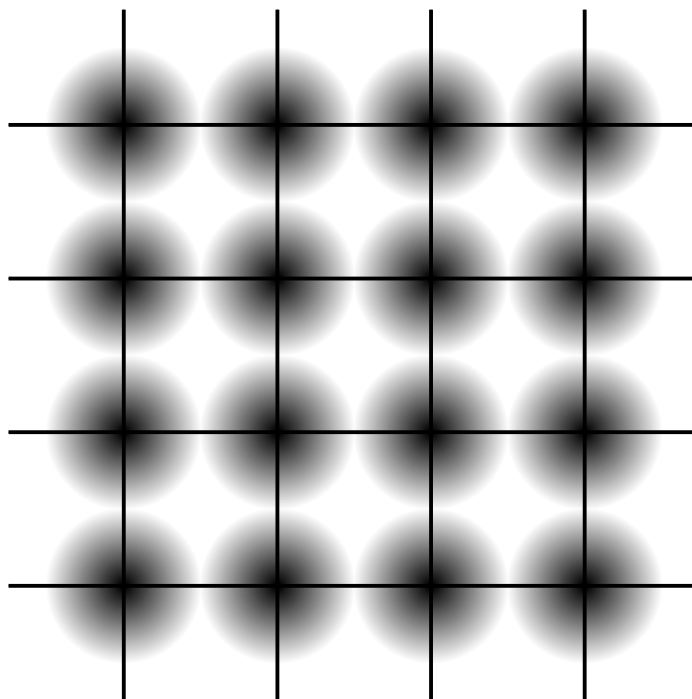**COHERENCE DOMAINS in MATTER INTERACTING with RADIATION**

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1) Imagine a piece of matter, either gas, liquid or solid

2) Imagine a series of excitation levels, atomic, molecular

3) Imagine the interaction of this substance with the EM radiation (dipolar int)

Now, you are ready to hearing me telling you something SURPRISING!

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Electromagnetic Field

Vector potential

$$\mathbf{A}(\mathbf{r}) = \sum_{\alpha\mathbf{k}} \sqrt{\frac{2\pi\hbar c^2}{V\omega_k}} \left[\mathbf{e}_\alpha(\mathbf{k})a_{\alpha\mathbf{k}}e^{i\mathbf{k}\mathbf{r}} + \mathbf{e}_\alpha^*(\mathbf{k})a_{\alpha\mathbf{k}}^*e^{-i\mathbf{k}\mathbf{r}}\right]$$

The fields: $\mathbf{E} = -(1/c)\partial\mathbf{A}/\partial t$, $\mathbf{H} = curl\mathbf{A}$

Three Maxwell's eqs: $curl\mathbf{E} = -\frac{1}{c}\partial\mathbf{H}/\partial t$, $div\mathbf{H} = 0$, $div\mathbf{E} = 0$

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The free Lagrangian:

$$L_f = \tfrac{1}{8\pi} \int d\mathbf{r} \left(E^2 - H^2\right) =$$

$$= \sum_{\alpha\mathbf{k}} \frac{\hbar}{4\omega_k} \left(\dot{a}_{\alpha\mathbf{k}}\dot{a}_{-\alpha-\mathbf{k}} + \dot{a}_{\alpha\mathbf{k}}^*\dot{a}_{-\alpha-\mathbf{k}}^* + \dot{a}_{\alpha\mathbf{k}}\dot{a}_{\alpha\mathbf{k}}^* + \dot{a}_{\alpha\mathbf{k}}^*\dot{a}_{\alpha\mathbf{k}}\right) -$$

$$- \sum_{\alpha\mathbf{k}} \frac{\hbar\omega_k}{4} \left(a_{\alpha\mathbf{k}}a_{-\alpha-\mathbf{k}} + a_{\alpha\mathbf{k}}^*a_{-\alpha-\mathbf{k}}^* + a_{\alpha\mathbf{k}}a_{\alpha\mathbf{k}}^* + a_{\alpha\mathbf{k}}^*a_{\alpha\mathbf{k}}\right)$$

The interacting Lagrangian:

$$L_{int} = \frac{1}{c} \int d\mathbf{r} \cdot \mathbf{j}\mathbf{A} = \sum_{\alpha\mathbf{k}} \sqrt{\frac{2\pi\hbar}{\omega_k}} \left[\mathbf{e}_\alpha(\mathbf{k})\mathbf{j}^*(\mathbf{k})a_{\alpha\mathbf{k}} + \mathbf{e}_\alpha^*(\mathbf{k})\mathbf{j}(\mathbf{k})a_{\alpha\mathbf{k}}^*\right]$$

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Eq of motion:

$$\ddot{a}_{\alpha\mathbf{k}} + \ddot{a}_{-\alpha-\mathbf{k}}^* + \omega_k^2 \left(a_{\alpha\mathbf{k}} + a_{-\alpha-\mathbf{k}}^*\right) = \sqrt{\frac{8\pi\omega_k}{\hbar}}\mathbf{e}_\alpha^*(\mathbf{k})\mathbf{j}(\mathbf{k})$$

The fourth Maxwell's eq:

$$curl\mathbf{H} = (1/c)\partial\mathbf{E}/\partial t + 4\pi\mathbf{j}/c$$

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Matter Field (Substance)

Atoms, molecules $i = 1, 2...N$ (non-interacting)

$$H_s = \sum_i H_s(i)$$

Wavefunctions:

$$H_s(i)\varphi_n(j) = \varepsilon_n\delta_{ij} \;, \; \int d\mathbf{r}\varphi_n^*(i)\varphi_m(j) = \delta_{ij}\delta_{nm}$$

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The field:

$$\psi_n = \sum_i c_{ni}\varphi_n(i) \ , \ H_s\psi_n = \varepsilon_n\psi_n$$

$$\sum_i |c_{ni}|^2 = 1 \ , \ \psi_n = \frac{1}{\sqrt{N}}\sum_i e^{i\theta_{ni}}\varphi_n(i)$$

The quantization of the field:

$$\Psi = \sum_n b_n\psi_n$$

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Boson operators: $N \to \infty$ (any occupancy)

$$N = \sum_n b_n^* b_n$$

The Lagrangian:

$$L_s = \frac{1}{2}\int d\mathbf{r} \left(\Psi^* \cdot i\hbar\partial\Psi/\partial t - i\hbar\partial\Psi^*/\partial t \cdot \Psi\right) - \int d\mathbf{r}\Psi^* H_s\Psi$$

$$L_s = \frac{1}{2}\sum_n i\hbar \left[b_n^*\dot{b}_n - \dot{b}_n^* b_n\right] - \sum_n \varepsilon_n b_n^* b_n$$

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Schroedinger's equation:

$$H_s = \sum_n \varepsilon_n b_n^* b_n \ , \ i\hbar\dot{b}_n = \varepsilon_n b_n$$

The current density:

$$\mathbf{j}(\mathbf{r}) = \sum_i \mathbf{J}(i)\delta(\mathbf{r} - \mathbf{r}_i) = \frac{1}{V}\sum_{i\mathbf{k}} \mathbf{J}(i)e^{-i\mathbf{k}\mathbf{r}_i}e^{i\mathbf{k}\mathbf{r}} = \frac{1}{\sqrt{V}}\sum_{\mathbf{k}} \mathbf{j}(\mathbf{k})e^{i\mathbf{k}\mathbf{r}}$$

Remember the interaction with the EM field:

$$L_{int} = \sum_{nm\alpha\mathbf{k}} \sqrt{\frac{2\pi\hbar}{V\omega_k}} \left[\mathbf{e}_\alpha(\mathbf{k})\mathbf{I}_{mn}^*(\mathbf{k})a_{\alpha\mathbf{k}} + \mathbf{e}_\alpha^*(\mathbf{k})\mathbf{I}_{nm}(\mathbf{k})a_{\alpha\mathbf{k}}^*\right] b_n^* b_m$$

Note this matrix:

$$\mathbf{I}_{nm}(\mathbf{k}) = \frac{1}{N}\sum_i \mathbf{J}_{nm}(i)e^{-i(\theta_{ni}-\theta_{mi})}e^{-i\mathbf{k}\mathbf{r}_i}$$

Schroedinger's equation again, with interaction:

$$i\hbar\dot{b}_n = \varepsilon_n b_n - \sum_{m\alpha\mathbf{k}} \sqrt{\frac{2\pi\hbar}{V\omega_k}} \left[\mathbf{e}_\alpha(\mathbf{k})\mathbf{I}_{mn}^*(\mathbf{k})a_{\alpha\mathbf{k}} + \mathbf{e}_\alpha^*(\mathbf{k})\mathbf{I}_{nm}(\mathbf{k})a_{\alpha\mathbf{k}}^*\right] b_m$$

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And Maxwell's equation:

$$\ddot{a}_{\alpha\mathbf{k}} + \ddot{a}_{-\alpha-\mathbf{k}}^* + \omega_k^2\left(a_{\alpha\mathbf{k}} + a_{-\alpha-\mathbf{k}}^*\right) = \sum_{nm} \sqrt{\frac{8\pi\omega_k}{V\hbar}} \mathbf{e}_\alpha^*(\mathbf{k})\mathbf{I}_{nm}(\mathbf{k})b_n^* b_m$$

The most common from of the interacting hamiltonian (Quantum Electrodynamics):

$$H_{int} = -\sum_{nm\alpha\mathbf{k}} \sqrt{\frac{2\pi\hbar}{V\omega_k}}[\mathbf{e}_\alpha(\mathbf{k})\mathbf{I}_{mn}^*(\mathbf{k})a_{\alpha\mathbf{k}}e^{\frac{i}{\hbar}(\varepsilon_n-\varepsilon_m-\hbar\omega_k)}+$$

$$+\mathbf{e}_\alpha^*(\mathbf{k})\mathbf{I}_{nm}(\mathbf{k})a_{\alpha\mathbf{k}}^*e^{\frac{i}{\hbar}(\varepsilon_n-\varepsilon_m+\hbar\omega_k)}]b_n^* b_m$$

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Coherence Domains

$$L_{int} = \sum_{nm\alpha\mathbf{k}} \sqrt{\frac{2\pi\hbar}{V\omega_k}} F_{nm}(\alpha\mathbf{k})\left(a_{\alpha\mathbf{k}} + a_{-\alpha-\mathbf{k}}^*\right)b_n^* b_m$$

$$F_{nm}(\alpha\mathbf{k}) = \frac{1}{N}\sum_i \mathbf{e}_\alpha(\mathbf{k})\mathbf{J}_{nm}(i)e^{i\mathbf{k}\mathbf{r}_i - i(\theta_{ni}-\theta_{mi})}$$

What I want? A classical dynamics!

Note the RANDOM PHASE $i\mathbf{k}\mathbf{r}_i - i\left(\theta_{ni} - \theta_{mi}\right)$! Vanishing interaction!

We may arrange, perhaps, for some $\mathbf{k}$'s, but no thermodynamics!

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Way out: A LATTICE!

For any pair (nm) of energy levels: $\mathbf{r}_i = \mathbf{R}_p + \mathbf{r}_{pi}$, spatial lattice $\mathbf{R}_p$, $\mathbf{r}_{pi}$ restricted to the first Wigner-Seitz cell

$\mathbf{R}_p$ such that the magnitudes of its shortest reciprocal vectors $\mathbf{k}_r$, $r = 1, 2, 3$, are equal with the magnitude of the relevant wavevectors $\mathbf{k}$, *i.e.* those wavevectors which satisfy $\hbar\omega_k = \varepsilon_n - \varepsilon_m > 0$; and $\mathbf{k}_r\mathbf{R}_p = 2\pi \times integer$

Only a cubic and a trigonal (rhombohedral) symmetry is thus allowed

A cubic lattice: a periodicity length $\lambda = 2\pi/k$, where k is the magnitude of the relevant wavevector

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Again

$$F_{nm}(\alpha\mathbf{k}_r) = \frac{1}{N}\sum_{pi} \mathbf{e}_\alpha(\mathbf{k}_r)\mathbf{J}_{nm}(i)e^{i\mathbf{k}_r\mathbf{r}_{pi} - i(\theta_{ni}-\theta_{mi})}$$

Coherence condition:

$$\mathbf{k}_r \mathbf{r}_{pi} - (\theta_{ni} - \theta_{mi}) = K$$

The subsets $N_{nm}(\alpha \mathbf{k}_r)$: $\mathbf{e}_\alpha(\mathbf{k}_r)\mathbf{J}_{nm}(i) = J_{nm}$

$$F_{nm}(\alpha \mathbf{k}_r) = J_{nm} N_{nm}(\alpha \mathbf{k}_r)/N$$

$$\sum_{(nm)\alpha \mathbf{k}_r} N_{nm}(\alpha \mathbf{k}_r) = N$$

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The phase of the internal motion of the i-th particle is correlated to the position of that particle

Long-range order, a cooperative phenomenon

The phase of the internal motion "feels" the particle position

Various pairs (nm): a superposition of such lattices of coherence domains

These lattices can also be one- or two-dimensional

A one-dimensional lattice of coherence domains: a set of parallel sheets (layered structure), with the relevant periodicity length λ

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Classical Dynamics

Ground-state $n = 0$, the first excited state $n = 1$

Macroscopic occupation: use c-numbers $\beta_{0,1}$ for operators $b_{0,1}$

The occupation number has no definite value, its conjugate phase is well defined

These are coherent states defined by $b_{0,1}|\beta_{0,1}\rangle = \beta_{0,1}|\beta_{0,1}\rangle$

$\varepsilon_1 - \varepsilon_0 = \hbar\omega_0$, where $\omega_0 = ck_0$

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Limit the wavevectors to the basic reciprocal vectors $\mathbf{k}_r$ of magnitude $k_r = k_0 = 2\pi/\lambda_0$

Use c-numbers α for the photon operators $a_{\alpha \mathbf{k}_r}$

$$L_{int} = \sqrt{\frac{2\pi\hbar}{V\omega_0}} J_{01} \left(\alpha + \alpha^*\right)\left(\beta_1^* \beta_0 + \beta_1 \beta_0^*\right)$$

$$L_f = \frac{\hbar}{4\omega_0}\left(\dot{\alpha}^2 + \dot{\alpha}^{*2} + 2\left|\dot{\alpha}\right|^2\right) - \frac{\hbar\omega_0}{4}\left(\alpha^2 + \alpha^{*2} + 2\left|\alpha\right|^2\right)$$

$$L_s = \frac{1}{2}i\hbar\left(\beta_0^* \dot{\beta}_0 - \dot{\beta}_0^* \beta_0 + \beta_1^* \dot{\beta}_1 - \dot{\beta}_1^* \beta_1\right) - \left(\varepsilon_0\left|\beta_0\right|^2 + \varepsilon_1\left|\beta_1\right|^2\right)$$

$$L_{int} = \frac{g}{\sqrt{N}}\left(\alpha + \alpha^*\right)\left(\beta_0 \beta_1^* + \beta_1 \beta_0^*\right)$$

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$$g = \sqrt{\pi\hbar/6a^3\omega_0}J_{01} = \sqrt{\pi\hbar\omega_0(e^2/6a_0)}(a_0/a)^{3/2}$$

$\varepsilon_1 - \varepsilon_0 = \hbar\omega_0 = 10eV$, $\lambda_0 = 10^3\text{Å}$, $g \sim 0.8eV$ $(a_0 = 0.53\text{Å})$

Equations of Motion

$$\ddot{A} + \omega_0^2 A = \frac{2\omega_0 g}{\hbar\sqrt{N}}\left(\beta_0\beta_1^* + \beta_1\beta_0^*\right)$$

$$i\hbar\dot{\beta}_0 = \varepsilon_0\beta_0 - \frac{g}{\sqrt{N}}A\beta_1$$

$$i\hbar\dot{\beta}_1 = \varepsilon_1\beta_1 - \frac{g}{\sqrt{N}}A\beta_0$$

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The Hamiltonian:

$$H_f = \frac{\hbar}{4\omega_0}\dot{A}^2 + \frac{\hbar\omega_0}{4}A^2$$

$$H_s = \varepsilon_0\left|\beta_0\right|^2 + \varepsilon_1\left|\beta_1\right|^2$$

$$H_{int} = -\frac{g}{\sqrt{N}}A\left(\beta_0\beta_1^* + \beta_1\beta_0^*\right)$$

Conservation laws:

$$H_f + H_s + H_{int} = E \ , \ \left|\beta_0\right|^2 + \left|\beta_1\right|^2 = N$$

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Solutions: Ground-State $\beta_{0,1} = B_{0,1}e^{i\Omega t}$

$$A = \frac{2g}{\hbar\omega_0}\sqrt{N}\left[1 - (\hbar\omega_0/2g)^4\right]^{1/2}$$

$$B_0^2 = \frac{1}{2}N\left[1 + (\hbar\omega_0/2g)^2\right]$$

$$B_1^2 = \frac{1}{2}N\left[1 - (\hbar\omega_0/2g)^2\right]$$

$$\Omega = \omega_0\left[-\frac{1}{2} + \frac{2g^2}{\hbar^2\omega_0^2}\right]$$

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Critical coupling:

$$g > g_{cr} = \hbar\omega_0/2$$

Ground-state energy:

$$E = -\frac{g^2}{\hbar\omega_0}N\left[1 - (\hbar\omega_0/2g)^2\right]^2 = -\hbar\Omega B_1^2$$

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Some consequences

Electric field vanishing

Magnetic field quite high $H \sim \sqrt{\hbar\omega_0/a^3} \sim 10^6 Gs$

Polarization $\mathbf{P} = \frac{1}{V}\sum_i \mathbf{p}(i)\cos\left(\theta_{1i} - \theta_{0i}\right)\left[1 - \left(\hbar\omega_0/2g\right)^4\right]^{1/2}$

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Elementary excitations

$A \to A + \delta A$, $\beta_{0,1} \to \beta_{0,1} + \delta\beta_{0,1}$, $\delta\beta_{0,1} = \left(\delta B_{0,1} + iB_{0,1}\delta\theta_{0,1}\right)e^{i\Omega t}$

Solutions of the form $(\delta A, \delta B_1, \delta\varphi)e^{i\omega t}$

$$\omega_{1,2}^2 = \frac{1}{2}\omega_0^2\left[\lambda^4 + 1 \pm \sqrt{(\lambda^4 - 1)^2 + 4}\right] \ , \ \lambda = 2g/\hbar\omega_0$$

Elementary excitations $\Omega_{1,2} = \Omega \pm \omega_{1,2}$

Weak coupling limit these frequencies behave as $\omega_1 \simeq \sqrt{2}\omega_0$ and $\omega_2 \simeq \sqrt{\lambda^2 - 1}\omega_0$ $(\Omega_{1,2} \simeq \omega_{1,2})$.

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Thermodynamics

No thermodynamics

$$Z \simeq tr e^{\beta(\mu N - H)} = \int d\rho \cdot \frac{e^{\beta N\mu\rho}}{\sqrt{\hbar\omega_0\left(\hbar\omega_0 - \mu\right) - 4g^2\varrho}} \simeq e^{\beta N\mu\hbar\omega_0\left(\hbar\omega_0 - \mu\right)/4g^2}$$

(Compute tr by $\int d\beta_{0x}d\beta_{0y}...$)

Thermodynamic potential $\Omega = N\mu\hbar\omega_0\left(\hbar\omega_0 - \mu\right)/4g^2$

Ordered phase, vanishing entropy

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Super-Radiant Phase Transition

$$H_f = \hbar\omega_0\sum_\mu\left(a_\mu^* a_\mu + 1/2\right) \ , \ H_s = \hbar\omega_0 b_1^* b_1$$

$$H_{int} = -\frac{1}{\sqrt{N}}\left(Gb_1^* b_0 + G^* b_0^* b_1\right)$$

μ stands for the pair $\alpha\mathbf{k}_r$, $G = \sum_\mu g_\mu a_\mu$ and $g_\mu = \sqrt{2\pi\hbar/V\omega_0}J_{01}N(\mu)/\sqrt{N}$

Compute the partition function by introducing spin variables

$$S_z = b_0^* b_0 - b_1^* b_1 = \sum_i\left(b_{0i}^* b_{0i} - b_{1i}^* b_{1i}\right) = \sum_i s_{zi}$$

$$S_+ = b_0^* b_1 = \sum_i b_{0i}^* b_{1i} = \sum_i s_{+i}$$

$$S_- = b_1^* b_0 = \sum_i b_{1i}^* b_{0i} = \sum_i s_{-i}$$

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Free ensemble for $g < \hbar\omega_0$, at any temperature

For $g > \hbar\omega_0$ there exists a critical temperature T_c given by $\hbar^2\omega_0^2/g^2 = \tanh\beta_c\hbar\omega_0/2$ (or $\beta_c \simeq 2\hbar\omega_0/g^2$)

For $T > T_c$ free ensemble, for $T < T_c$ a non-trivial thermodynamics

In the former case the ensemble of particles is in the normal state, with a free energy per particle

$$f_0 = \hbar\omega_0/2 - \beta^{-1}\ln\left[2\cosh\beta\hbar\omega_0/2\right]$$

For T slightly below T_c the free energy per particle is

$$f \simeq f_0 - \frac{\hbar\omega_0}{4}\left(1 - T/T_c\right)^2$$

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The entropy is continuous at the critical temperature

The specific heat has a discontinuity $C = C_0 + \hbar\omega_0/2T_c$

The transition is of the second kind, with the order parameter the photon occupation number

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Conclusions

A new state of matter interacting with EM radiation

Macroscopic occupation of the atomic, molecular energy levels

Macroscopic occupation of the photon field, classical field

All due to a correlation between the internal phases and spatial positions

Pattern: coherence domains

Providing certain critical conditions on the coupling strength, temperature

New collective excitations, measurable

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