

**(ELI-NP at Magurele - "Pulse and Impulse of ELI")**

**Extensive Light Investigations-ELI-apoma Laboratory**

1) "**Polaritonic pulse** and coherent X- and gamma rays from Compton (Thomson) backscattering" (MA&MGan), J. Appl. Phys. **109** 013307 (2011) (1-6)

2)"Dynamics of **electron–positron pairs** in a vacuum polarized by an external radiation field" (MA), Journal of Modern Optics, **58** 611 (2011)

3)"**Classical interaction** of the electromagnetic radiation with two-level polarizable matter" (MA), Optik **123** 193 (2012)

4)"**Coherent polarization** driven by external electromagnetic fields" (MA&MGan), Physics Letters **A374** 4848 (2010)

5) "Coupling of **(ultra-) relativistic atomic nuclei** with photons" (MA&MGan), AIP Advances **3** 112133 (2013)

6) "Propagation of **electromagnetic pulses** through the surface of dispersive bodies" (MA), Roum J. Phys. **58** 1298 (2013)

7) "**Giant dipole oscillations** and ionization of heavy atoms by intense electromagnetic pulses" (MA), Roum. Reps. Phys. **67** 837 (2015)

8) "**Parametric resonance**" in molecular rotation spectra" (MA&LC), Chem. Phys. **472** 262 (2016)

9) "**Motion of an electric charge in laser fields**" (CM&MA),  
Roum. J. Phys. **62** 117 (2017)

10) "**Scattering** of non-relativistic charges by electromagnetic  
radiation" (MA) Z. Naturforsch. **A72** 1173 (2017)

11) "**Fast atom ionization** in strong electromagnetic radiation"  
(MA) - 2017

12) "Electromagnetic-radiation effect on **alpha decay**" (MA) -  
2017

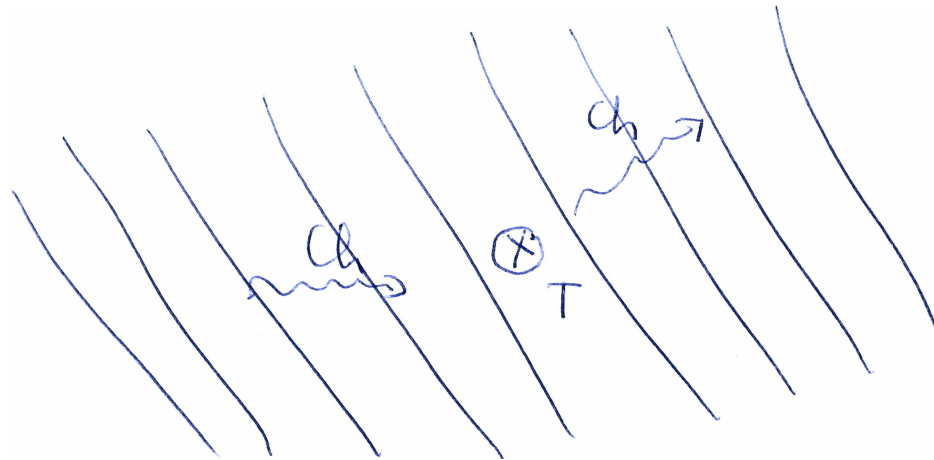
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**Charges Scattered by Laser Pulses**

**M Apostol**

**2018**

## Charge Scattering Immersed in Radiation



## Charge Scattering Immersed in Radiation

-Kroll-Watson (1973)

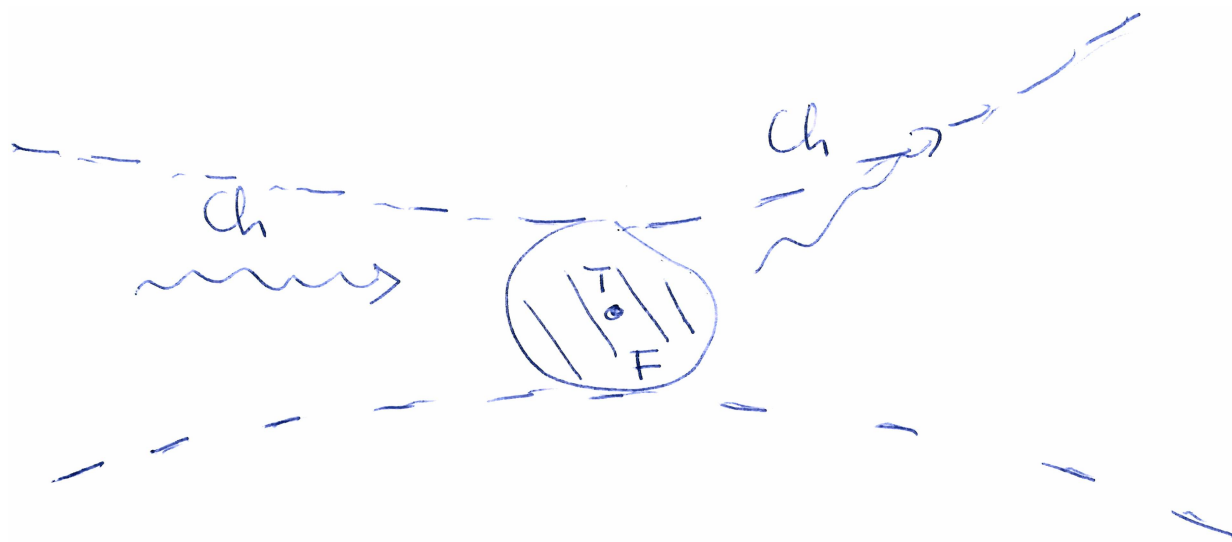
$$d\sigma_n = \frac{p_f n}{p_i} J_n^2(e\mathbf{p}\mathbf{E}_0/m\hbar\omega^2) d\sigma_B$$

$$\mathbf{p} = \mathbf{p}_i - \mathbf{p}_f, \quad p_{fn}^2/2m = p_i^2/2m + n\hbar\omega$$

$$\theta_n \simeq \frac{\hbar\omega^2}{ev_i E_0} \alpha_n$$

-Low fields; importance: multiple photons

## Focused Beams



-mainly heavy ions, electrons

## Scattering in Focused Beams

-non-rel particle, mass  $m$ , charge  $e$ ; em field  $\mathbf{A} = \mathbf{A}_0 \cos(\omega t - \mathbf{k}\mathbf{r})$   
(lin pol)

-approx  $\mathbf{A} \simeq \mathbf{A}_0 \cos \omega t$ ,  $\mathbf{E}(t) = \mathbf{E}_0 \sin \omega t$ ,  $\mathbf{E}_0 = \omega \mathbf{A}_0 / c$

-dipole hamiltonian

$$H = \frac{1}{2m} p^2 - e\mathbf{r}\mathbf{E}(t) + V(\mathbf{r})$$

-  $eA_0/mc^2 \ll 1$ ; satisfied for heavy ions, strong fields



## Schrodinger Equation

$$i\hbar \frac{\partial \psi_i}{\partial t} = (H_0 + U + V)\psi_i, \quad U = -e\mathbf{r}\mathbf{E}(t)$$

-Incoming, outgoing states

$$\psi_i^0 = \frac{1}{\sqrt{v}} e^{-\frac{i}{\hbar} E_i t + \frac{i}{\hbar} \mathbf{p}_i \mathbf{r}}, \quad \psi_f = \frac{1}{\sqrt{v}} e^{-\frac{i}{\hbar} E_f t + \frac{i}{\hbar} \mathbf{p}_f \mathbf{r}}$$

-Solution

$$\psi_i = \psi_i^0 - \frac{i}{\hbar} e^{-\frac{i}{\hbar} H_0 t} \int_{-\infty}^t dt' e^{\frac{i}{\hbar} H_0 t'} (U + V) \psi_i$$

-Scatt amplitude ( $S$ -matrix)

$$a_{fi} = -\frac{i}{\hbar} \int_{-\infty}^{+\infty} dt (\psi_f, (U + V) \psi_i)$$

## Canonical Transformations

-GM

$$\psi_i = e^{iS_1} \phi_i, \quad S_1 = -\frac{e}{\hbar c} \mathbf{r} \mathbf{A}$$

$$i\hbar \frac{\partial \phi_i}{\partial t} = \left[ \frac{1}{2m} (\mathbf{p} - \frac{e}{c} \mathbf{A})^2 + V \right] \phi_i$$

-KH

$$\phi_i = e^{iS_2} \chi_i, \quad S_2 = \frac{e}{\hbar m c \omega} \mathbf{p} \mathbf{A}_0 \sin \omega t - \frac{e^2}{8\hbar m c^2 \omega} A_0^2 (\sin 2\omega t + 2\omega t)$$

$$i\hbar \frac{\partial \chi_i}{\partial t} = (H_0 + \tilde{V}) \chi_i, \quad \tilde{V}(\mathbf{r}) = V(\mathbf{r} - e\mathbf{E}/m\omega^2)$$

-For  $V$  - Born approximation; consequently

$$\chi_i = \psi_i^0 - \frac{i}{\hbar} e^{-\frac{i}{\hbar} H_0 t} \int_{-\infty}^t dt' e^{\frac{i}{\hbar} H_0 t'} \tilde{V} \psi_i^0$$

$$a_{fi} = -\frac{i}{\hbar} \int_{-\infty}^{+\infty} dt (\psi_f, (U + V) e^{iS_1} e^{iS_2} \psi_i^0) +$$

$$+ \left(-\frac{i}{\hbar}\right)^2 \int_{-\infty}^{+\infty} dt (\psi_f, U e^{iS_1} e^{iS_2} e^{-\frac{i}{\hbar} H_0 t} \int_{-\infty}^t dt' e^{\frac{i}{\hbar} H_0 t'} \tilde{V} \psi_i^0)$$

-Three obs: laser scatt ( $U$ ),  $\tilde{V}$  is rad-dressed, interf terms

## Simplifications

- $S_1$ : order  $(r/\lambda)(eA_0/\hbar\omega) \gg eA_0/\hbar\omega$  ( $E_0 = 10^8 \text{esu}$ ,  $I = 10^{18} \text{w/cm}^2$ ,  $eA_0 = 10^6 \text{eV}$ )

- $S_2$ : order  $(p/mc, eA_0/mc^2)(eA_0/\hbar\omega) \ll eA_0/\hbar\omega$

-consequently, we may neglect  $S_2$

- $\tilde{V}$ : order  $r - \lambda_c(eA_0/\hbar\omega)$ , very large ( $\lambda_c = \hbar/mc$  Compton,  $3 \times 10^{-11} \text{cm}$ ); neglect  $\tilde{V}$

## Scatt Amplitude

$$a_{fi} = \frac{1}{v} \int_{-\infty}^{+\infty} dt d\mathbf{r} e^{\frac{i}{\hbar}(E_f - E_i)t} \frac{\partial}{\partial t} e^{-\frac{ie}{\hbar\omega} \mathbf{r} \mathbf{E}_0 \cos \omega t} e^{\frac{i}{\hbar} \mathbf{p} \mathbf{r}} -$$

$$-\frac{i}{\hbar v} \int_{-\infty}^{+\infty} dt d\mathbf{r} e^{\frac{i}{\hbar}(E_f - E_i)t} V e^{-\frac{ie}{\hbar\omega} \mathbf{r} \mathbf{E}_0 \cos \omega t} e^{\frac{i}{\hbar} \mathbf{p} \mathbf{r}}$$

- mom transfer  $\mathbf{p} = \mathbf{p}_i - \mathbf{p}_f$

-Use

$$e^{-\frac{ie}{\hbar\omega} \mathbf{r} \mathbf{E}_0 \cos \omega t} = \sum_{n=-\infty}^{+\infty} (-i)^n J_n(er\mathbf{E}_0/\hbar\omega) e^{-in\omega t}$$

$$a_{fi} = -\frac{2\pi i}{v} \sum_{n=-\infty}^{+\infty} (-i)^n \delta(E_f - E_i - n\hbar\omega) \cdot$$

$$\int d\mathbf{r} [n\hbar\omega + V(\mathbf{r})] J_n(er\mathbf{E}_0/\hbar\omega) e^{\frac{i}{\hbar} \mathbf{p} \mathbf{r}}$$

## Cross-Section

-Trans prob

$$w_{fi} = \frac{2\pi}{\hbar v^2} \sum_{n=-\infty}^{+\infty} \delta(E_f - E_i - n\hbar\omega) \cdot \left| \int d\mathbf{r} [n\hbar\omega + V(\mathbf{r})] J_n(e\mathbf{r}\mathbf{E}_0/\hbar\omega) e^{i\mathbf{p}\mathbf{r}/\hbar} \right|^2$$

-Multiply by density of fin sts  $v p_f^2 dp_f d\Omega / (2\pi\hbar)^3$

-Divide by the current density  $v_i/v$

-Integrate over the final mom  $p_f$

-Get diff cross-section for the  $n$ -process

$$d\sigma_n = \frac{p_{fn}}{p_i} \left| \frac{m}{2\pi\hbar^2} \int d\mathbf{r} [n\hbar\omega + V(\mathbf{r})] J_n(e\mathbf{r}\mathbf{E}_0/\hbar\omega) e^{i\frac{m}{\hbar}\mathbf{p}\mathbf{r}} \right|^2 d\Omega$$

-energy conservation  $p_{fn}^2/2m = p_i^2/2m + n\hbar\omega$

## Relation to Kroll-Watson

We remove  $S_2$  and keep  $S_1$  - KW keep  $S_2$  and has not  $S_1$

KW slow modulation of the Born ampl - we get a strong modulation (well-separated diffraction max)



## Comments

-En cons  $p_{fn}^2/2m = p_i^2/2m + n\hbar\omega$ ; (mom cons!),  $\mathbf{p}$  along  $\mathbf{E}_0$   
( $\mathbf{p} \simeq e\mathbf{E}_0/\omega$ )

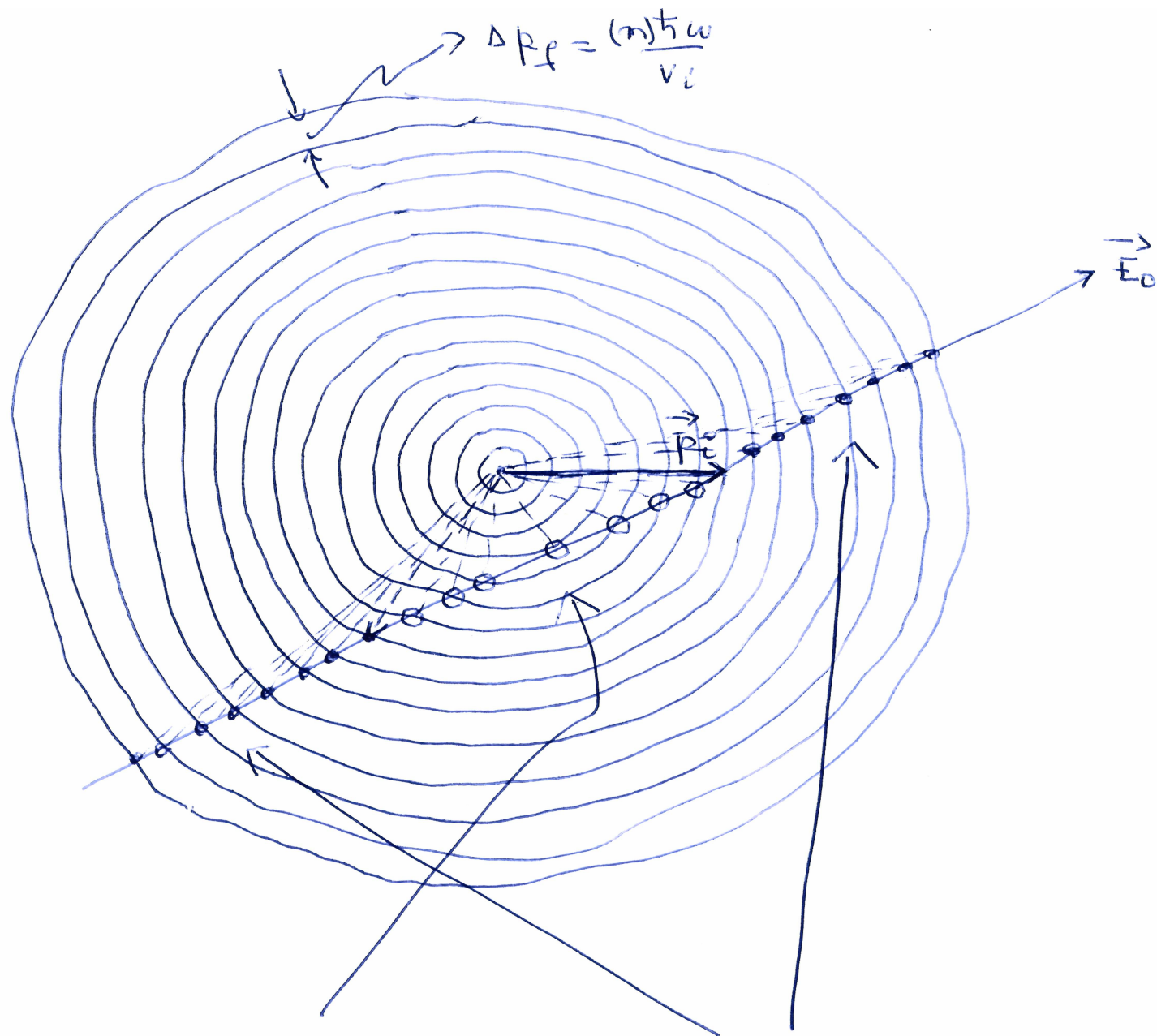
-Elastic scatt for  $n = 0$

-Cons laws:

$$\theta_n \simeq \sin(\mathbf{p}_i, \mathbf{E}_0) \frac{n\hbar\omega}{2E_i}$$

-Photon absorption ( $n > 0$ ), scatt angle  $\theta_n$  increases, photon emiss ( $n < 0$ ),  $\theta_n$  decreases

-Favoured almost transverse, low fields, both forward & “backward” scatt (small  $n$ )



Emission

Absorption

## Order of magnitude

(In order to compute  $\sigma_n$  use the gen fct of  $J_n$ !)

$\sigma_n \simeq d^2 \left[ \frac{n\hbar\omega}{\hbar^2/md^2} \right]^2 \left( \frac{\hbar\omega}{eE_0d} \right)^2$ ; for  $d = 1\mu m$ ,  $\omega = 10^{15} s^{-1}$ ,  $E_0 = 10^8 esu$   
( $I = 10^{18} w/cm^2$ ) cross-section  $\sigma_n \simeq 10^{-2} n^2 A^2 (cm^{-2})$ , heavy ions (small  $n$ , forward scatt)

-reduction  $(a/d)^4 \simeq 10^{-16}$  (normaliz wavefunction, trajectory)

-electrons, favoured

## Final obs

-Duration  $\tau$ , repetition time  $\Delta t$

-Reduction factor  $\tau/(\tau + \Delta t)$