Journal of Theoretical Physics

Founded and Edited by M. Apostol **104** (2005)

ISSN 1453-4428

Generalized Omori's law and a self-generating process

B.-F. Apostol
Department of Theoretical Physics, Institute of Atomic Physics,
Magurele-Bucharest MG-6, POBox MG-35, Romania
email: apoma@theory.nipne.ro

Abstract

A generalized Omori's law is derived for probability distributions which may govern, for instance, the seismic aftershocks. It is shown that Omori's law implies a self-replication process of the events accompanying main events, like the seismic activity associated with the occurrence of main seismic shocks. In general, it is shown that any finite probability distribution can be associated to an Omori-type singular distribution, and, conversely, any Omori-type singular distribution can be associated with a finite distribution, the two distributions being inter-related by Euler's transform. The finite distribution plays the role of a generating distribution for the self-replication process, and the self-consistency of the generating process requires this distribution be given by an exponential law.

Omori's empirical law[1] states that the temporal distribution of the seismic aftershocks goes like $\sim 1/\tau^{\gamma}$, where $\gamma = 1^{+}$, and τ denotes the time elapsed from the occurrence of the main seismic shock at $\tau = 0$. The law is widely used in analyzing the seismic activity accompanying a major earthquake, both for aftershocks and foreshocks, as well as in a variety of other situations.[2]-[10] It bears also relevance on a critical-point theory for the accompanying seismic activity, and other similar phenomena.[11]-[13] A generalized Omori's law is derived here, by assuming a self-replication process for the events acompanying main events, like the seismic activity associated with major seismic events. It is shown that Omori's law implies a generating probability distribution, the two distributions being inter-related by Euler's transform. The self-consistency of the generating process requires the generating distribution be given by an exponential law.

Let $p(x) = dN/dx$ be a finite probability distribution for $x > 0$, and let $dN_0 = p_0 dx$ be viewed as the number of main events placed at origin, where $p_0 = p(0)$. The rest of events can be viewed as accompanying events, so that they may be produced by the main events at a rate

$$r(x) = p(x)/p_0 \ . \tag{1}$$

It is assumed that these events self-replicate at this rate, so that the total number of resulted events is distributed according to $P(x)$ satisfying

$$P(x) = p(x) + \frac{p(x)}{p_0} P(x) \ . \tag{2}$$

It follows from (2) that the distribution $P(x)$ is given by

$$P(x) = \frac{p(x)}{1 - p(x)/p_0} \ , \tag{3}$$

which is Euler's transform between $p(x)/p_0$ and $-P(x)/p_0$. Equation (3) gives a generalized Omori's law. Indeed, $P(x)$ as given by (3) is singular at origin. We may consider the series expansion $p(x) = p_0 - p_1 x...$ of $p(x)$ in powers of x in the neighbourhood of origin, where $-p_1 = p'(0) < 0$ is the first derivative of $p(x)$ at $x = 0$, which leads to Omori's original law $P(x) = p_0 x_0/x$ for $x \ll x_0$, where $x_0 = p_0/p_1$. In general, equation (3) provides a representation $P(x) = p_0/h(x)$ for $P(x)$, where $h(0) = 0$ and $h(\infty) \rightarrow \infty$, such that $p(x) = p_0/[1 + h(x)] \simeq p_0[1 - h(x)]$ for $x \rightarrow 0$, which may be viewed as a generalized Omori's law. For $h(x) = (x/x_0)^\beta$, where $\beta > 0$ and x_0 denotes a scale parameter, power-law distributions $P(x) = p_0(x_0/x)^\beta$ are obtained. In view of their possible non-integrability, the parameter x_0 may be taken as a lower-bound cutoff x_c, $x_0 = x_c$, for $\beta \geq 1$, and an upper-bound cutoff $D \gg x_0$ may be employed for $0 < \beta \leq 1$.

The distribution $p(x)$ in (2) can be viewed as a generating distribution for the self-replicating events accompanying main events. Since the generating events are produced by main events at rate $r(x)$ given by (1), it may also be assumed that such events are also self-generated at the same rate, according to

$$p(x + \Delta x) = r(\Delta x) p(x) \ , \tag{4}$$

which leads to

$$dp/dx = -(p_1/p_0) p(x) \ . \tag{5}$$

It follows that $p(x)$ is given by an exponential law $p(x) = p_0 e^{-p_1 x/p_0}$, which may be normalized into the probability distribution $p(x) = p_0 e^{-p_0 x}$. Equation (4) expresses a self-consistency, or a self-generating, character of the accompanying events. Inserting the exponential distribution $p(x) = p_0 e^{-p_0 x}$ into (3) the distribution

$$P(x) = \frac{p_0}{e^{p_0 x} - 1} \ , \tag{6}$$

is obtained, which is the original Omori's law $P(x) = 1/x$ for $p_0 x \ll 1$. It is customary to extend the distribution $1/x$ to infinite as $P(x) = x_c^{\beta-1}/x^\beta$ for $\beta = 1^+$, such that

$$\int_{x_c}^{\infty} dx \frac{p_0}{e^{p_0 x} - 1} = \int_{x_c}^{\infty} dx (x_c^{\beta-1}/x^\beta) \ . \tag{7}$$

Equation (7) leads to the exponent $\beta = 1 - 1/\ln(p_0 x_c) = 1^+$ in the limit $x_c \rightarrow 0$. It might be noted that equation (6) resembles formally a Bose-Einstein-type occupation number (in two dimensions), for an exponential, Boltzmann-type, distribution $p(x)$. The self-replication equation (2), which describes a geometric series, looks also formally as a Dyson equation in the theory of many-particles ensembles.

In conclusion, it may be said that a self-replication process for a generating distribution of accompanying events leads to a generalized Omori's law, and the self-consistency character of the process requires an exponential law for the generating distribution.

Acknowledgments

The author is indebted to the members of the Institute of Earth's Physics, Magurele-Bucharest for enlightening discussions.

References

[1] F. Omori, J. Coll. Sci. Imper. Univ. Tokyo **7** 111 (1894)

[2] T. Utsu, Geophys. Mag. **30** 521 (1961)

[3] D. Sornette, C. Vanneste and L. Knopoff, Phys. Rev. **A45** 8351 (1992)

[4] A. Helmstetter and D. Sornette, Phys. Rev. **E66** 061104 (2002)

[5] A. Helstetter, S. Hergarten and D. Sornette, Phys. Rev. **E70** 046120 (2004)

[6] D. Sornette and G. Ouillon, Phys. Rev. Lett. **94** 038501 (2005)

[7] A. Saichev and D. Sornette, Phys. Rev. **E70** 046123 (2004); ibid, **71** 016608 (2005)

[8] R. Console, A. M. Lombardi, M. Murru and D. Rhoades, J. Geophys. Res. **108** 2128 (2003)

[9] A. Petri, G. Paparo, A. Vespignani, A. Alippi and M. Constantini, Phys. Rev. Lett. **73** 3423 (1994)

[10] C. Maes, A. Van Moffaert, H. Frederix and H. Strauven, Phys. Rev. **B57** 4987 (1998)

[11] See, for instance, D. Sornette, Phys. Reps. **378** 1 (2003); P. Bak, K. Christensen, L. Danon and T. Scanlon, Phys. Rev. Lett. **88** 178501 (2002)

[12] D. Sornette and C. G. Sammis, J. Physique I **5** 607 (1995); D. Sornette, Phys. Reps. **297** 239 (1998); ibid, **313** 238 (1999)

[13] B. Barriere and D. L. Turcotte, Phys. Rev. **E49** 1151 (1994)