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Principles of Earthquake Forecasting
Short-Term Prediction
Application to Vrancea, Romania

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Prediction is very difficult, especially if it's about the future (Nils Bohr).

Abstract

The mean recurrence time theory for regular earthquakes is briefly reviewed, as well as Omori's law for the seismic activity accompanying main seismic shocks. It is shown how the Gutenberg-Richter magnitude distribution, the corresponding logarithmic distribution and the cumulative recurrence law can be employed to characterize a particular seismic activity and region. The California model introduced recently for short-term prediction is analyzed (Gerstenberger et al, Nature, **435** 328 (2005)), with emphasis on its statistical character and time-decreasing sequences of clustering earthquakes described by Omori's law. A different approach to short-term earthquake prediction is put forward herein, based on statistical analysis of the time-magnitude distributions of the next earthquake. The method makes use of the general n-point correlation functions in statistical analysis. The next-earthquake model is applied to 1999 earthquakes recorded in Vrancea over the last 30 years with (moment) magnitude higher than $M > 3$. It is shown that the short-time Vrancea seismic activity is characterized by time-decreasing distributions of the next earthquake, possibly with a long tail extinguishing slowly in time, described by Omori-type power laws, as expected. The short-term Vrancea seismic activity exhibits a correlation time of roughly $20 - 25$ days for the next earthquake, and a similar size correlation for magnitudes $M < 4 - 5$. The null hypothesis is investigated for these distributions, and the confidence level is estimated to cca 77% for magnitudes $M < 4$. Unfortunately, the poor statistics prevents a confident prediction for stronger earthquakes, but data are given for Vrancea earthquakes with magnitude up to $M > 7$.

Introduction. Forecasting earthquakes is of utmost relevance for estimating seismic hazard and risk. The main question in forecasting earthquakes is to determine what earthquake will occur, and what characteristics will it have. For instance, the forecasting question may sound like "what earthquake will occur at time t, at location $\mathbf{r}$ in a particular seismic region, what its magnitude M will be, and what effects will it have". The depth of the focus producing that particular earthquake may be added to such characteristics. This is a particularly important parameter, since the effects of an earthquake may depend considerably on its depth. However, the variability of earthquake occurrence with respect to this parameter may be rather limited, so it may be left aside for the moment (though its taking into account does not imply any particular difficulty).

Lacking a deterministic causality for earthquake occurrence, we must resort to statistical analysis for their forecasting. Therefore, the above question must be answered in terms of the probability

$$\frac{d^3N}{N_0 dM dt d\mathbf{r}} = P(M, t, \mathbf{r}) \ , \tag{1}$$

where N is the number of earthquakes, N_0 is the total number of earthquakes and $d\mathbf{r}$ is the infinitesimal area at $\mathbf{r}$. Statistical forecasting is warranted by a statistically significant ensemble of data, *i.e.* a set of data as large as possible. Very likely, a previous "a priori" knowledge is needed for getting the probability P in (1), so the latter becomes in fact a conditioned probability, like, for instance, the probability $P(M, t, r \mid M_0, t_0, r_{0,}...)$ of an earthquake of magnitude M occurring at time t and location r, providing an earthquake of magnitude M_0 occurrs at previous time t_0, at location r_0, or a sequence of events with determined characteristics occurred already, etc. Earthquake forecasting becomes this way a Bayesian statistical analysis,[1] and conditioned probabilities may reveal statistical correlations.

There are a few semi-empirical, statistically documented, patterns in earthquake phenomenology, like the Gutenberg-Richter relationship between released energy and magnitude, the Gutenberg-Richter magnitude distribution, the logarithmic magnitude distribution, the recurrence law for the cumulative number of earthquakes,[2]-[6] Omori's law for the seismic activity accompanying a main shock (aftershocks and foreshocks),[7, 8] Bath's law for the average highest aftershock's magnitude,[9] etc. Since their significant variability, these patterns must be used in earthquake forecasting for any particular situation, the corresponding parameters being updated according to the evolution of that particular situation, such as to ensure the statistical significance of the data sets.

The effect of an earthquake is usually given by the (modified) Mercalli intensity I (or peak ground acceleration)[10] as a function $I(M, \mathbf{r} - \mathbf{r}')$ of earthquake magnitude M and distance $\mathbf{r} - \mathbf{r}'$, where $\mathbf{r}$ is the location of interest and $\mathbf{r}'$ is the earthquake location. Usually, it is an empirical function which is linear in magnitude (with coefficients still depending slightly on M). The rate of predicted effects of a seismic activity at t and $\mathbf{r}$ is therefore given by

$$I(t, \mathbf{r}) = \int dM d\mathbf{r}' \cdot I(M, \mathbf{r} - \mathbf{r}') P(M, t, \mathbf{r}') \ . \tag{2}$$

Usually, values greater than $I > VI$ on the modified Mercalli scale are of interest. A "shaking" map is thereby generated, for the seismic hazard (and risk) for any moment t.

A model of short-term earthquake forecasting for California has been recently put forward,[11] according to the general framework outlined above. It employs, essentially, analytical relationships, like the Gutenberg-Richter magnitude distribution and Omori's law, fitted to past statistical data, and permanently updated in real time. Beside giving herein a short account of this model, another, more general, model is introduced here for short-term earthquake forecasting, based on statistical analysis of next-earthquake time-magnitude distributions, and applied to Vrancea, Romania. Next-earthquake distributions (also called inter-occurrence or waiting time distributions) have been widely used recently for highlighting universal scale laws,[12] or self-similarity patterns, or clustering properties in time, space and magnitude.[13]

Gutenberg-Richter magnitude distribution. It is widely agreed that, in general, an earthquake needs time to building energy in its focus. It follows that earthquakes may first be described by a mean recurrence time t_r, which depends on their released energy, and magnitude. The time probability is then given immediately by $\sim 1/t_r^2$. On the other hand, it was shown recently[14, 15] that seismic energy E accumulating in focus is related to the mean recurrence time t_r by $E \sim t_r^{1/r}$,

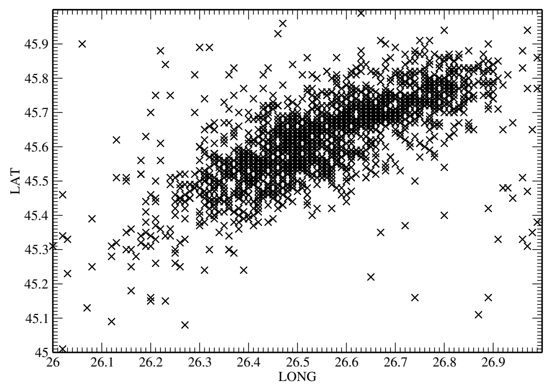

Figure 1: Geographical distribution of 1999 Vrancea earthquakes with (moment) magnitude $M > 3$ recorded between 1974 and 2004 (30 years) within $45° - 46°$N latitude and $26° - 27°$E longitude (Romanian Earthquake Catalogue)[29]

where r is a geometric parameter. For a point-like focus building seismic energy with a uniform velocity the parameter r acquires the value $r = 1/3$. For different geometries of seismic foci, and different seismic mechanisms, parameter r may acquire different values. For instance, for a seismic fault with a dominant two dimensional geometry the parameter r may take a value close to $r = 1/2$. Making use of this energy-time relationship given above, and using the Gutenberg-Richter energy-magnitude law $\ln(E/E_0) = bM$ (where E_0 is the well-known Gutenberg-Richter energy threshold), one obtains straightforwardly the Gutenberg-Richter magnitude distribution

$$P(M) = dN/N_0 dM = \beta e^{-\beta M} \quad , \tag{3}$$

where $\beta = br$.

The Gutenberg-Richter magnitude distribution given above is well documented, either in its logarithmic form

$$\ln(\Delta N/T) = \ln(\beta \Delta M/t_0) - \beta M \quad , \tag{4}$$

or in its cumulative, or recurrence, law

$$\ln(N_{ex}/T) = -\ln t_0 - \beta M \quad , \tag{5}$$

where T is the entire period of the analyzed statistical data, $1/t_0 = N_0/T$ is the seismicity rate, and N_{ex} is the number of earthquakes exceeding M in magnitude. For instance, an overall analysis[6] for $5.8 < M < 7.3$ indicates $\beta = 1.38$, which is in fair agreement with $\beta = br = 1.17$ obtained for the Gutenberg-Richter value[6] $b = 3.5$ and $r = 1/3$, as suggested by a point-like generic model of seismic focus. Similarly, the same set of data gives a global seismicity rate $1/t_0 \simeq 10^{5.5}$ per year, which is consistent with estimations of cca $10^5 - 10^6$ worldwide earthquakes per year, on average.[6] It seems that value $\beta \sim 1.38$, corresponding to $B = \beta/\ln 10 = \beta/2.3 = 0.6$, enjoys a certain universality.[16] On the other hand, a recent analysis of Vrancea earthquakes, for instance, indicates $\beta = 1.89$ (corresponding to $r = 0.54$, consistent with a geometry nearer to a two-dimensional one for Vrancea seismic fault), and a rate of seismicity $1/t_0 \simeq 10^{4.21}$ per year.[17] In addition, significant deviations from these parameter values in magnitude distribution are recorded for both small and high values of magnitude.[6] Due to such considerable variability in the parameter values of the magnitude distribution (3), it is advisable to employ those parameter values in the Gutenberg-Richter magnitude distribution (specifically β) which are obtained by fitting the particular data analysed for earthquake forecasting.

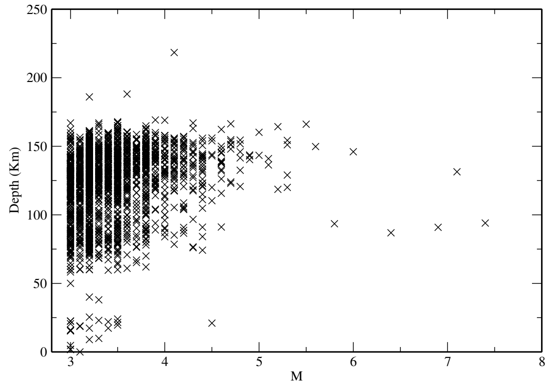

Figure 2: Depth distribution of 1999 Vrancea earthquakes with (moment) magnitude $M > 3$ recorded between 1974 and 2004 (30 years) within $45° - 46°$N latitude and $26° - 27°$E longitude (Romanian Earthquake Catalogue)[29]

It is a straightforward matter to derive the mean recurrence time

$$t_r(M) = (t_0/\beta\Delta M)e^{\beta M} \tag{6}$$

for ocurrence of an earthquake of magnitude M to $M + \Delta M$, and it is tempting to use this equation for earthquake prediction. However, this is only a statistical information, plagued with uncertainties. It is reasonable to assume an earthquake occurrence governed by randomness, such as to maximize the Boltzmann entropy, for instance, under the constraint of a fixed mean recurence time. This is a Poisson-like assumption, and it leads immediately to a temporal distribution $(1/t_r)e^{-t/t_r}$, whose deviation $(\bar{t^2})^{1/2} - t_r = 0.41t_r$ is too large (41%) to be of practical use.

Nevertheless, it is worth noting that the Gutenberg-Richter magnitude distribution given by (3) is not time-independent, as claimed sometimes, since to each magnitude M in (3) there corresponds a well-defined mean recurrence time. However, for earthquakes of high magnitude, which, beside those of surface, are among the most interesting for forecasting, the mean recurrence time is long, and making use of considerations as those given above would amount to a long-term prediction. As described here, it is usually associated with considerable uncertainties. Such uncertainties leave room for other possible particular patterns in earthquake occurrence over a shorter-time scale.

Omori's law. Accompanying seismic activity. Apart from regular earthquakes described above, characterized by a mean recurrence time, the earthquake occurrence exhibits another pattern, consisting of the seismic activiy accompaning a main shock, and described by Omori's law. This accompanying seismic activity amounts to identifying a main seismic shock, of magnitude, say M_0, and associating to it a whole seimic activity, decreasing in time on departing from the occurrence time of the main shock, both in the future and in the past, and decreasing in magnitude M. This associated seismic activity looks like a clustering feature of the main shocks, with two sequences, one decreasing in time, and corresponding to aftershocks, another increasing in time on approaching the moment of the main shock's occurrence, and corresponding to foreshocks.

Originally, Omori's law has been suggested for aftershocks,[7, 8] but, very likely, it holds for foreshocks too.[18]-[20] A significant increase in the seismic activity preceding a main shock may serve as a useful tool in earthquake forecasting, and a critical-point theory, or other theoretical approaches, associated mainly to a self-organized criticality, have been put forward,[21]-[23] in order to characterize the accompanying seismic activity occurring in the proximity of a critical seismic event. In some particular situations such an accompanying seismic activity is well documented, while the lack of data in other situations leaves it still to the status of a theoretical assumption.

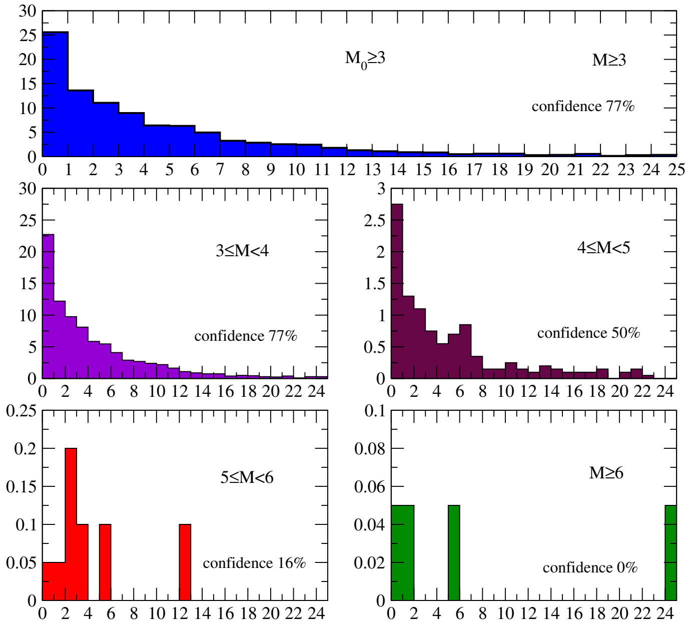

Figure 3: Time probability distribution $P(t)$ (upper panel) and time-magnitude probabilities $P(M,t)$ for the next Vrancea earthquake (probabilities are given in % on the coordinate axis and time is measured in days on the abscissa)

Omori's law states that the aftershocks following a main shock are distributed by a rate $dn/d\tau \sim 1/\tau^{\gamma}$, where τ is the time elapsed from the main shock and $\gamma = 1^{+}$. A similar law holds for foreshocks, where τ is the time preceding the occurrence of the main shock. It was shown recently[24] that such power-law distributions may originate in a self-replication process underwent by an original, generating distribution, the two distributions being inter-related by Euler's transform. For self-consistency reasons the orginal generating distribution is an exponential one, so that Omori's law reads very likely $dn/d\tau \sim 1/\tau$ for small values of τ. It is usually extended to $\tau \to \infty$ as $1/\tau^{\gamma}$ with $\gamma = 1^{+}$, and used in conjunction with a lower-bound cutoff τ_c, such that the normalized probability reads $dn/n_0 d\tau = (\gamma-1)\tau_c^{\gamma-1}/\tau^{\gamma}$. Parameter γ is related to the cutoff τ_c and the parameter $\alpha = 1/t_c'$ of the original exponential generating distribution, through $\gamma = 1 - 1/\ln(\alpha\tau_c)$, where t_c' is a characteristic time for the aftershocks activity's decay.[24] Such a decay time may possibly be very long, it may be estimated, for instance, to $t_c' = t_r/2$, where t_r is the main recurrence time of the main shock.[15] Omori's distribution is generally characterized by a sudden fall for small values of τ, and, possibly, a very long tail.

It is often more convenient to use

$$P(\tau) = A \cdot \frac{1}{\tau_c + \tau} \tag{7}$$

for Omori's law, where τ goes from zero to a certain upper bound cutoff, which determines the data employed for the normalization constant A. Such a fitting relationship is particularly useful for short-term earthquakes forecasting, which also justifies the exponent $\gamma = 1$ in (7).

Omori's law holds also for other statistical variables defining the accompanying seismic activity, like the difference in magnitude $m = M_0 - M$ between an aftershock or a foreshock (M), and the main shock (M_0), or for the released energy in the accompanying seismic activity.[14, 15] The magnitude of the accompanying seismic activity is time-dependent, according to[14, 15]

$$M = M_0 - \alpha\tau/\beta \ , \tag{8}$$

day	N=1998	P(t)%	N(3<M<4)=1769	P(3<M<4,t)%	N(4<M<5)=211	P(4<M<5,t)%	N(5<M<6)=13	P(5<M<6,t)%	N(6<M<8)=5	P(6<M<8,t)%
0	511	25.58	454	22.72	55	2.75	1	0.05	1	0.05
1	272	13.61	244	12.21	26	1.30	1	0.05	1	0.05
2	221	11.06	195	9.76	22	1.10	4	0.20	0	0.00
3	179	8.96	162	8.11	15	0.75	2	0.10	0	0.00
4	128	6.41	117	5.86	11	0.55	0	0.00	0	0.00

Table 1: Event distribution for Vrancea next-earthquake (1999 Vrancea earthquakes with (moment) magnitude $M > 3$, recorded between 1974 and 2004 (30 years) within $45° - 46°$N latitude and $26° - 27°$E longitude, Romanian Earthquake Catalogue[29])

and it may hold for long, as Omori's law has a long tail, though with a very low probability. It can be shown, by using Euler's relationship,[14, 15] that Omori's law in magnitude implies an exponential distribution $\sim e^{-\beta M}$ for magnitude values close to the magnitude M_0 of the main shock, which is identical with the Gutenberg-Richter magnitude distribution described above. Therefore, the Gutenberg-Richter magnitude distribution for regular earthquakes, as given by (3), can also be used for accompanying seismic events close in magnitude to the main shock, as noted previously.[25]

Moreover, since the magnitude distribution for the accompanying seismic events is symmetrical in τ for aftershocks and foreshocks, it follows that the mean difference vanishes, $\bar{m} = 0$, so that it is reasonable to use the deviation $(\overline{m^2})^{1/2} = \sqrt{2}/\beta$ as a measure of the average difference in magnitude of the greatest aftershock (or foreshock) with respect to the main shock. For $\beta = 1.17$ indicated above, one obtains $(\overline{m^2})^{1/2} \simeq 1.2$, in fair agreement with Bath's empirical law.[26] Similar results are obtained recently, though on a different conceptual basis.[27, 28]

It is also worth noting that for values E of the seismic energy released by the accompanying seismic activity close to the energy of the main shock, Omori's distribution can be written for variable $1/E$.[15] By comparing the corresponding generating distribution with the temporal one, the rate of released energy $dE/d\tau \sim -1/\tau^2$ is obtained for the accompanying seismic activity, which seems to be supported by empirical data.[8, 22]

Short-term prediction. California model.[11] California model for short-term earthquake forecasting[11] identifies mainshocks of magnitude, say, M in a certain range (within ± 0.5 around $M = 4, 5, 6, 7$) in the past data, and look for time-decreasing sequences of earthquake clustering in the same range of magnitude, within a few months, untill the number of earthquakes becomes comparable with the average background. It is worth noting that "aftershocks" greater in magnitude than the main shock (within the magnitude *ecart*) are allowed in the same sequence in this model.

Omori's law identifies a particular pattern in the short-term seismic activity accompanying a main seismic shock. However, the seismic activity over a short-time scale is more complex. The scarcity of data regarding aftershocks or foreshocks (*i.e.* the poor statistics) may render Omori's law of

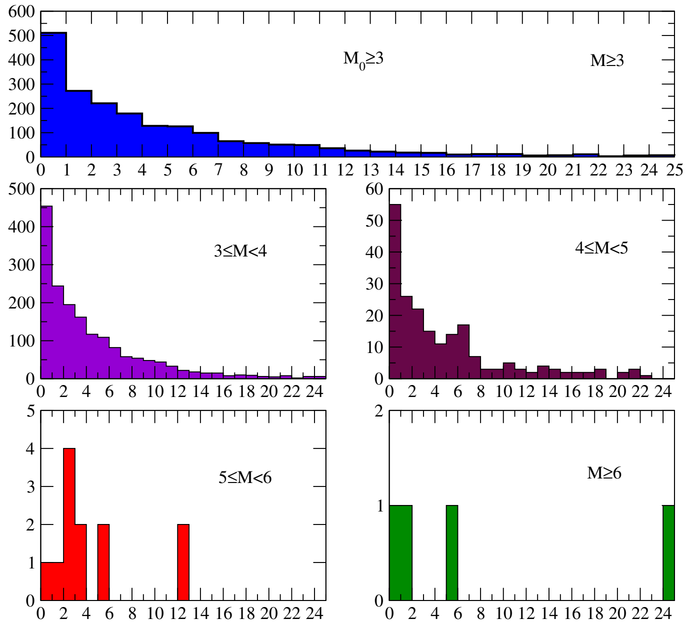

Figure 4: Events distributions for Vrancea next earthquake (time is measured in days on the abscissa). The upper panel shown the total (cumulative) number of events (1998 out of 1999; the reference earthquake is not included). The analysis is performed over a set of 1999 Vrancea earthquakes with (moment) magnitude $M > 3$ recorded between 1974 and 2004 (30 years) within $45° - 46°$N latitude and $26° - 27°$E longitude (Romanian Earthquake Catalogue[29])

little practical use. On the other hand, it is still lacking a practical method of disentangling the truly accompanying seismic activity from other, regular seismic activity, without any relationship with the main shock, especially for the long tail of Omori's law. In addition, the short-time seismic activity may exhibit a more complex pattern, of a multiple-branch character. Indeed, a mainshock may produce aftershocks, according to Omori's law, whose magnitude decreases in time according to (8). It may sound reasonable to assume that each aftershock in such a sequence may itself be a main shock, which produces in turn its own aftershocks. It is worth noting that aftershocks magnitude depends on the mainshock's magnitude, through the α-coefficient in (8). A series of sub-branches is then conceivable to appear under such an assumption, removing thereby the differences between a main shock and aftershocks. Similarly, an earthquake may not be viewed as only belonging to a certain branch of aftershocks, but pertaining also to a certain branch of foreshocks, corresponding to some forthcoming main shock. It is also conceivable that such foreshocks may generate other branches of foreshocks, according to (8), with various α-slopes. The whole picture obtained this way is that of various earthquakes occurring at any time with various magnitudes, *i.e.* the magnitude is decoupled in fact from time. Similarly, these two independent variables may also be viewed as being decoupled from location $\mathbf{r}$ in probability (1). It is then natural to write down $P(M, t, r)$ as a product of three probabilities, corresponding to independent statistical variables M, t and $\mathbf{r}$, and use (7) and (3) for magnitude and temporal distributions. Moreover, a similar Omori's law may be assumed for the r-distribution, which in area variable becomes $\sim 1/r^2$. Therefore, the short-term prediction California model assumes[11]

$$P(M, t, r) = Ae^{-\beta M} \cdot \frac{1}{t_c + t} \cdot \frac{1}{(r_c + r)^2} \tag{9}$$

for the probability of occurring an earthquake, where A is a constant of normalization, Omori's law is written in t-notation, t_c is a temporal cutoff, r_c is a spatial cutoff, and the distribution is taken

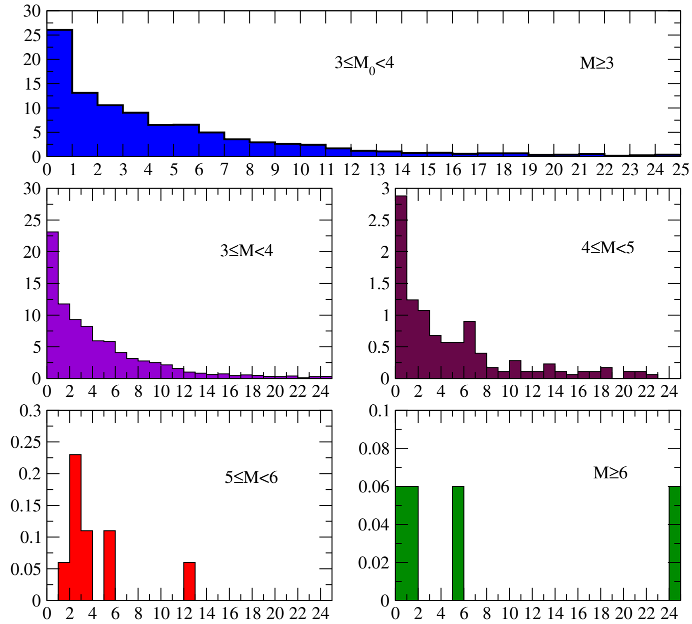

Figure 5: Vrancea next-earthquake time-magnitude probabilities $P(M,t \mid M_0)$ for magnitude $3 < M_0 < 4$ of the former earthquake (probabilities are given in % on the coordinate axis, and time is measured in days on the abscissa). The analysis is performed over a set of 1999 Vrancea earthquakes with (moment) magnitude $M > 3$ recorded between 1974 and 2004 (30 years) within $45° - 46°$N latitude and $26° - 27°$E longitude (Romanian Earthquake Catalogue[29])

isotropic with respect to location $\mathbf{r}$ (actually, the daily rate is used for the probability distribution given above, with $\Delta M = 1$ and $\Delta\mathbf{r}$ corresponding to a grid of 5km square size, and the exponent in Omori's law is taken slighlty greater than unity). Equation (9) above is fitted to empirical data, and employed for short-term prediction. For a particular earthquake sequence in evolution parameters in (9) may be updated in real time, and employed for the next-day prediction. The procedure was tested against the null hypothesis, by employing fits to previous data for predicting a posteriori, known, data, with satisfactory results. The final results are cast into the form of earthquake effects as given by (2). Though slight increases are sometimes indicated in temporal sequences, time-increasing series are not included in the model.

Next-earthquake prediction. In view of the highly complex statistical behaviour of the short-term seismic activity, more general appraoches are needed, of more practical relevance, without resorting too much to patterns established statistically on general sets of data. From the practical standpoint the most relevant question in short-term earthquake forecasting seems to be "what happens next?". The general approach described herein is focused on this main question. Suppose that an earthquake occurs a time t_0 and the next one occurs at some time t measured with respect to t_0. Then, we may define a distribution $P(t)$ of these next earthquakes, and determine it from a set of relevant statistical data. Once determined, it may be used for forecasting the time probability of occurrence of the next earthquake, based on the principle "what happened will happens again".

Let the earthquakes be labelled by some generic parameter x, like magnitude, location, depth, etc. Then, we may distribute the next earthquakes with respect to x, and introduce the time probability distribution $P(x,t)$ of the next earthquake characterized by parameter x occurring at time t. Of course, $\int dx \cdot P(x,t) = P(t)$. Another distribution $P(x,t \mid x_0)$ may also be introduced with respect to the former earthquake labelled by parameter x_0, so that $\int dx_0 \cdot P(x,t \mid x_0) = P(x,t)$

and $\int dx \cdot P(x, t \mid x_0) = P(t \mid x_0)$, where the latter is the probability distribution of the next earthquake occurring at time t providing the former is characterized by x_0.

The procedure may obviously be detailed, by introducing similarly the probability distributions $P(x, t \mid x_{01}, x_{02}, ...)$, or $P(x_1, x_2, ..., t \mid x_{01}, x_{02}, ...)$, which resemble the hierarchy of n-point correlation functions in statistical analysis. Characteristic scale time or size, or correlations range, could be identified from the statistical analysis of such functions, providing the statistical set of data is large enough, which may shed light on the statistical patterns of a seismic activity. Unfortunately, the statistics is rather poor, in general, precisely for those range of x where prediction is most interesting, like, for instance, for x corresponding to high values of magnitude M. Nevertheless, the next-earthquake approach to short-term earthquake prediction is applied here to Vrancea earthquakes, in order to illustrate its predictive capabilities and limits.

Brief characterization of Vrancea earthquakes. Vrancea is a seismic region located approximately at 45.7°N latitude and 26.6°E longitude (Romania). The geographical distribution of Vrancea seismic foci is shown in Fig. 1 for 1999 earthquakes with (moment) magnitude $M > 3$ recorded between 1974 and 2004 (30 years) within $45° - 46°$N latitude and $26° - 27°$E longitude.[29] This basic data set is used for the statistical analysis reported here (magnitude accuracy $\Delta M = 0.1$). Vrancea exhibits mainly middle-depth earthquake ($\sim$ 80km to $\sim$ 150km), and occasionally a few crustal, or surface, earthquake. The depth distribution of Vrancea earthquakes is shown in Fig. 2. It may be seen in Figs. 1 and 2 a Vrancea seismically active focal zone resembling a fault with a geometry close to a two-dimensional one. Strong earthquakes occur sometime in Vrancea, as, for instance, $M = 7.4$, March 4, 1977, depth 94km, or $M = 7.1$, August 30, 1986, depth 131km. Nine earthquakes with magnitude $M > 7$ have been recorded in the past two centuries in Vrancea.

As noted above, the magnitude distribution of these Vrancea earthquakes has been analyzed by using the Gutenberg-Richter distribution, the logarithmic distribution and the cumulative recurrence law, as given by equations (3) to (5). The parameter β in these distributions has been found to acquire the value $\beta = 1.89$, and the seismicity rate $1/t_0 = 10^{4.21}$ per year, on average.[17] These values correspond to a parameter $r = 0.54$ in the energy distribution (for the Gutenberg-Richter parameter $b = 3.5$), suggesting a fault geometry nearer to a two-dimensional one rather than a localized point-like seimic zone ($1/r = 1.85$),[14, 15] and a mean recurrence time $t_r = 34.9 \pm 14.3$ years for earthquakes with magnitude $M > 7$.

Next-earthquake prediction for Vrancea. For the sake of simplicity we neglect here the geographical and depth distribution of Vrancea earthquakes, so the generic parameter x in the next-earthquake prediction procedure described above amounts to magnitude M. The location parameters may be introduced in a refined analysis, without any technical difficulties, though they may reduce considerably the size of the statistical set. The time probability distribution $P(t)$ for Vrancea next-earthquake (*i.e.* $P(M > 3, t \mid M_0 > 3)$) is shown in the upper panel in Fig. 3, and the time-magnitude distributions $P(M, t)$ for Vrancea next earthquake with magnitudes $3 < M < 4$, $4 < M < 5$, $5 < M < 6$ and $M > 6$ (all for $M_0 > 3$) are also shown in Fig. 3 (probability is given in % on the coordinate axis and time is measured in days on the abscissa). We note first that Vrancea next-earthquake distributions $P(M, t)$ exhibit a characteristic decrease in time, with the highest probability of next-earthquake occurrence in the same day as the reference earthquake, at least for small magnitudes ($M < 5$). The mean time for $P(t)$ is cca 5.89 days, and the variance $\sigma = 9.55$ days. Then, we note the maximum values of these probabilities $\sim 22.7\%$ for $3 < M < 4$, $\sim 2.75\%$ for $4 < M < 5$, while probability $P(M, t)$ vanishes practically for $M > 5$. It is also worth noting that $P(t)$ and $P(3 < M < 4, t)$ are similar, obeying, very likely, Omori-type power laws, at least for short times,[12, 13] while the distributions become gradually irregular, exhibiting large fluctuations on increasing magnitude above $M = 4 - 5$. The statistics becomes

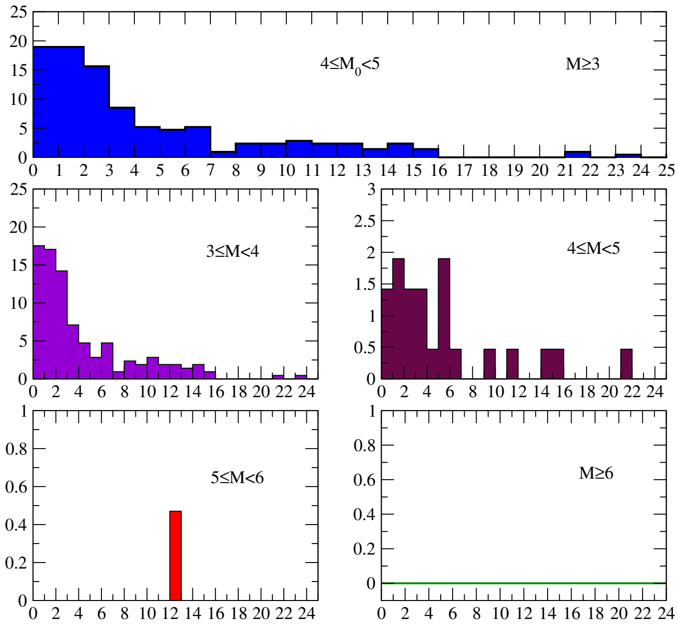

Figure 6: Vrancea next-earthquake time-magnitude probabilities $P(M, t \mid M_0)$ for magnitude $4 < M_0 < 5$ of the former earthquake (probabilities are given in % on the coordinate axis, and time is measured in days on the abscissa). The analysis is performed over a set of 1999 Vrancea earthquakes with (moment) magnitude $M > 3$ recorded between 1974 and 2004 (30 years) within $45° - 46°$N latitude and $26° - 27°$E longitude (Romanian Earthquake Catalogue[29])

extremely poor for higher magnitude ($M > 5$), as expected. A correlation time of cca $20 - 25$ days can be estimated, after which the probabilities decrease drastically (below 1%), as well as a size correlation of cca $M = 4 - 5$, above which the distributions acquire very small values, and are very irregular. The distribution of events is given in Table 1 and shown in Fig. 4.

The null hypothesis was tested on these distributions, by comparing the results of the first half of data with those derived from the second half of data. The result of this comparison was estimated in terms of relative mean square deviation, and it is indicated as the confidence level in Fig. 3. It can be seen in Fig. 3 that this confidence level is cca 77% for $P(t)$ and $P(3 < M < 4, t)$, and decreases considerably for $M > 5$.

The above analysis does not include the distribution with respect to the magitude M_0 of the former earthquake. When included, we obtain the time-magnitude conditioned probabilities (or two-point correlation functions) $P(M, t \mid M_0)$, as shown in Figs. 5 and 6 for $3 < M_0 < 4$ and $4 < M_0 < 5$, respectively. The corresponding cumulative distributions $P(t \mid M_0)$ are also shown in Figs. 5 and 6, in the upper panel (for $M > 3$). The first observation is that distributing the time-magnitude events with respect to the former earthquake magnitude M_0 does not change practically the characteristic time-decreasing behaviour of the next-earthquake activity, at least for small magnitudes. It can be seen in Fig. 3 and Fig. 5 that $P(M, t)$ and $P(M, t \mid 3 < M_0 < 4)$ are very similar, while considerable differences appear for $P(M, t \mid 4 < M_0 < 5)$, even for small magnitudes $3 < M < 4$. This reflects again the size correlation $M = 4 - 5$, and makes useless the estimation of the confidence levels for higher magnitudes, as the corresponding distributions are affected by large fluctuations. Unfortunately, higher-order correlation functions (as well as higher-magnitude analysis, or sharpening the magnitude gap $\Delta M = 1$) reduce considerably the statistical set, thus exhibiting a poor confidence and being practically irrelevant.

Time-magnitude next-earthquake probabilities $P(M, t \mid M_0 > 3)$ are shown in Fig. 7 for Vrancea

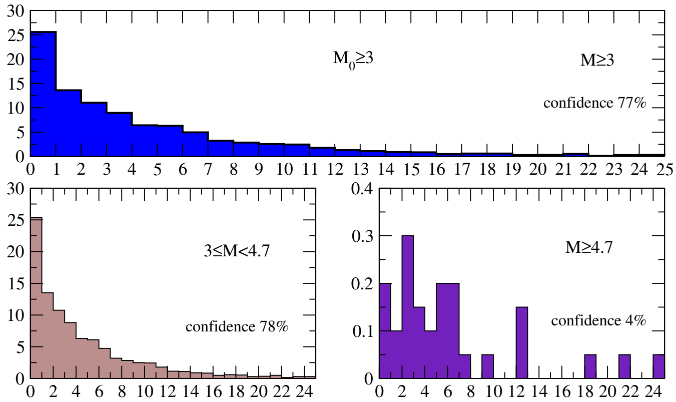

Figure 7: Vrancea next-earthquake time-magnitude probabilities $P(M, t \mid M_0 > 3)$ for magnitudes $M > 3$, $3 < M < 4.7$ and $M > 4.7$ (probabilities are given in % on the coordinate axis, and time is measured in days on the abscissa). The analysis is performed over a set of 1999 Vrancea earthquakes with (moment) magnitude $M > 3$ recorded between 1974 and 2004 (30 years) within $45° - 46°$N latitude and $26° - 27°$E longitude (Romanian Earthquake Catalogue[29])

earthquakes, for $3 < M < 4.7$ and $M > 4.7$. The threshold $M = 4.7$ is chosen since there is a partial consensus that a typical Vrancea earthquake is felt in Bucharest for magnitude above $M = 4.7$. The overall probability for $M > 4.7$ is very small (less than 2%), and the curve $P(M > 4.7, t \mid M_0 > 3)$ shown in Fig. 7 is very irregular. There are 35 Vrancea earthquakes with $M > 4.7$ in the whole data set, most of them placed in the first half of data, which leads to a very low confidence level (4%).

The probability $P(M, t \mid M_0)$ may be taken as the probability $P(M, t)$ in (1) (irrespective of other parameters, like $\mathbf{r}$, for instance), and used in (2) in order to determine the "shaking" maps, according to

$$I(t, \mathbf{r} \mid M_0) = \int dM \cdot I(M, \mathbf{r}) P(M, t \mid M_0) \ . \tag{10}$$

The empirical determination of the seismic effects function $I(M, \mathbf{r})$ for Vrancea is underway, by ongoing macro- and microzonation studies.

Conclusion. A new approach to short-term earthquake prediction is put forward here, and applied to Vrancea seismic activity, as based on the statistical analysis of the next-earthquake distributions. The main distribution function is the time-magnitude probability $P(M, t \mid M_0)$ of the next earthquake of magnitude M occurring at time t elapsed from the occurrence of the former with magnitude M_0. It may be viewed as a two-point correlation function, or a conditioned probability. This distribution is obtained by analyzing 1999 earthquakes with (moment) magnitude $M > 3$ recorded in Vrancea over the last 30 years (since 1974 to 2004). The result shows a characteristic time-decreasing behaviour of the occurrence of the next earthquake, at least for small magnitueds. Correlations are identified for lower-magnitude earthquakes ($M, M_0 < 4 - 5$), extending roughly over $20 - 25$ days. Statistics becomes extremely poor for strong earthquakes ($M > 5$), preventing thus any confident prediction. It is shown also how to use such prediction information for constructing "shaking" maps of (Mercally modified) intensity $I(t, \mathbf{r} \mid M_0)$ (or peak ground acceleration), of particular relevance in estimating the seismic hazard and risk.

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