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# Short-Term Seismic Activity. Next-Earthquake Time-Magnitude Distributions

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## Abstract

Short-term seismic activity in Vrancea is analyzed by means of the next-earthquake (or inter-occurrence, or waiting time) distributions. It is shown that this seismic activity obeys Omori-type time-power laws, with possible correlations over a range of cca $20-25$ days, and for magnitudes $M<4-5$. Such temporal patterns may be employed in seismic hazard and risk estimation, providing the data set is statistically significant. Unfortunately, the statistics is very poor, precisely for high-magnitude earthquakes, which are most interesting in this respect.

**Introduction.** Recently, there is a great deal of interest in statistical analysis of short-term seismic activity, with the general goal of revealing possible space-time and magnitude regular patterns of earthquake occurrence.[1]-[4] Needless to say, such information is of great relevance for seismic hazard and risk estimation. A short-term earthquake prediction model was recently put forward.[5] Many of such studies are focused on next-earthquake (or inter-occurrence, or waiting time) distributions, and revealed, for instance, universal scaling laws,[1] possibly of a limited validity,[2] or clustering, and time-decreasing sequences, governed by Omori-type power laws.[3, 4]

Though of a more restricted statistical size in comparison with these previous studies, we report herein upon similar statistical distributions for next earthquakes occurring in Vrancea, focusing especially on their temporal behaviour over short times. A general statistical analysis of the seismic activity in Vrancea has been reported previously.[5, 6]

**General considerations upon temporal distributions of earthquakes.** It is generally believed that earthquakes may conveniently be divided into regular earthquakes and earthquakes accompanying main seismic shocks, as aftershocks and foreshocks. The regular earthquakes are characterized by a mean recurrence time, depending on their energy (and magnitude), and their temporal distributions are governed by Poisson-like randomess processes, as for rare events. They exhibit the well-known Gutenberg-Richter magnitude distribution $\beta e^{-\beta M}$, which is well documented for various types of seismic activities, periods and regions. A typical value $\beta=1.17$ is provided for parameter $\beta$ by the point-like seismic focus model, though different values are also known, like, for instance, $\beta=1.89$ for Vrancea seismic fault-like region.[7] Similarly, data given in Ref. 8 indicates $\beta=1.38$.

Regular earthquakes represent a distinct seismic activity over long times. The seimic activity over short times is dominated, typically, by seisms accompanying main shocks. Their temporal distributions are power laws of Omori's type,[9, 10] like $1/\tau$, where $\tau$ is time measured with respect

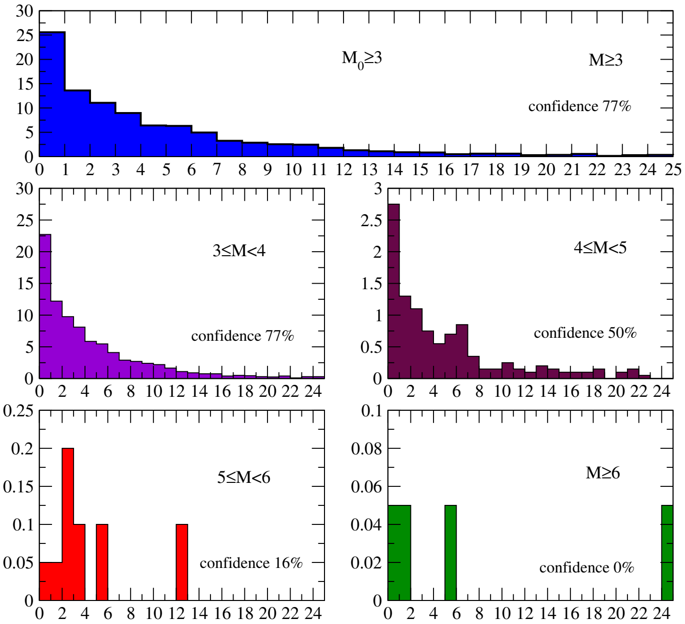

Figure 1: Time probability distribution $P(t)$ (upper panel) and time-magnitude probabilities $P(M,t)$ for the next Vrancea earthquake (probabilities are given in % on the coordinate axis and time is measured in days on the abscissa)

to the main shock, both in the future, as for aftershocks, and in the past, as for foreshocks. It is likely that the accompanying seismic activity is symmetrical under time reversal with respect to the main shock,[11]-[13] and the analysis of the foreshock activity may shed light upon the occurrence of the main shock, as a critical point within the theory of the self-organized criticality, for instance.[14]-[16] It was shown recently[7] that similar Omori's laws may hold for the difference in magnitude between the main shock and an aftershock (or a foreshock), or for the inverse of the seismic energy released in the accompanying seismic activity (which leads to a rate $d\varepsilon/d\tau \sim -1/\tau^2$ for the released energy $\varepsilon$, a relationship which seems to be supported by empirical data[10, 15]). Such Omori laws may arise from a self-replication phenomenon underwent by an original, generating distribution, which is of exponential type,[7] so that aftershocks and foreshocks are distributed by a similar Gutenberg-Richter law in magnitude (as suggested previously[17]), and, usually, the magnitude of the accompanying seisms decreases linearly with increasing time $\tau$. The generating, exponential distributions for the accompanying seismic activity have also been used to analyze Bath's law.[18]-[20]

However, Omori's law represents only a particular pattern in the short-term seismic activity accompanying a main seismic shock, the seismic activity over a short-time scale being more complex. First, the scarcity of data regarding aftershocks or foreshocks (*i.e.* the poor statistics) may render Omori's law of little practical use. On the other hand, it is still lacking a practical method of disentangling the truly accompanying seismic activity from other, regular seismic activity, without any relationship with the main shock, especially for the long tail of Omori's law. In addition, the short-time seismic activity may exhibit a more complex pattern, of a multiple-branch character. Indeed, a mainshock may produce aftershocks, according to Omori's law, whose magnitude decreases in time. It may sound reasonable to assume that each aftershock in such a sequence may itself be a main shock, which produces in turn its own aftershocks. It is worth noting that aftershocks magnitude depends on the mainshock's magnitude, so that a series of sub-branches is then conceivable to appear under such an assumption, removing thereby the differences between a main

| day | N=1998 | P(t)% | N(3<M<4)=1769 | P(3<M<4,t)% | N(4<M<5)=211 | P(4<M<5,t)% |
|---|---|---|---|---|---|---|
| 0 | 511 | 25.58 | 454 | 22.72 | 55 | 2.75 |
| 1 | 272 | 13.61 | 244 | 12.21 | 26 | 1.30 |
| 2 | 221 | 11.06 | 195 | 9.76 | 22 | 1.10 |
| 3 | 179 | 8.96 | 162 | 8.11 | 15 | 0.75 |
| 4 | 128 | 6.41 | 117 | 5.86 | 11 | 0.55 |

| N(5<M<6)=13 | P(5<M<6,t)% | N(6<M<8)=5 | P(6<M<8,t)% |
|---|---|---|---|
| 1 | 0.05 | 1 | 0.05 |
| 1 | 0.05 | 1 | 0.05 |
| 4 | 0.20 | 0 | 0.00 |
| 2 | 0.10 | 0 | 0.00 |
| 0 | 0.00 | 0 | 0.00 |

Table 1: Event distribution for Vrancea next-earthquake (1999 Vrancea earthquakes with (moment) magnitude $M > 3$, recorded between 1974 and 2004 (30 years) within $45° - 46°$N latitude and $26° - 27°$E longitude, Romanian Earthquake Catalogue[21])

shock and aftershocks. Similarly, an earthquake may not be viewed as only belonging to a certain branch of aftershocks, but pertaining also to a certain branch of foreshocks, corresponding to some forthcoming main shock. It is also conceivable that such foreshocks may in turn generate other branches of foreshocks, so that the whole picture obtained this way is that of various earthquakes occurring at any time with various magnitudes, *i.e.* the magnitude is decoupled, in fact, from time. The short-time seismic activity may then be described by a set of independent statistical variables, like, for instance, magnitude, time, location, etc. Such a decoupling is used in the short-term sequence-clustering prediction model proposed recently for California,[5] which introduces a probability $P(M,t,\mathbf{r})$ for earthquakes characterized by magnitude $M$, occurrence time $t$, and location $\mathbf{r}$, and write down this probability like a product $P(M,t,\mathbf{r}) \sim e^{-\beta M} \cdot 1/(t_c+t) \cdot 1/(r_c+r)^2$ of Gutenberg-Richter magnitude distribution and Omori-type distributions, where $t_c$ and $r_c$ are time and spatial cutoffs, respectively (for an isotropic spatial distribution). Such a probability law is fitted to empirical data, and employed for short-term prediction of daily rates of earthquakes.[5] For a particular earthquake sequence in evolution parameters may be updated in real time, and employed for the next-day prediction.

**Next-earthquake correlation functions.** In view of the complexity of the statistical behaviour of the short-term seismic activity, more general approaches may be more convenient. Such approaches should be of more practical relevance, without resorting too much to patterns established statistically on general sets of data. From the practical standpoint the most relevant question in short-term earthquake forecasting seems to be "what happens next?". The general approach described herein is focused on this main question. It is based on next-earthquake distributions employed recently for statistical analysis of short-term seismic activity.[1]-[4] Suppose that an earthquake occurs a time $t_0$ and the next one occurs at some time $t$ measured with respect to $t_0$. Then, we may define a distribution $P(t)$ of these next earthquakes, and determine it from a set of relevant statistical data. Once determined, it may be used for estimating the time probability of occurrence of the next earthquake, based on the principle "what happened will happens again".

Let the earthquakes be labelled by some generic parameter $x$, like magnitude, location, depth, etc. Then, we may distribute the next earthquakes with respect to $x$, and introduce the time probability distribution $P(x,t)$ of the next earthquake characterized by parameter $x$ occurring at time $t$. Of

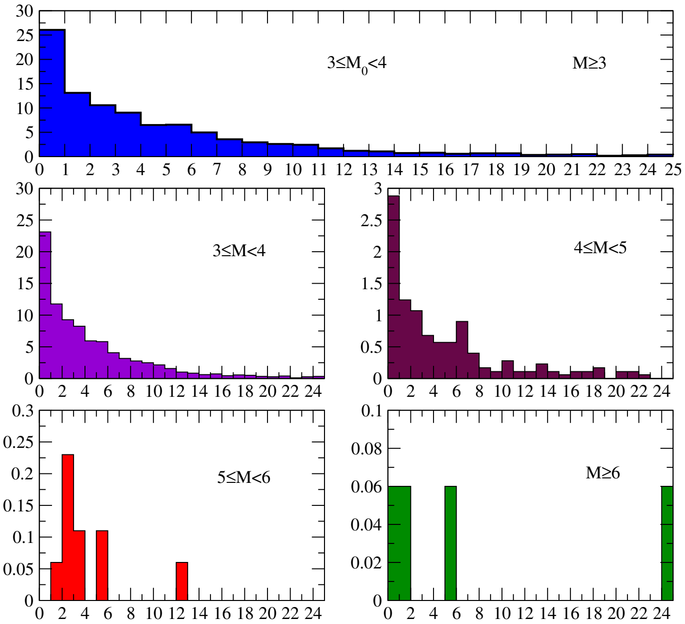

Figure 2: Vrancea next-earthquake time-magnitude probabilities $P(M, t \mid M_0)$ for magnitude $3 < M_0 < 4$ of the former earthquake (probabilities are given in % on the coordinate axis, and time is measured in days on the abscissa). The analysis is performed over a set of 1999 Vrancea earthquakes with (moment) magnitude $M > 3$ recorded between 1974 and 2004 (30 years) within $45° - 46°$N latitude and $26° - 27°$E longitude (Romanian Earthquake Catalogue[21])

course, $\int dx \cdot P(x, t) = P(t)$. Another distribution $P(x, t \mid x_0)$ may also be introduced with respect to the former earthquake labelled by parameter $x_0$, so that $\int dx_0 \cdot P(x, t \mid x_0) = P(x, t)$ and $\int dx \cdot P(x, t \mid x_0) = P(t \mid x_0)$, where the latter is the probability distribution of the next earthquake occurring at time $t$ providing the former is characterized by $x_0$.

The procedure may obviously be detailed, by introducing similarly the probability distributions $P(x, t \mid x_{01}, x_{02}, ...)$, or $P(x_1, x_2, ..., t \mid x_{01}, x_{02}, ...)$, which resemble the hierarchy of $n$-point correlation functions in statistical analysis. Characteristic scale time or size, or correlation range, could be identified from the statistical analysis of such functions, providing the statistical set of data is large enough, which may shed light on the statistical patterns of a seismic activity. Unfortunately, the statistics is rather poor, in general, precisely for those range of $x$ where estimation of seismic hazard and risk is most interesting, like, for instance, for $x$ corresponding to high values of magnitude $M$. Nevertheless, the next-earthquake approach is applied here to Vrancea earthquakes, in order to illustrate its predictive capabilities and limits.

**Brief characterization of Vrancea earthquakes.** Vrancea is a seismic region located approximately at 45.7°N latitude and 26.6°E longitude (Romania). The geographical and depth distributions of Vrancea seismic foci have been given elsewhere,[7] for 1999 earthquakes with (moment) magnitude $M > 3$ recorded between 1974 and 2004 (30 years) within $45° - 46°$N latitude and $26° - 27°$E longitude.[21] This basic data set is used for the statistical analysis reported here (magnitude accuracy $\Delta M = 0.1$). Vrancea exhibits mainly middle-depth earthquake ($\sim$ 80km to $\sim$ 150km), and occasionally a few crustal, or surface, earthquake. The seismically active focal zone of Vrancea exhibits a fault-like, near two-dimensional geometry. Strong earthquakes occur sometime in Vrancea, as, for instance, $M = 7.4$, March 4, 1977, depth 94km, or $M = 7.1$, August 30, 1986, depth 131km. Nine earthquakes with magnitude $M > 7$ have been recorded in Vrancea in the past two centuries.

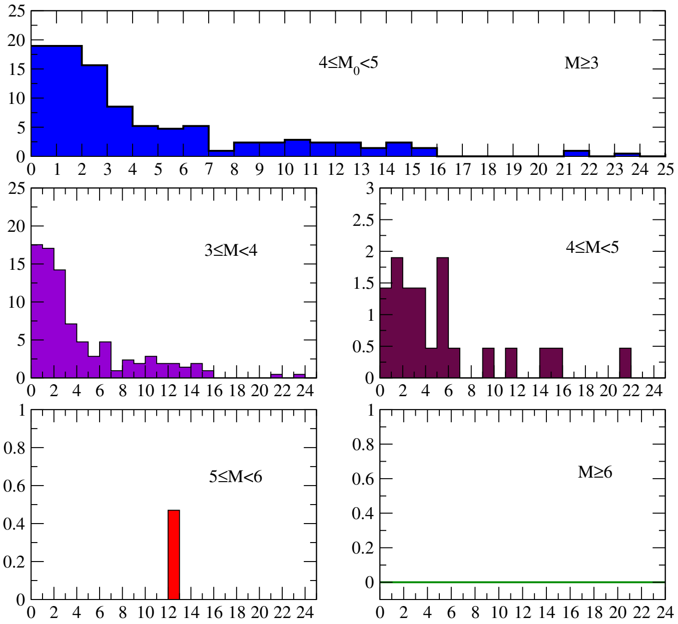

Figure 3: Vrancea next-earthquake time-magnitude probabilities $P(M, t \mid M_0)$ for magnitude $4 < M_0 < 5$ of the former earthquake (probabilities are given in % on the coordinate axis, and time is measured in days on the abscissa). The analysis is performed over a set of 1999 Vrancea earthquakes with (moment) magnitude $M > 3$ recorded between 1974 and 2004 (30 years) within $45° - 46°$N latitude and $26° - 27°$E longitude (Romanian Earthquake Catalogue[21])

The magnitude distribution of these Vrancea earthquakes has been analyzed by using the Gutenberg-Richter law.[7] As noted above, the parameter $\beta$ in these distributions has been found to acquire the value $\beta = 1.89$, and the seismicity rate $1/t_0 = 10^{4.21}$ per year, on average.

**Next-earthquake distributions analysis for Vrancea.** For the sake of simplicity we neglect here the geographical and depth distribution of Vrancea earthquakes, because, on one hand, Vrancea earthquakes do not exhibit a large variability with respect to location coordinates, and, on the other hand, their including may reduce considerably the size of the data set. Therefore, the generic parameter $x$ in the next-earthquake analysis procedure described above amounts to magnitude $M$. The next-earthquake time-magnitude probabilities $P(M, t)$ for Vrancea are shown in Fig. 1 for $3 < M < 4$, $4 < M < 5$, $5 < M < 6$ and $M > 6$ (all for $M_0 > 3$), while the cumulative distribution $P(t)$ (*i.e.* $P(M > 3, t \mid M_0 > 3)$) is also shown in the top panel in Fig. 1 (probability is given in % on the coordinate axis and time is measured in days on the abscissa). We note first that Vrancea next-earthquake distributions $P(M, t)$ exhibit a characteristic decrease in time, with the highest probability of next-earthquake occurrence in the same day as the reference earthquake, at least for small magnitudes ($M < 5$). The mean time for $P(t)$ is cca 5.89 days, and the variance $\sigma = 9.55$ days. Then, we note the maximum values of these probabilities $\sim 22.7\%$ for $3 < M < 4$, $\sim 2.75\%$ for $4 < M < 5$, while probability $P(M, t)$ vanishes practically for $M > 5$. It is also worth noting that $P(t)$ and $P(3 < M < 4, t)$ are similar, obeying, very likely, Omori-type power laws, at least for short times,[1]-[4] while the distributions become gradually irregular, exhibiting large fluctuations on increasing magnitude above $M = 4 - 5$. The statistics becomes extremely poor for higher magnitude ($M > 5$), as expected. A correlation time of cca $20 - 25$ days can be estimated, after which the probabilities decrease drastically (below 1%), as well as a size correlation of cca $M = 4 - 5$, above which the distributions acquire very small values, and are very irregular. The distribution of events is given in Table 1.

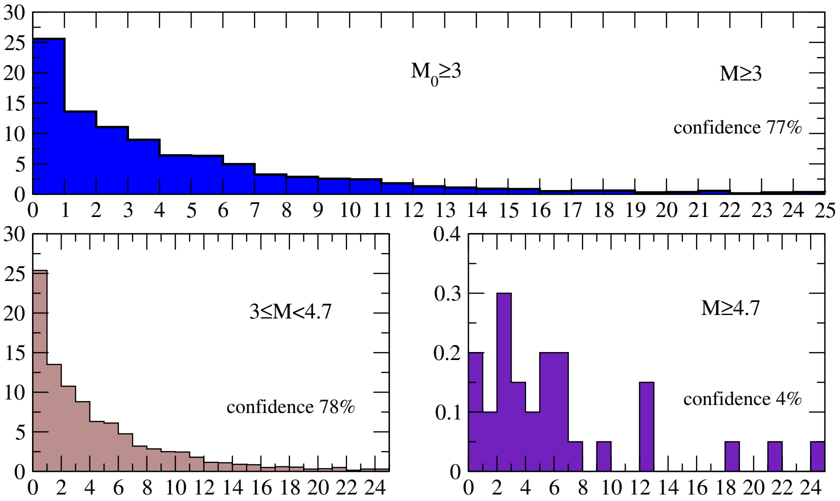

Figure 4: Vrancea next-earthquake time-magnitude probabilities $P(M, t \mid M_0 > 3)$ for magnitudes $M > 3$, $3 < M < 4.7$ and $M > 4.7$ (probabilities are given in % on the coordinate axis, and time is measured in days on the abscissa). The analysis is performed over a set of 1999 Vrancea earthquakes with (moment) magnitude $M > 3$ recorded between 1974 and 2004 (30 years) within $45° - 46°$N latitude and $26° - 27°$E longitude (Romanian Earthquake Catalogue[21])

The null hypothesis was tested on these distributions, by comparing the results of the first half of data with those derived from the second half of data. The result of this comparison was estimated in terms of relative mean square deviation, and it is indicated as the confidence level in Fig. 1. It can be seen in Fig. 1 that this confidence level is cca 77% for $P(t)$ and $P(3 < M < 4, t)$, and decreases considerably for $M > 5$.

The above analysis does not include the distribution with respect to the magitude $M_0$ of the former earthquake. When included, we obtain the time-magnitude conditioned probabilities (or two-point correlation functions) $P(M, t \mid M_0)$, as shown in Figs. 2 and 3 for $3 < M_0 < 4$ and $4 < M_0 < 5$, respectively. The corresponding cumulative distributions $P(t \mid M_0)$ are also shown in the top panel in Figs. 2 and 3 (for $M > 3$). The first observation is that distributing the time-magnitude events with respect to the former earthquake magnitude $M_0$ does not change practically the characteristic time-decreasing behaviour of the next-earthquake activity, at least for small magnitudes. It can be seen in Fig. 1 and Fig. 2 that $P(M, t)$ and $P(M, t \mid 3 < M_0 < 4)$ are very similar, while considerable differences appear for $P(M, t \mid 4 < M_0 < 5)$, even for small magnitudes $3 < M < 4$. This reflects again the size correlation $M = 4 - 5$, and makes useless the estimation of the confidence levels for higher magnitudes, as the corresponding distributions are affected by large fluctuations. Unfortunately, higher-order correlation functions (as well as higher-magnitude analysis, or sharpening the magnitude gap $\Delta M = 1$) reduce considerably the statistical set, thus exhibiting a poor confidence, and being practically irrelevant.

Time-magnitude next-earthquake probabilities $P(M, t \mid M_0 > 3)$ for Vrancea are shown in Fig. 4 for $3 < M < 4.7$ and $M > 4.7$. The threshold $M = 4.7$ is chosen since there is a partial consensus that a typical Vrancea earthquake is felt in Bucharest for magnitude above $M = 4.7$. The overall probability for $M > 4.7$ is very small (less than 2%), and the curve $P(M > 4.7, t \mid M_0 > 3)$ shown in Fig. 4 is very irregular. There are 35 Vrancea earthquakes with $M > 4.7$ in the whole data set, most of them placed in the first half of data, which leads to a very low confidence level (4%).

**Conclusion.** A new approach to short-term earthquake analysis is used here for Vrancea seismic activity, as based on the statistical analysis of the next-earthquake distributions. The main distribution function is the time-magnitude probability $P(M,t \mid M_0)$ of the next earthquake of magnitude $M$ occurring at time $t$ elapsed from the occurrence of the former with magnitude $M_0$. It may be viewed as a two-point correlation function, or a conditioned probability. This distribution is obtained by analyzing 1999 earthquakes with (moment) magnitude $M > 3$ recorded in Vrancea over the last 30 years (since 1974 to 2004). The result shows a characteristic time-decreasing behaviour of the occurrence of the next earthquake, at least for small magnitueds. Correlations are identified for lower-magnitude earthquakes ($M, M_0 < 4-5$), extending roughly over $20-25$ days. Statistics becomes extremely poor for strong earthquakes ($M > 5$), preventing thus any confident estimation.

The result of the present report is rather negative, in the sense that employing the next-earthquake probability distributions for estimating the short-term seismic hazard and risk for Vrancea earthquakes of interest (*i.e.* earthquakes of higher magnitude) is of little practical use, as due to the poor statistics. However, the approach may turn out to be useful for those seismic activities, periods and regions of interest, characterized by rich statistics (as, for instance, crustal, or surface, moderate earthquakes).

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