Journal of Theoretical Physics

Founded and Edited by M. Apostol **112** (2005)

ISSN 1453-4428

# Scaling and universal temporal distribution of nearest-neighbours pairs of earthquakes

**apoma Laboratory**
Magurele-Bucharest MGg-6, POBox MG-35, Romania
email: apoma@theory.nipne.ro

**Abstract**

The universal function introduced recently for the temporal distribution of nearest-neighbours pairs of earthquakes (see, for instance, A. Corral, Phys. Rev. Lett. **92** 108501 (2004)) is derived by general scaling arguments and seismicity characteristics of earthquakes distributions and focal mechanisms. An application is made to the temporal distribution of pairs for Vrancea earthquakes.

PACS: 89.75.Da; 91.30.Dk; 64.60.Ht; 05.65.+b

Temporal distribution of earthquakes is of particular importance for assessing seismic risk and hazard. In the absence of more specific knowledge regarding earthquake generating mechanisms, statistical analysis is employed in order to detect possible regular patterns in their spatial, temporal and magnitude distributions.[1]-[4] Conventionally, the earthquakes are divided into regular seisms, like main shocks, characterized by a mean recurrence time and quasi-randomly distributed in time by Poisson-like distributions,[5] and accompanying seims, like aftershocks and foreshocks, distributed in time according to Omori's law.[6] Recently, it was becoming increasingly apparent that the picture is more complex. It was recognized, at least for small and moderate earthquakes, that their magnitude and occurrence time are distinct, independent statistical variables.[7] In addition, correlations effects of various sorts, including clustering[8, 9] or self-organized seismic criticality,[10]-[13] seem to be present in statistical distributions of earthquakes, beside random occurrence.

In this context, the temporal pair distribution $D(\tau)$ of nearest-neighbouring earthquakes[14]-[16] acquired recently a pre-eminence in statistical studies of earthquakes. This function is also known as the recurrence, or waiting time distribution, or next-earthquake distribution. It is defined by

$$D(\tau) = \frac{dN}{Nd\tau} = \frac{1}{N}\sum_i \delta(t_{i+1} - t_i - \tau) \ , \tag{1}$$

where $N$ is the total number of earthquakes and $t_i$ denotes the occurrence time of the $i$-th earthquake in the temporal series. Usually, $N$ represents the number $N_{ex}$ of earthquakes with magnitude $M$ greater than a certain cutoff magnitude $M_c$, and it is given by the excedence rate (or recurrence law)

$$N_{ex}/T = t_0^{-1} e^{-\beta M_c} \ , \tag{2}$$

where $T$ is the time interval spanned by the total number $N_0$ of earthquakes (*i.e.* with magnitude greater than zero), $t_0^{-1} = N_0/T$ is the seismicity rate, and $\beta$ is the slope parameter of the recurrence law $\ln(N_{ex}/T) = -\ln t_0 - \beta M$.

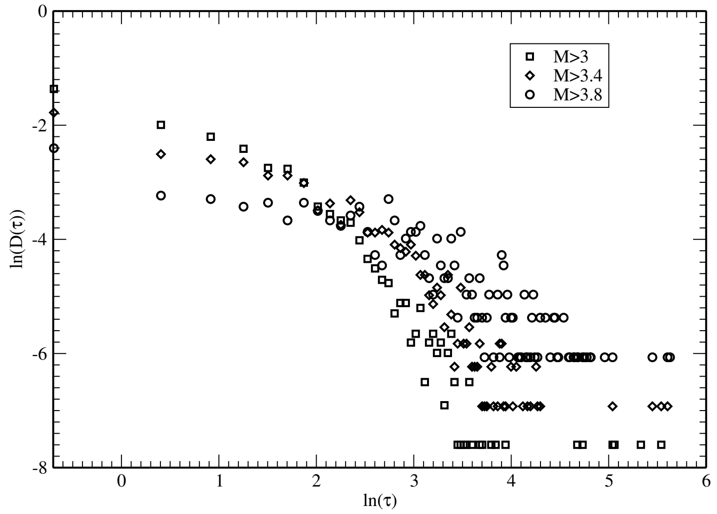


Figure 1: Pair distribution *vs* time (measured in days) for various cutoff magnitudes ($M_c = 3$, 3.4, 3.8) for 1999 earthquakes recorded in Vrancea between 1974 and 2004[20]

The seismicity rate and the slope parameter $\beta$ are well documented for a variety of regions, time intervals and ranges of magnitude. For instance, $\beta = 1.38$ and $-\ln t_0 = 12.65$ (where $t_0$is measured in years) for a world wide analysis of earthquakes with magnitude $5.8 < M < 7.3$.[17] Similarly, data for Southern California[18] indicate $\beta = 2.3$ and $-\ln t_0 = 17.25$. Recently, an analysis was made[19] for 1999 earthquakes with magnitude $M > M_c = 3$ recorded in Vrancea between 1974 and 2004,[20] which led to average values $\beta = 1.89$ and $-\ln t_0 = 9.68$ (the fit to the recurrence law given above for Vrancea indicates $\beta = 1.76$ and $-\ln t_0 = 8.99$ with an error 12%; $t_0$ is measured in years).

Recently, a model of seismic focus was put forward,[21] which relates the accumulating time $t$ and the seismic energy $E$ by $t \sim E^r$, where $r$ is a parameter depending on the geometry of the seismic focus and the mechanism of accumulating seismic energy. For instance, for a localized, point-like, focus and a uniform mechanism of energy accumulation the parameter $r$ acquires the value $r = 1/3$. It turns out that the parameter $r$ contributes to the exponent of the Gutenberg-Richter magnitude distribution, through $\beta = br$, where $b = 3.5$ is the coefficient in the well-known Gutenberg-Richter energy-magnitude relationship. For $r = 1/3$ we obtain $\beta = 1.17$, in agreement with $\beta = 1.38$ given above (corresponding to $r = 0.39$). The Southern California value $\beta = 2.3$ corresponds to $r = 0.66$. Data for Vrancea indicate $r = 0.54$ ($\beta = 1.89$), or $r = 0.50$ ($\beta = 1.76$).[19]

The seismicity parameters $t_0^{-1}$(seismicity rate), $\beta$ (slope parameter in the recurrence law) and $r$ (related to the mechanism of accumulating energy in the seismic focus) are relevant for the form of the pair distributon function $D(\tau)$ given by (1). Indeed, we note first that function $D(\tau)$ must depend on a dimensionless variable $R\tau$, where $R^{-1}$ is a characteristic time scale. Since the only time scale in the problem is the excedence rate (2) it follows that $R = t_0^{-1}e^{-\beta M_c}$. For reasons of normalization the pair distribution can therefore be written as $D(\tau) = Rf(R\tau)$, where $f$ is a function which remains to be determined. The scaling equation $D(\tau) = Rf(R\tau)$ implies that on changing the cutoff parameter $R$, as, for instance, $R \rightarrow pR$, where $p = e^{-\beta(M_c'-M_c)}$, corresponding to a change in the cutoff magnitude $M_c \rightarrow M_c'$, the pair distribution changes according to $D(\tau) \rightarrow \widetilde{D}(\tau) = pD(p\tau)$. It was shown recently[22] that the general form of the solution of such a scaling equation is a superposition of power-law elementary solutions for the Laplace transform, which may lead, for instance, to $D(\tau) \sim (1/\tau^{1-\alpha})e^{-\tau}$, where $0 < \alpha < 1$. On the other hand, if the accumulating energy $E$, in time-energy relationship $t \sim E^r$, is released in time

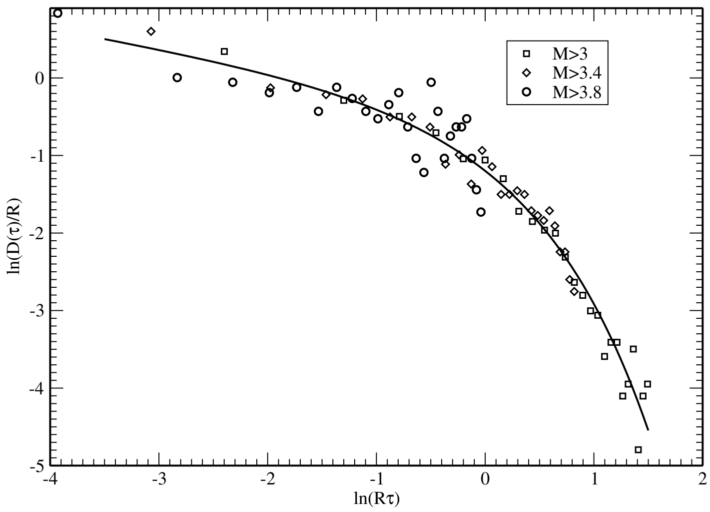


Figure 2: Rescaled pair distributions and the fit provided by equation (3) (solid curve, $r = 0.25$, $B = 1./17$, $C = 0.71$) for 1999 earthquakes recorded in Vrancea between 1974 and 2004[20]

$\tau$, then its releasing rate leads to $E \sim 1/\tau$, according to Omori's law.[6, 12, 23] For small values of $\tau$, the pair distribution $D(\tau)$ is proportional to time $t$, *i.e.* $D(\tau) \sim t \sim E^r \sim 1/\tau^r \rightarrow 1/(R\tau)^r$, which corresponds to a power law with the exponent equal to the parameter $r$. This behaviour indicates correlations in pair distribution, in agreement with clustering effects.[8] For large values of $\tau$ it is natural to expect an exponential behavior $\sim e^{-R\tau/B}$, in agreement with an uncorrelated, quasi-random, Poisson-like distribution of pairs.[5, 24] The parameter $B$ in the exponential above may originate in the exact relationship between the accumulating time and energy, which is $1 + t/t_0 = (1 + E/E_0)^r$, where $E_0$ is the threshold energy in the Gutenberg-Richter energy-magnitude law, which leads to an excedence rate $N_{ex}/T = t_0^{-1}(1 + e^{bM})^{-r}$, in contrast with the simplified form $N_{ex}/T = t_0^{-1}e^{-\beta M}$ (where $\beta = br$).[21] The difference between the two relationships can be seen for small values of $t$, *i.e.* large values of $\tau$, which means vanishing magnitudes. Indeed, the exact relationship given above yields a correction factor of the order $\sim 2^r$, which amounts to 1.26, for instance, for $r = 1/3$. This correction factor may account for the value of the parameter $B$ in the exponential given above, which means that the cutoff parameter $R$ is larger, in fact, than its simplified value $R = t_0^{-1}e^{-\beta M_c}$.

Therefore, we may suggest that the pair distribution (1) can be written as

$$D(\tau) = CR \cdot \frac{1}{(R\tau)^r} \cdot e^{-R\tau/B} \quad , \tag{3}$$

such that the normalization constant $C$ satisfies $CB^{1-r}\Gamma(1 - r) = 1$, where $\Gamma$is Euler's gamma function. It is worth noting that exponent $r$ is a fitting parameter for the class of functions given by equation (3).

The universal function given by (3) has been established recently[25]-[27] for $C = 1/2$, $B = 1.58$ and $r = 0.33$, by an extensive analysis of earthquakes recorded in a large variety of world wide regions, spanning various time intervals and magnitude ranges. It was also discussed recently in connection with correlation effects.[28] The exponent $r = 0.33$ corresponds to rather moderate values $R\tau < 1$, *i..e.* for time $\tau \leq R^{-1}$ comparable with the mean seismicity rate $R^{-1}$, in agreement with moderate and strong earthquakes statistics, and possibly suggesting a generic model of localized, point-like seismic focus. In the limit $R\tau \rightarrow 0$ the exponent $r$ in (3) exhibits a certain

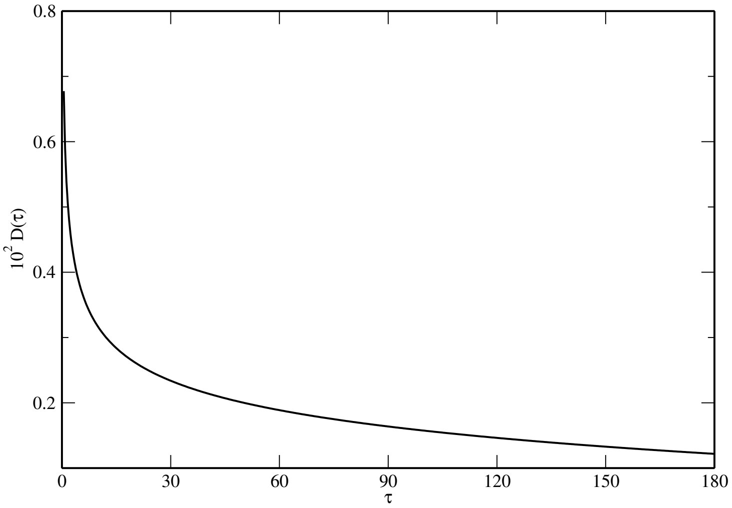


Figure 3: Temporal distribution for successive pairs of earthquakes with magnitude $M > 5$ as estimated by equation (3) for Vrancea ($r = 0.25$, $B = 1.17$, $C = 0.71$; time $\tau$ on the abscissa is measured in days)

variability, acquiring values that are close to the values obtained from the fit of the recurrence law (2). For instance, $r = 0.57$ for Vrancea earthquakes in this limit, in agreement with $r = 0.54$ and $r = 0.50$ indicated above. The pair distribution undergoes a non-stationary criticality[25, 26] in the limit $R\tau \to 0$, where aftershocks tend to increase the value of the exponent $r$ toward Omori's law exponent $r = 1$.

The pair distribution $D(\tau)$ given by (1) has been analyzed for 1999 earthquakes with magnitude $M > M_c = 3$ recorded in Vrancea between 1974 and 2004.[19, 20] The function $D(\tau)$ *vs* $\tau$ is shown in Fig. 1 (on a logarithmic scale) for various cutoff magnitudes $M_c$. It exhibits a rather large dispersion, especialy for greater cutoff magnitudes ($M_c = 3.8$) and large values of $R\tau$. The data collapse on rescaling with cutoff parameter $R$, as shown in Fig. 2, except for the limit $R\tau \to 0$. The fit employing the universal function given by (3) gives $r = 0.25$, $B = 1.17$ and $C = 0.71$ (with an error 13%), in fair agreement with the fitting parameters given in Refs. 25 and 26. The poor statistics for Vrancea, especially for earthquakes with higher magnitude, prevents a more reliable analysis.

One of the most interesting applications of the pair distribution is the computation of the next-earthquake probability for earthquakes with magnitude greater than $M$, which, according to its definition (1) and (2), is given by $D(\tau)$, where the cutoff parameter $R$ in equation (3) is given by $R = t_0^{-1}e^{-\beta M}$. This probability is shown in Fig. 3 for Vrancea, for $M = 5$ and $r = 0.25$, $B = 1.17$, $C = 0.71$. It corresponds to $R = 1.6 \times 10^{-3}$ day$^{-1}$, *i.e.* 18 earthquakes with $M > 5$ for 30 years, which differs slightly from $R = t_0^{-1}e^{-\beta M} = 5 \times 10^{-4}$day$^{-1}$ obtained by fitting the recurrence law for all earthquakes with magnitude $M > M_c = 3$ ($-\ln t_0 = 8.99$ and $\beta = 1.76$). By making use of (3), it may be estimated, for instance, that the probability of having two earthquakes in the same day, in Vrancea, with magnitude greater than $M = 5$ is $\sim 0.8\%$. In general, the probability of having the next-earthquake in time $\tau$ is given by $\int_0^\tau d\tau' \cdot D(\tau') = CB^{1-r}\gamma(1-r, R\tau/B)$, where $\gamma$ is Euler's incomplete gamma function.

In conclusion, the temporal pair distribution of nearest-neighbouring earthquakes is derived herein by general scale arguments and seismicity parameters, in agreement with the universal function established recently by statistical analysis of extensive empirical data sets.[25]-[27] The results are applied to Vrancea earthquakes, especially in connection with assessing short-term seismicity.

# References

[1] B. Gutenberg and C. F. Richter, *Seismicity of the Earth*, Hafner Publishing Co, New York (1965)

[2] Y. Y. Kagan and L. Knopoff, Geophys. J. R. Astron. Soc. **62** 303 (1980)

[3] Y. Y. Kagan, Physica (Amsterdam) **D77** 160 (1994)

[4] D. L. Turcotte, *Fractals and Chaos in Geology and Geophysics*, Cambridge University Press, 2nd ed, Cambridge (1997)

[5] J. K. Gardner and L. Knopoff, Bull. Seismol. Soc. Am. **64** 1363 (1974)

[6] T. Utsu, Geophys. Mag. **30** 521 (1961)

[7] M. C. Gerstenberger, S. Wiemer, L. M. Jones and P. A. Reasenberg, Nature, **435** 328 (2005)

[8] Y. Y. Kagan and D. D. Jackson, Geophys. J. Int. **104** 117 (1991)

[9] P. M. Davis, D. D. Jackson and Y. Y. Kagan, Bull. Seismol. Soc. Am. **79** 1439 (1989)

[10] D. Sornette and C. G. Sammis, J. Physique I **5** 607 (1995)

[11] D. Sornette, Phys. Reps. **297** 239 (1998)

[12] D. Sornette, C. Vanneste and L. Knopoff, Phys. Rev. **A45** 8351 (1992)

[13] A. Helmstetter and D. Sornette, J. Geophys. Res. **107** 2237 (doi:10.1029/2001JB001580) (2002)

[14] D. Sornette and L. Knopoff, Bul. Seismol. Soc. Am. **87** 789 (1997)

[15] J.-H. Wang and C.-H. Kuo, J. Seismol. **2** 351 (1998)

[16] R. F. Smalley, J.-L. Chatelain, D. L. Turcotte and R. Prevot, Bull. Seismol. Soc. Am. **77** 1368 (1987)

[17] K. E. Bullen, *An Introduction to the Theory of Seismology*, Cambridge University Press, London (1963)

[18] P. Bak, K. Christensen, L. Danon and T. Scanlon, Phys. Rev. Lett. **88** 178501 (2002)

[19] B.-F. Apostol, J. Theor. Phys. **108** 1 (2005)

[20] Romanian Earthquake Catalogue, 2005 (updated), Institute for Earth's Physics, Magurele-Bucharest, Romania, http:www.infp.ro/eqsinfo.php/ (continuously updated Catalogue first published by M. C. Oncescu, V. I. Rizescu and M. Popa, 1999, in *Vrancea Earthquakes and Tectonics, Hazard and Mitigation*, eds. F. Wenzel and D. Lungu and co-ed. O. Novak, pp.43-47, Kluwer, Dordrecht, Netherlands.

[21] B.-F. Apostol, J. Theor. Phys. **105** 1 (2005)

[22] **apoma Laboratory**, J. Theor.Phys. **111** 1 (2005)

[23] B.-F. Apostol, J. Theor. Phys. **102** 1 (2005)

[24] G. Molchan, Pure Appl. Geophys. **162** 1135 (2005)

[25] A. Corral, Phys. Rev. Lett. **92** 108501 (2004)

[26] A. Corral, Phys. Rev. **E71** 017101 (2005)

[27] A. Corral, Phys. Rev. Lett. **95** 028501 (2005)

[28] M. Lindman, K. Jonsdottir, R. Roberts, B. Lund, R. Bodvarsson, Phys. Rev. Lett. **94** 108501 (2005)