

**Scaling and universal power laws in time series of seismic events**
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Magurele-Bucharest MG-6, POBox MG-35, Romania**Abstract**

It is shown that functions obeying scale invariance can be conveniently represented as linear superpositions of power laws. Seismicity characteristics of earthquake distributions and focal mechanisms are employed to derive such a power law for the temporal distribution of nearest-neighbours pairs of earthquakes in the limit of short times. The universal function introduced recently for such pair distribution (A. Corral, Phys. Rev. Lett. **92** 108501 (2004)) is obtained and an application is made for Vrancea earthquakes.

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Temporal distribution of earthquakes is of particular importance for assessing seismic risk and hazard. In the absence of more specific knowledge regarding earthquake generating mechanisms, statistical analysis is employed in order to detect possible regular patterns in their spatial, temporal and magnitude distributions. It is widely agreed that temporal series of earthquakes exhibit two main patterns. First, they may appear as "regular" earthquakes,[1]-[3] characterized by a mean recurrence time  $t_r$ , and secondly each main seismic shock may be accompanied by foreshocks and aftershocks, as described by Omori's law.[4]-[9] According to this wisdom, earthquakes are conventionally divided into "regular" earthquakes, quasi-randomly distributed in time by Poisson-like distributions, and accompanying earthquakes governed by Omori's law. The basic equation in describing earthquakes is the Gutenberg-Richter equation  $E = E_0 \exp(bM)$ , where  $E$  is the energy released in an earthquake,  $E_0$  is a threshold energy and  $M$  is the earthquake's magnitude (usually defined as seismic moment).[10]-[12] Parameter  $b$  has the value  $b = 3.5$ . [12]

A model has been put forward recently[13] for accumulating seismic energy in a localized focus, which relates the accumulating time  $t$  to energy  $E$  through the relationship  $t/t_0 = (E/E_0)^r$ , where  $t_0$  is a threshold time and  $E \gg E_0$ ,  $t \gg t_0$ . The exponent  $r$  is a parameter accounting for the particular non-uniformity of the critical focal zone and the particular mechanism of accumulating energy. It was shown on geometrical grounds that parameter  $r$  acquires the value  $r = 1/3$  for a uniform mechanism of accumulating energy in a point-like localized focus, or  $r = 1/2$  for a fault-planes critical zone with a geometry close to a two-dimensional one. In general, the inverse  $1/r$  may be viewed as a fractal dimension[14] in the relationship  $E/E_0 \sim (R/R_0)^{1/r}$ , where  $R$  and  $R_0$  are corresponding scale lengths characterizing the focal zone. The particular values of the parameter  $r$ , as depending on the seismic region and the set of data analyzed, can be obtained by statistical fits to data.

Making use of this model, and identifying the accumulation time with the mean recurrence time, the earthquakes distribution in energy  $rE_0^r/E^{1+r}$  has been derived, as well as the well-known

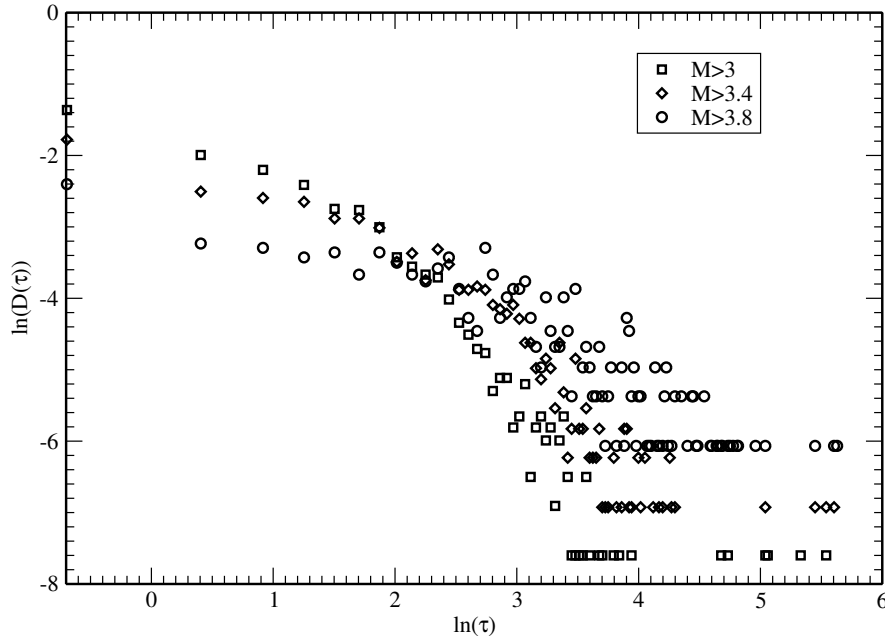


Figure 1: Pair distribution *vs* time (measured in days) for various cutoff magnitudes ( $M_c = 3, 3.4, 3.8$ ) for 1999 ( $M_c = 3$ ) earthquakes recorded in Vrancea between 1974 and 2004.[37]

magnitude distribution  $P(M) = \beta \exp(-\beta M)$ , where  $\beta = br$  ( $\beta = 1.17$  for  $r = 1/3$ ). It follows immediately the well-known Gutenberg-Richter distribution law[15]-[17]  $\ln(\Delta N/N_0 \Delta M) = \ln \beta - \beta M$ , and the Gutenberg-Richter cumulative distribution  $\ln(N_{ex}/T) = -\ln t_0 - \beta M$  (recurrence law, or exceedance rate), where  $\Delta N$  is the number of earthquakes with magnitude  $M$  to  $M + \Delta M$  out of a total of  $N_0$  earthquakes occurring in (a long) time  $T$ . It turns out that  $1/t_0 = N_0/T$  is the seismicity rate. The Gutenberg-Richter laws given above are well documented for a variety of regions, time intervals and magnitudes. For instance, a worldwide analysis for  $5.8 < M < 7.3$  indicates  $r = 0.39$  ( $\beta = 1.38$ , compares with  $\beta = 1.17$  corresponding to  $r = 1/3$ ; and  $-\ln t_0 = 12.65$  for  $t_0$  measured in years).[12] Data for Southern California[18] seem to indicate  $r = 0.66$  ( $\beta = 2.3$ ; and  $-\ln t_0 = 17.25$  for  $t_0$  in years). Similarly, a recent analysis for 1999 earthquakes with magnitude  $M > 3$  recorded in Vrancea between 1974 and 2004[13] indicates the average values  $r = 0.54$  ( $\beta = 1.89$ ; and  $-\ln t_0 = 9.68$  for  $t_0$  in years; the fit to the Gutenberg-Richter recurrence law gives  $\beta = 1.76$  and  $-\ln t_0 = 8.99$ ; in general, there are appreciable deviations from these linear laws for extreme values of the magnitude). In addition, as it follows from the distributions given above, the model of accumulation of seismic energy gives a mean recurrence time  $t_r = (t_0/\beta \Delta M) \exp(\beta M)$  for earthquakes with magnitude  $M$  to  $M + \Delta M$ , which enter the Poissonian distribution[19]  $(1/t_r) \exp(-t/t_r)$  for such earthquakes characterized by a (fixed) mean recurrence time  $t_r$ .

The accompanying seismic activity of foreshocks and aftershocks enjoyed extensive investigations, in special in connection with the critical-point theory and self-organized criticality.[20]-[22] It was shown recently that such associated earthquakes may appear by a self-replication mechanism with leads to Omori's law.[23] According to Omori's law, the rate of accompanying earthquakes is given by  $dn/d\tau \sim 1/|\tau|$ , and the released energy goes like  $dE/d|\tau| \sim -1/\tau^2$  (or  $E \sim 1/|\tau|$ ), where  $\tau$  denotes time measured with respect to the occurrence of the main shock ( $\tau < 0$  for foreshocks and  $\tau > 0$  for aftershocks). Both relationships seem to be supported by empirical evidence.[5, 7]

Recently, it was becoming increasingly apparent that the picture is more complex. It was recognized, at least for small and moderate earthquakes, that their magnitude and occurrence time

are distinct, independent statistical variables.[24] In addition, correlations effects of various sorts, including clustering[25, 26] or self-organized seismic criticality,[7],[20],[27],[28] seem to be present in statistical distributions of earthquakes, beside random occurrence. In this context, the temporal pair distribution  $D(\tau)$  of nearest-neighbouring earthquakes[29]-[31] received recently a great deal of attention in statistical studies of earthquakes.[32]-[35] In general, the probability density of  $N$  serial events denoted by  $i$  and occurring at time  $t_i$  can be written as  $\rho(t) = N^{-1} \sum_i \delta(t_i - t)$ . Similarly, the pair distribution of nearest-neighbours separated by time  $\tau$  is given

$$D(\tau) = \frac{dN}{Nd\tau} = \frac{1}{N} \sum_i \delta(t_{i+1} - t_i - \tau) . \quad (1)$$

This function is also known as the recurrence, or waiting time distribution, or next-earthquake distribution. Usually,  $N$  represents the number  $N_{ex}$  of earthquakes with magnitude  $M$  greater than a certain cutoff magnitude  $M_c$ . According to the Gutenberg-Richter recurrence law given above it can be written as  $N/T = t_0^{-1} e^{-\beta M_c}$ . Function  $D(\tau)$  must depend on a dimensionless variable  $R\tau$ , where  $R^{-1}$  is a characteristic scale time. Since the only characteristic time in the problem is the inverse of the exceedence rate it follows that  $R = t_0^{-1} e^{-\beta M_c}$ . The pair distribution can therefore be written as  $D(\tau) = Rf(R\tau)$ , where  $f$  is a function which remains to be determined. It was recently claimed[33, 34] that the pair distribution of nearest-neighbouring earthquakes can be written in this form, where  $f$  is a universal function (in the sense that it does not depend on the cutoff parameter  $M_c$ ). It can be said that functions  $D(\tau)$  which satisfy the relationship  $D(\tau) = Rf(R\tau)$ , where  $f(\tau)$  is a universal function, are scale-invariant. Such a function  $D(\tau)$  can also be represented as  $R^{-1}D(R^{-1}\tau) = f(\tau)$ , and, since  $f(\tau)$  does not depend on  $R$ , it follows that function  $D(R^{-1}\tau)$  is given by a function  $F(R^{-1}, R^{-1}\tau) = D(R^{-1}\tau) = Rf(\tau)$  of two variables, which satisfy the equation

$$F + x\partial F/\partial x + u\partial F/\partial u = 0 , \quad (2)$$

where  $x = R^{-1}$  and  $u = R^{-1}\tau$ . The elementary solutions of this equation are  $F(x, u) = x^{-1+\alpha}u^{-\alpha}$ , *i.e.*  $D(\tau) = R(R\tau)^{-\alpha}$  and universal power laws  $f(\tau) = \tau^{-\alpha}$ , where  $\alpha$  is any (real) exponent. The general solution is a superposition of such power laws.

According to the model of accumulation of seismic energy, if the accumulating energy  $E$ , in time-energy relationship  $t \sim E^r$  given above, is released in time  $\tau$ , then its releasing rate leads to  $E \sim 1/\tau$ , according to Omori's law. On the other hand, for small values of  $\tau$ , the pair distribution  $D(\tau)$  is proportional to time  $t$ , *i.e.*  $D(\tau) \sim t \sim E^r \sim 1/\tau^r$ , which leads to the power law  $D(\tau) \sim 1/(R\tau)^r$  for  $\tau \rightarrow 0$ , with the exponent  $\alpha$  equal to the parameter  $r$ . This behaviour indicates correlations in pair distribution, in agreement with clustering effects.[25] On the other hand, for large values of  $\tau$  it is natural to expect an exponential behavior  $D(\tau) \sim e^{-R\tau/B}$ , in agreement with an uncorrelated, quasi-random, Poisson-like distribution of pairs.[19, 36] The parameter  $B$  in the exponential above may originate in the corrections to the simplified relationship  $t \sim E^r$  for small values of  $t$  and  $E$ . Indeed, if the cutoff parameters  $t_0$  and  $E_0$  are introduced explicitly by  $t \rightarrow t + t_0$  and  $E \rightarrow E + E_0$ , the above relationship reads  $1 + t/t_0 = (1 + E/E_0)^r$ , which leads to an exceedence rate  $N_{ex}/T = t_0^{-1}(1 + e^{bM})^{-r}$ , in contrast with the simplified form  $N_{ex}/T = t_0^{-1}e^{-\beta M}$  (where  $\beta = br$ ). The difference between the two relationships can be seen for small values of  $t$ , *i.e.* large values of  $\tau$  (where parameter  $B$  is effective in (3)), which means vanishing magnitudes. Indeed, the relationship given above yields a correction factor of the order  $\sim 2^r$  for  $M = 0$ , which amounts to 1.26, for instance, for  $r = 1/3$ . This correction factor may account for the value of the parameter  $B$  in the exponential given above, which means that the cutoff parameter  $R = t_0^{-1}(1 + e^{bM_c})^{-r}$  is larger, in fact, than its simplified value  $R = t_0^{-1}e^{-\beta M_c}$ .

Taking into account the asymptotic behaviour of  $D(\tau)$  given above, both for  $\tau \rightarrow 0$  and  $\tau \rightarrow \infty$ ,

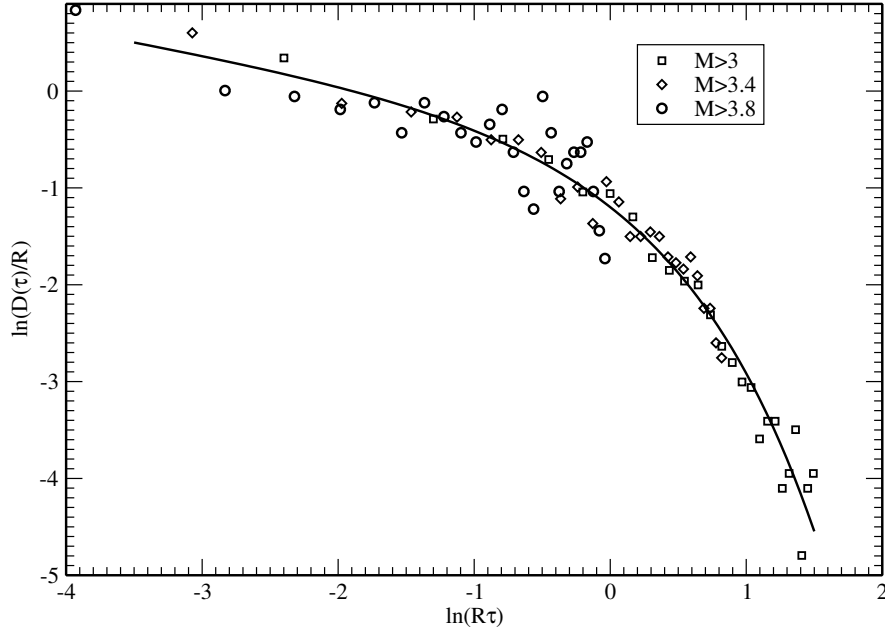


Figure 2: Rescaled pair distributions and the fit provided by equation (3) (solid curve,  $\alpha = 0.25$ ,  $B = 1./17$ ,  $C = 0.71$ ) for 1999 earthquakes recorded in Vrancea between 1974 and 2004 ( $M > M_c = 3$ ).[37]

we may suggest that the pair distribution (1) can be written as

$$D(\tau) = CR \cdot \frac{1}{(R\tau)^\alpha} \cdot e^{-R\tau/B} , \quad (3)$$

such that the normalization constant  $C$  satisfies  $CB^{1-\alpha}\Gamma(1-\alpha) = 1$ , where  $\Gamma$  is Euler's gamma function, as it was pointed out recently.[32]-[34] The exponent  $\alpha$  is tentatively identified here with the parameter  $r$ , and is viewed as a fitting parameter for the class of functions given by equation (3).

The values  $C = 1/2$ ,  $B = 1.58$  and  $\alpha = 0.33$  in (3) have been established recently,[32]-[34] by an extensive analysis of earthquakes recorded in a large variety of world wide regions, spanning various time intervals and magnitude ranges. The universal function in (3) has also been discussed recently in connection with correlation effects.[35] The exponent  $\alpha = 0.33$  corresponds to rather moderate values  $R\tau < 1$ , *i.e.* for time  $\tau \leq R^{-1}$  comparable with the mean seismicity rate  $R^{-1}$ , in agreement with moderate and strong earthquakes statistics, and possibly suggesting a generic model of localized, point-like seismic focus (compare with  $r = 1/3$ ). In the limit  $R\tau \rightarrow 0$  the exponent  $\alpha$  in (3) exhibits a certain variability, acquiring values that are close to the values of the parameter  $r$  obtained from the fit of the Gutenberg-Richter recurrence law. For instance,  $\alpha = 0.57$  for Vrancea earthquakes in this limit, in agreement with  $r = 0.54$  indicated above. The pair distribution undergoes a non-stationary criticality[32, 33] in the limit  $R\tau \rightarrow 0$ , where aftershocks tend to increase the value of the exponent  $\alpha$  toward Omori's law exponent  $\alpha = 1$ .

The pair distribution  $D(\tau)$  given by (1) has been analyzed for 1999 earthquakes with magnitude  $M > M_c = 3$  recorded in Vrancea between 1974 and 2004.[37] The function  $D(\tau)$  vs  $\tau$  is shown in Fig. 1 (on a logarithmic scale) for various cutoff magnitudes  $M_c$ . It exhibits a rather large dispersion, especially for greater cutoff magnitudes ( $M_c = 3.8$ ) and large values of  $R\tau$ . The data collapse on rescaling with cutoff parameter  $R$ , as shown in Fig. 2, except for the limit  $R\tau \rightarrow 0$ .

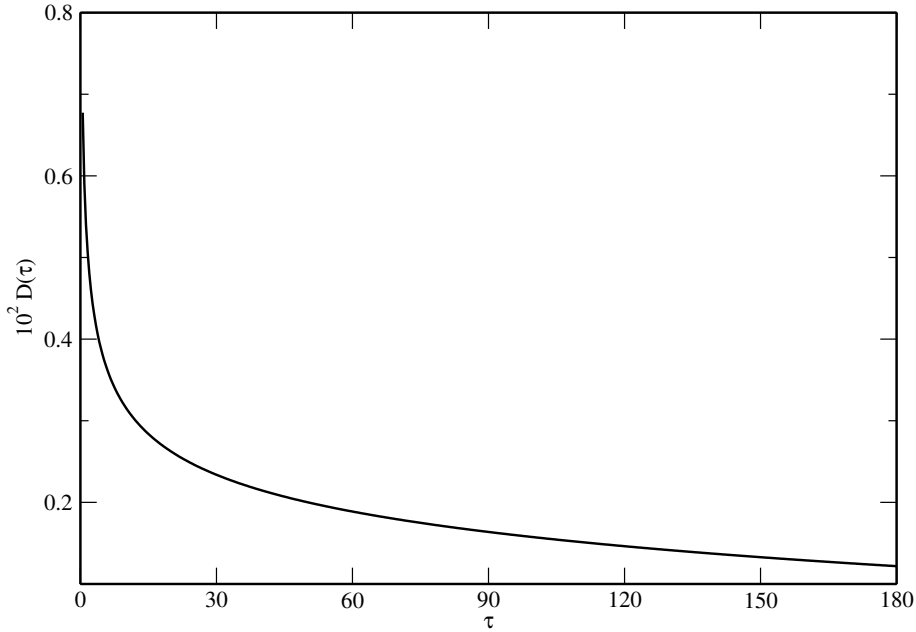


Figure 3: Temporal distribution for successive pairs of earthquakes with magnitude  $M > 5$  as estimated by equation (3) for Vrancea ( $\alpha = 0.25$ ,  $B = 1.17$ ,  $C = 0.71$ ; time  $\tau$  on the abscissa is measured in days).

The fit employing the universal function given by (3) gives  $\alpha = 0.25$ ,  $B = 1.17$  and  $C = 0.71$  (with 13% error), in fair agreement with the fitting parameters given in Refs. 32 and 33. The poor statistics for Vrancea, especially for earthquakes with higher magnitude, prevents a more reliable analysis.

One of the most interesting applications of the pair distribution is the computation of the next-earthquake probability for earthquakes with magnitude greater than  $M$ , which, according to its definition (1), is given by  $D(\tau)$  for the cutoff parameter  $R = t_0^{-1}e^{-\beta M}$  in equation (3). This probability is shown in Fig. 3 for Vrancea, for  $M = 5$  and  $\alpha = 0.25$ ,  $B = 1.17$ ,  $C = 0.71$ . It corresponds to  $R = 1.6 \times 10^{-3} \text{ day}^{-1}$ , *i.e.* 18 earthquakes with  $M > 5$  for 30 years, which differs slightly from  $R = t_0^{-1}e^{-\beta M} = 5 \times 10^{-4} \text{ day}^{-1}$  obtained by fitting the Gutenberg-Richter recurrence law to all earthquakes with magnitude  $M > M_c = 3$  ( $-\ln t_0 = 8.99$  and  $\beta = 1.76$ ). By making use of (3), it may be estimated, for instance, that the probability of having two earthquakes in the same day, in Vrancea, with magnitude greater than  $M = 5$  is  $\sim 0.8\%$ . In general, the probability of having the next-earthquake in time  $\tau$  is given by  $\int_0^\tau d\tau' \cdot D(\tau') = CB^{1-r}\gamma(1-\alpha, R\tau/B)$ , where  $\gamma$  is Euler's incomplete gamma function.

In conclusion, the temporal pair distribution of nearest-neighbouring earthquakes is derived herein by general scale arguments and seismicity parameters, in agreement with the universal function established recently by statistical analysis of extensive empirical data sets.[32]-[34] The results are applied to Vrancea earthquakes, especially in connection with assessing short-term seismicity.

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