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On composite particles in relativistic mechanics, asymptotic freedom and nuclear forces

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It is notorious that the relativistic mechanics imposes certain non-trivial constraints and prescriptions.

The relativistic mechanics started by noticing that Maxwell equations are invariant under Lorentz transformations, which mix up space and time. The Lorentz transformations relate the coordinates $\mathbf{r}$ and time t in one frame to the coordinates and time in another frame moving with constant velocity $\mathbf{v}$ with respect to the former. They contain the velocity c of light, which is then recognized as being universal, and make sense only for $v < c$. On the other hand, Newton's equations are not invariant under Lorentz transformations, but instead they are invariant under Galilei transformations, which are the non-relativistic limit $v/c \ll 1$ of the Lorentz transformations. Obviously, Newton's equations had to be modified such as to obey the Lorentz invariance. This was done by Einstein. This requirement was called the principle of relativity. It implies the inertial frames moving with constant velocity, according to the principle of inertia. It was later on extended to requiring invariance under any point transformations of coordinates and time, implying non-inertial frames. In this form it was called the principle of general relativity.

It was immediately noticed that Lorentz transformations leave invariant the square length $ds^2 = c^2dt^2 - dr^2$, as if $\mathbf{r}, ict$ would be a cuadrivector subjected to Lorentz rotations. The basic definition of mechanics $\mathbf{v} = \partial E/\partial\mathbf{p}$, where E is energy and $\mathbf{p}$ is momentum, is maintained. The principle of relativity requires obviously that $E^2 - c^2p^2$ be a positive constant on passing from a frame to another, since $\mathbf{p}$, iE/c is a cuadrivector like $\mathbf{r}$, ict. Let C^2 be this constant, so that $E = \sqrt{C^2 + c^2p^2} \simeq C + p^2c^2/2C$ in the limit of small momenta. The Newton's mechanics is recovered for $C = mc^2$, where m is the mass of the body. The rest energy mc^2 in $E = \sqrt{m^2c^4 + c^2p^2}$ is probably the most important consequence of the relativistic mechanics. One gets $\mathbf{v} = \mathbf{p}c^2/E$, and $\mathbf{p} = m\mathbf{v}$ for $v/c \ll 1$, $v = c$ for $p \to \infty$; and $\mathbf{p} = m\mathbf{v}/\sqrt{1 - v^2/c^2}$, which indicates the relativistic mass $m/\sqrt{1 - v^2/c^2}$; it increases indefinitely for v approaching the velocity of light. Only massless particles can move with the velocity of light.

One of the first disturbing consequence of the relativistic mechanics is the negative energy $E = -\sqrt{m^2c^4 + c^2p^2}$ allowed by the quadratic invariance. It led later on to admit the existence of antiparticles and of an infinite energy in the vacuum; still, the world may well be unstable with such antimatter.

The relativistic mechanics has no room for kinetic or potential energy. This comes from mixing up coordinates and time. Therefore, we cannot have bound, or composite, relativistic particles, at least not in the usual sense. Instead, the relativistic particles interact, and during this interaction the ingoing free particles change their motion into outgoing free particles, acording to relativistic formula of energy $E = \sqrt{m^2c^4 + c^2p^2}$. However, this relativistic energy has another, profound implication: according to the principle of transformation of the energy, the rest energy mc^2 may become motion energy cp, for instance, and, conversely, the latter may transform into the former, and so on. The rest energy is a transformable energy. This means that during interaction particles may be destroyed or created. Since interaction proceeds in space and time, it follows that particles, which may be destroyed or created by interaction, must be described by fields, which must obey their own equation of motion; like the electromagnetic field, not unexpectedly. However, the classical physics does not know of any field for matter like electrons, for instance.

Nevertheless, such field equations are usually eigenvalues equations, decomposable in plane waves for instance. On the other hand, such plane waves must have attached energy and momentum, if they are going to describe particles. Since the space homogeneity conserves the wavevector $\mathbf{k}$, and the time uniformity conserves the frequency ω, it is obvious by mechanical analogy that energy and momentum must be "quantized" by $\varepsilon = \hbar\omega$ and, respectively, $\mathbf{p} = \hbar\mathbf{k}$, where $\hbar$ is Planck's constant of action; and they are the eigenvalues of operators $i\hbar\partial/\partial t$ and, respectively, $-i\hbar\partial/\partial\mathbf{r}$. This is the quantization of the motion, and if we are going to attach fields to particles, these fields must inevitably be quantal fields. Some of them may subsist in the classical limit, *i.e.* for processes implying a large amount of action, by a macroscopic ocupation of their strength, some others may not. The former must be bosons, the latter fermions. The relativistic mechanics entails therefore the spin and the quantal behaviour, via fields. The quantization establishes also the relativistic field equations, by quantization rules ($\mathbf{p} \rightarrow -i\hbar\partial/\partial\mathbf{r}$, $E \rightarrow i\hbar\partial/\partial t$) and relativistic invariance. The states described by quantal fields are statistical, and they obey the uncertainty relationships $\delta x\delta p > \hbar$, $\delta E\delta t > \hbar$, as the waves do. Interaction processes and the nature's laws do not admit action lesser than quanta $\hbar$. Moreover, in the static limit we recover the non-relativistic form of quantal and classical mechanics, potential and kinetic energies and bound states included. Thus, via fields, the relativistic mechanics implies the quantal physics. Matter is created or destroyed in interaction by quanta, no matter whether particles or fields; quanta of Maxwell's electromagnetic field are particles as well.

This point deserves probably more talking. I imply herein that, as long as particles are created and destroyed by interaction extended in space and time, they necessarily are subjected to a statistical description. However, charges and currents are statistical only if a statistical distribution is given for them, which means it be independent of them. Now, we would have two distinct objects, charges and currents on one hand, and statistical distributions on the other. This is not satisfactory, and leads us not far. It is by far better to incorporate the statistical behaviour in the very nature of charges and currents. This requires quantal fields, and quantal wavefunctions in the non-relativistic limit. The probabilistic existence of particles is then obtained by means of the fields and wavefunctions, the attached charge follows immediately, the currents through the rule of motion for such fields, *i.e.* their equation of motion, all can be expressed with bilinear forms constructed out of fields, according to statistical nature of the quanta.

Unfortunately, there seems to be a deep difference between the two philosophies, relativity and mechanics, as inter-connected as they still are. The former deals with transformations, the latter with motion. We want to describe motion, but science requires also to preserve the results of measurements on passing from a frame to another. Transformations are not motion, through they may be possibly made to agree with each other, with limitations. The rest frame of a moving particle leads inevitably to the motion of that particle in that frame, which is a contradiction; or the motion of the internal structure of that particle is to be considered as the equivalent, transformed, motion of the particle in the moving frame. After all, the rest energy may have a motion-like nature. And an ultrarelativist particle does not exhibit a mass anymore. The outside and the inside of the particle are to be considered then, both in relation of transformation and of motion to each other, which is beyond our actual physical reach. (For instance, a moving nucleon looks like an indefinite number of partons, while a resting nucleon seems to have a different, triplet-quark structure. A similar problem is going to face the string theory of composite particles).

The way of accommodating the two distinct things involves many embarassing difficulties.

One of such difficulties consists of the famous infinities in field theories. Though generaly valid, the uncertainty relationships get more restrictive in relativistic mechanics, because of the finite c. Indeed, the localization cannot be less than $\delta x \sim \hbar/mc$, or $\delta x \sim \hbar/p$, which is precisely the (Compton) wavelength. It follows that for sufficiently high energies either the existence or non-existence of particles will introduce an ultraviolet cutoff in the divergencies, which makes the theory relativistically non-invariant; or the point-like fields in their actual form are not the right thing. The renormalization of the divergencies is just the recognition of this dichotomy. Quantal motion needs a minimal extension in space and time, which is refused by relativity. In particular, this is the main obstacle in quantizing the gravitation.

This basic limitation shows again that relativistic bound states cannot exist. If a relativistic particle would be composite we were not be able to probe distances shorter than its own length of existence, *i.e.* its own Compton wavelength, as its composite nature would require; or the way of thinking about composite particles needs to be quite different, essentially with no definite structure.

Another difficulty about the composite relativistic particles is that the composite may have a mass m and a rest energy mc^2 which cannot be accounted by the mass of its components. Usually, the mass of a bound state is slightly less than the sum of masses of its constituents, the mass defect being accounted by the binding energy. But this is a relativistic correction, and Bethe-Salpeter equation (or Breit equation), which describes such a bound state, is in fact a non-relativistic approximation. Its only merit is to show that the non-relativistic bound states are related to poles in the full S-matrix, but such poles do not exist except for the non-relativistic limit. If we assume them, the dispersion relations would require fantastically inconsistent experimental data, beyond any physical imagination. The S-matrix is made for extended states, its non-existence on its poles is just only naturally to be associated with bound states: it allows the latter, but declines any functionality in describing them.

Yet, relativistic composite particles seem to exist, as, for instance, the hadrons in the quantal chromodynamics, made up of quarks and gluons. The systematics of the hadrons imposes such a composite description. For instance, the nucleons are viewed as composite particles made up of three quarks (two quarks and an antiquark). Similarly, the mesons are made up of one quark and an antiquark. The way out of the difficulties noted above seems to be in this case the asymptotic freedom and the confinement. The quarks are free inside the localized hadrons, but they are subjected to strong forces, mediated by gluons, when trying to escape from the hadrons. This quantal chromodynamics looks like a quantal field which, however, has no room for working. (In particular, we have reduced probably the number of "elementary" particles, but have increased in turn enormously the number of quantal numbers). It has some basic points in common wth the gravitation field, through non-linearities, and it might not come as much of a surprise to learn that some sort of confinement would work also for matter interacting through gravitons. The confining forces do not imply necessarily a potential energy, except for the idealized case when the quarks are moved out slowly, which is non-relativistic. Obviously, such an idealized experiment shows that the rest energy mc^2 of the hadron is accounted by the confinement force, it is equal (and of opposite sign) to the binding energy of the quarks. When the quarks are stretched far apart from each other as to reach the borders of the hadron, the latter ceases in fact to exist anymore, its rest energy is now on the confining force, the mechanical work provided from the outside compensates the binding energy, and everything look consistently, except that the quarks are not free in fact. Or, equally well, the hadron rest energy compensates the binding energy, but the quarks are not free, and the mechanical work is responsible for their departure against the confining forces. Of course, if a lot of energy is provided, the hadrons are destroyed, and the quarks may move relatively free over short distances, but there are larger distances over which they feel the confining force. If the energy is provided quickly, many other particles are produced in the deconfinement process, especially a lot of interacting gluons, which in turn give rise to a lot of quarks, and a quark-gluon plasma is conceivable to exist for short times, made up of quarks and gluons in equilibrium. The subsequent moments of time bring the hadronization of this plasma, and a yield of hadronic output is expected.

The strong attraction force between the quarks may result in a destruction of the nucleons when they get too close to each other in the nucleus. This destruction is followed immediately by a reconstruction, by virtue of the same confining force, with possible emission of mesons. Such a process is equivalent with a repulsive effective strong nuclear force at small distances.

Now, I may notice that this article has some points in common with the vision put forward byWeinberg in his book on Quantal Fields. I apologize for writing it more for me, than for the reader.

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