Journal of Theoretical Physics

Founded and Edited by M. Apostol **151** (2007)

ISSN 1453-4428

Hadronization of the quark-gluon plasma

M. Apostol
Department of Theoretical Physics, Institute of Atomic Physics,
Magurele-Bucharest Mg-6, POBox MG-35, Romania
email: apoma@theory.nipne.ro

Hadronization of the Quark-Gluon Plasma

apoma Laboratory,

by special arrangement with **NIHAM Group**

Magurele-Bucharest

email: apoma@theory.nipne.ro
(May 2006)

The Generic Nucleus

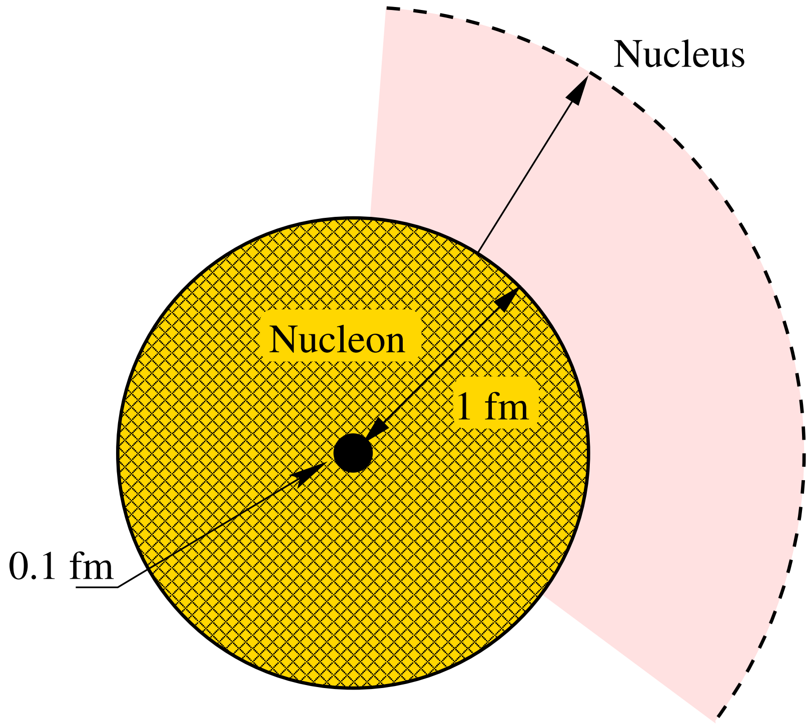
Nucleon binding energy $\varepsilon \sim 8MeV$

$a = 1.5fm\,(10^{-15}m)$, $R_0 = aN^{1/3}$ (Rutherford)

Nucleon rest energy $E_b = Mc^2 \sim 1GeV$, $\lambda = \hbar/Mc \sim 0.1fm$

$p \sim \hbar/a$, $\tau \sim \hbar/\varepsilon \sim 10^{-22}s$, $v \sim a/\tau \sim \varepsilon/p$, $v^2/c^2 \sim 10^{-3}$

Atomic Nucleus



4

Atomic Nucleus is Cold

One-particle model $p \sim (\hbar/R_0)n = \hbar n/aN^{1/3}$, $n_F \sim N^{1/3}(\sim 5-6)$

$\varepsilon \sim \hbar^2 n^2/MR_0^2$, $\varepsilon_F \sim 15MeV$, $\delta\varepsilon \sim 2-3MeV \sim \varepsilon_F$

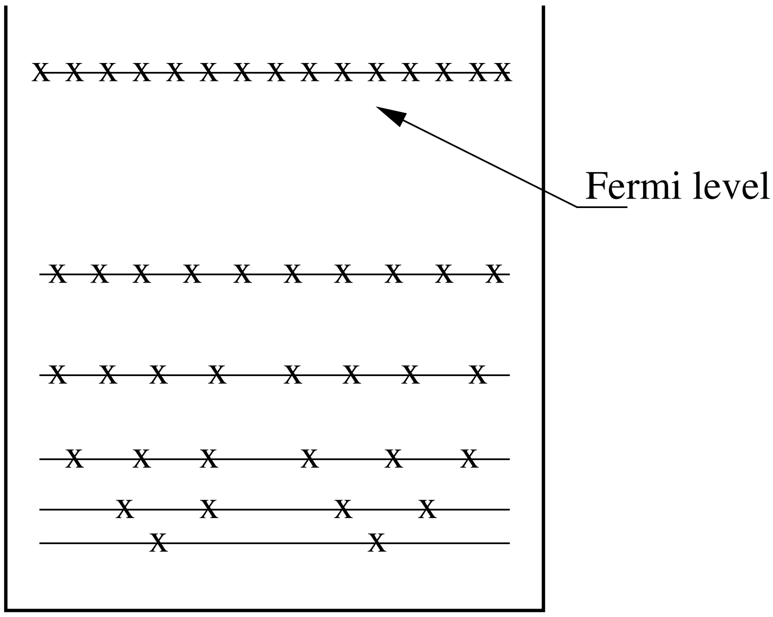
No statistical equilibrium; high spatial degeneracy

Similar for any other mean-field one-particle model (7 nuclear shells)

The nucleus is too small to sustain thermal excitations from its one-particle ground state; a purely quantal ensemble

5

Square potential well



6

One-particle ground-state is however good

Nuclear Shells, magic numbers, etc (even gentle excitations like radiative capture of neutrons)

Mass formula, etc

Higher excitations (threshold $\varepsilon \sim 8MeV!$) would vaporize the quantal nucleus

7

The nucleus still sustains high excitations, with a high density of states

How?

Under excitations the mean field is completely spoiled out

Local nucleonic vibrations, Nuclear Liquid?

Too little, the natural threshold $\varepsilon \sim 8MeV$ would vaporize the nucleus

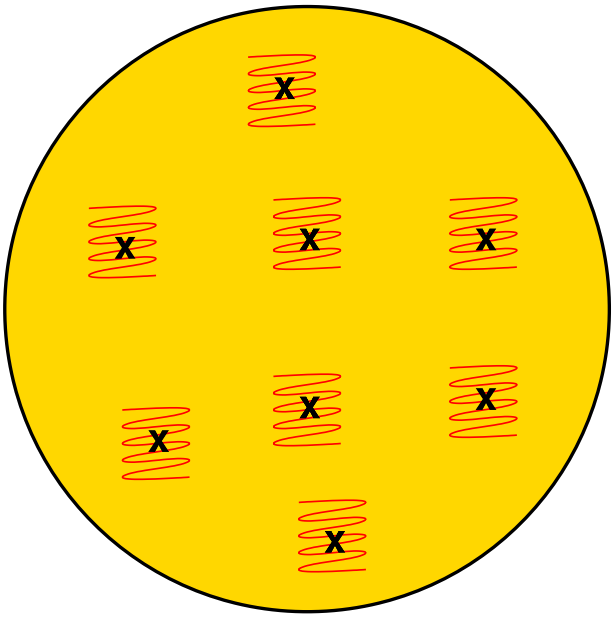
Change the ground-state to a **NUCLEAR SOLID**!

Rigid, finite size, amorphous, with a lot of global energy levels

Stable, equilibrium attainable, dense states, etc

8

Nuclear Solid



9

Statistical and Thermal Equilibrium

$$\varepsilon_{eq} > T > \delta\varepsilon_f > \delta\varepsilon_{ex} \gg \delta\varepsilon_q > \delta\varepsilon_{obs}$$

$\delta\varepsilon_f \sim T(\partial\varepsilon/\partial T)^{1/2}$, fluctuations

$\delta\varepsilon_{ex}$ (their lifetime, $\hbar/\delta\varepsilon_{ex}$)

$\delta\varepsilon_q$- spacing between the quantal levels

$\delta\varepsilon_{obs}$- uncertainty in measuring the energy

$$\tau_{eq} < \tau_{th} < \tau_f < \tau_{ex} \ll \tau_q < \tau_{obs}$$

10

Quark-Gluon Plasma. Ignition Threshold Energy

$E > E_b$, asymptotic freedom, release quarks (uud, udd) and gluons

Radiation and ultrarelativistic fermions ($m_u \sim 4MeV$, $m_d \sim 8MeV$)

Few nucleons destroyed - no plasma (quick recovery of the nucleons)

All nucleons in nucleus with equally shared (slightly above E_b) energy - no plasma, again

11

Threshold Energy - Fermi Energy

$$\varepsilon_{thr} \sim \hbar c/a \sim 125MeV$$

(or $180MeV$ for 3 quarks per nucleon)

($\sim (3/4)\varepsilon_{thr}$ per nucleon, beside, and above, E_b) (ultrarelativistic gas of quarks)

Thermalization, poor: $\tau \sim \hbar/\varepsilon_{thr} \sim 10^{-23}s$

Quantal fluctuations: $\delta\varepsilon_q \sim \hbar c/R_0 = \hbar c/aN^{1/3} \sim \varepsilon_{thr}$

Hot and dense quark-gluon plasma over the entire nucleus

12

Hot and Dense Quark-Gluon PLasma

$$E_p = E_q + E_g$$

$$E_g \simeq VT^4/(\hbar c)^3$$

$E_q = \frac{3}{4}N_q\varepsilon_F + \frac{3\pi^2}{2}N_q(T^2/\varepsilon_F) + ...$; $E_q \simeq N_qT$; $\varepsilon_F \sim \hbar c n_q^{1/3}$, $n_q = N_q/V$

The gluon energy dominates the plasma energy

What is wrong? The fixed number of quarks (which doesnt give any rich hadronization output!)

13

Quark-Gluon Equilibrium

Pair production? Non-linear quark-gluon, gluon-gluon interaction?

$\mu_q = 0$

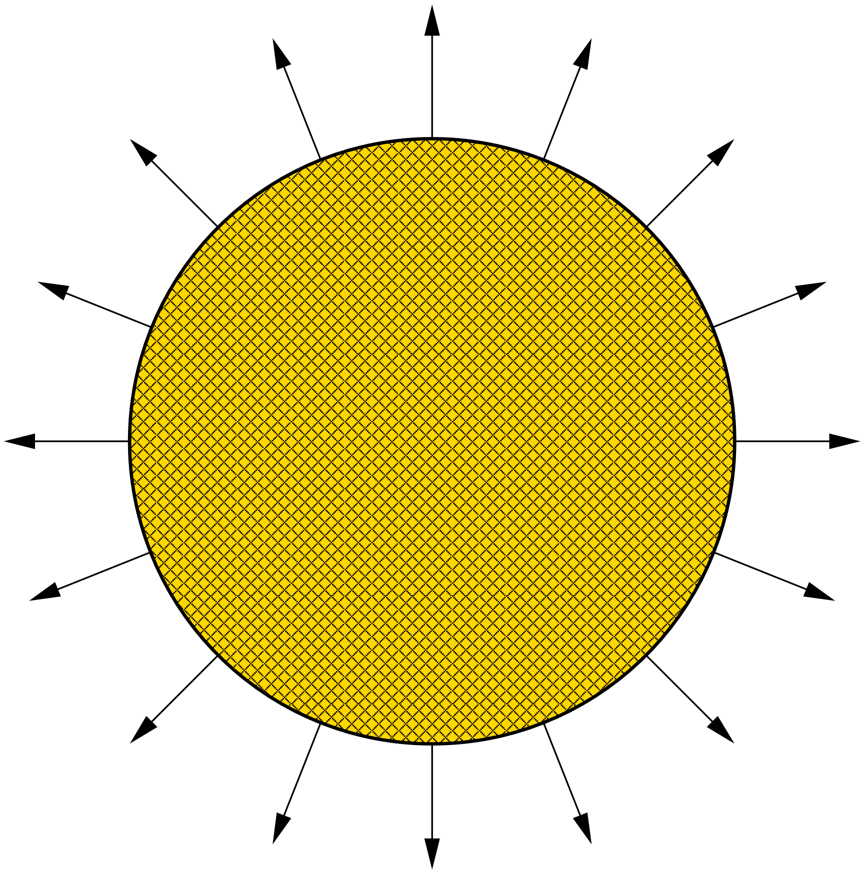
$E_q \simeq VT^4/(\hbar c)^3$, $E_g \simeq VT^4/(\hbar c)^3$, $N_q \simeq VT^3/(\hbar c)^3$, $N_g \simeq VT^3/(\hbar c)^3$

Plasma energy $E_p \simeq VT^4/(\hbar c)^3$

Number of quark-gluon quanta $N_q \simeq N_g \sim VT^3/(\hbar c)^3$

14

Quark-Gluon Plasma



15

$$E_p \longleftrightarrow 10^3 GeV/fm^3$$

Temperature $T \sim 1GeV$

Number of quarks-gluons $N_q \sim 10^3 N$ (N number of nucleons)

Quark production: $T > mc^2$; $m_u \sim 4MeV$, $m_d \sim 8MeV$, $m_s \sim 150MeV$; not $m_c \sim 1.5GeV$, $m_b \sim 4.7GeV$, $m_t \sim 176GeV$

Hot and Dense plasma thermalizes OK: $\tau \sim \hbar/T \sim 10^{-24}s$, $T \sim 1GeV \gg \delta\varepsilon_q$ ($\sim 10MeV$)

16

Quark-Gluon Plasma Expands and Gets "Cool"

$$R = R_0(1 + ct/aN^{1/3})$$

Energy conservation: $T = T_0(1 + ct/aN^{1/3})^{-3/4}$, $T_0 = E_p^{1/4}(\hbar c/R_0)^{3/4}\,(1GeV)$

$$N_q = N_{q0}(1 + ct/aN^{1/3})^{3/4} \ , \ N_{q0} = (R_0 T_0/\hbar c)^3 = N(T_0 a/\hbar c)^3 \, (10^3 N)$$

Non-equlibrium, irreversible, process; entropy $S \sim N_q$ increases

(rate of quark-gluon equilibrium slows down; density decreases)

17

Hadronization. Classical Statistics

"Surface" quarks $N'_q = N_q(\Delta R/R) = N_q(\Delta R_0/R_0) = N_q/N_{q0}^{1/3} = fN_q$

$$f = 1/N_{q0}^{1/3} = 1/10N^{1/3} \ll 1, \text{ for } 10^3 GeV/fm^3$$

Their Fermi energy $T_q = \varepsilon'_F = \hbar c N_q'^{1/3}/V^{1/3} = f^{1/3}\hbar c N_q^{1/3}/V^{1/3} = f^{1/3}(\hbar^3 c^3 N_q/V)^{1/3} = f^{1/3}T$

$$T/\varepsilon'_F = 1/f^{1/3} \gg 1$$

Classical statistics

Hadrons: $\sqrt{m^2c^4 + (\hbar c/r)^2} - mc^2 \ll \hbar c/r = \varepsilon'_F \ll T$, better!

18

Note

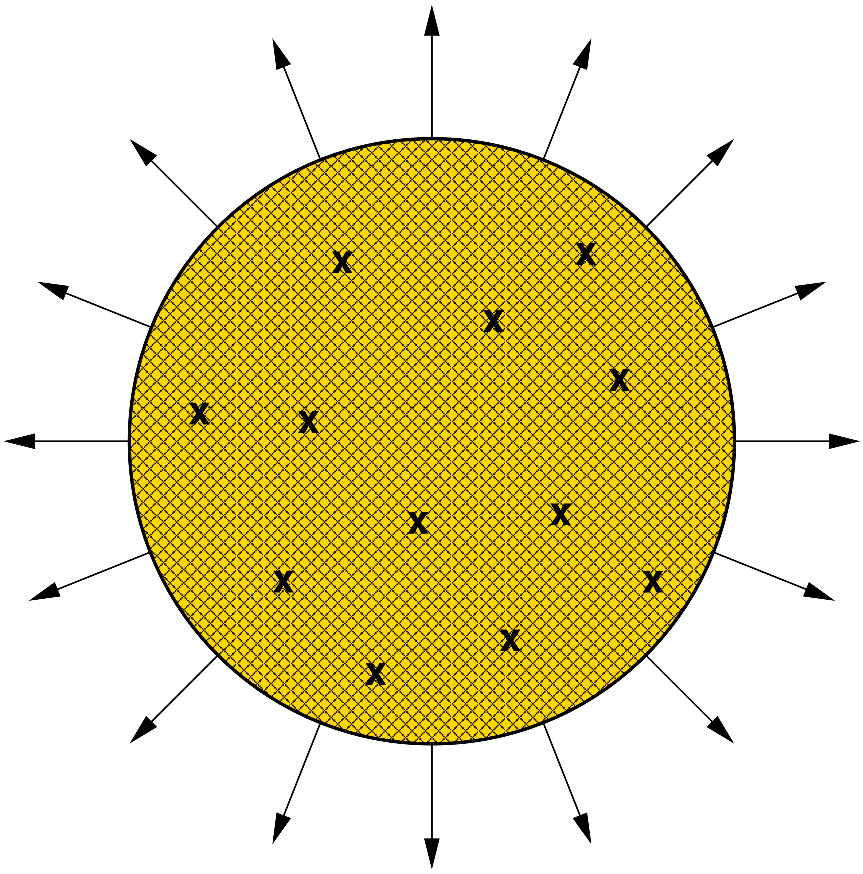
$E'_q = 3N'_q T = 3fN_q T = 3fVT^4/(\hbar c)^3$

Conservation of energy $VT^4/(\hbar c)^3 \, (gluons - quarks) + E'_q = E_p$

Same time-dependence of temperature and number of quarks-gluons

19

Quark-Gluon Plasma with Grains of Popcorn



20

Classical Gas of Ultrarelativistic Quarks

$$dN = [gV/(2\pi\hbar)^3]e^{\mu/T}e^{-\varepsilon/T}d\mathbf{p}$$

$\varepsilon = \sqrt{m^2c^4 + c^2p^2}$, in general; $\varepsilon_q = cp$

Chemical potential $\mu_q \simeq -3T\ln(T/T_q)$, $\quad T_q = f^{1/3}T$, $f \ll 1$

Energy $E_q \simeq 3N_qT$, pressure $\Omega_q = -p_qV_q = -N_qT$ (partial presure)

21

Hadronic Condensation

$$N_q = \sum n_jp_j \ , \ E_h = \sum \varepsilon_jp_j \ , \ N_h = \sum p_j \ , \ \varepsilon_j = \sqrt{m_j^2c^4 + c^2p_j^2}$$

$$S = -\sum p_j \ln(p_j/e)$$

22

Hadronic Output

$$dN_j = \frac{g_j V_h}{(2\pi\hbar)^3 m_0} e^{\mu_h n_j/T} e^{-\varepsilon_j/T} dn_j dm_j d\mathbf{p}_j$$

$j = 1, 2, 3, ...$ species; quark constituency $n_j = 2, 3, ...$; mass m_j; energy $\varepsilon_j = \sqrt{m_j^2 c^4 + c^2 p_j^2}$; momentum $\mathbf{p}_j$

m_j and n_j related? Resonances

Continuous mass spectrum, minimal mass cutoff m_0

Empirical distributions, discrete mass, sparse n_j, etc

23

Hadronic Gas

$$dN_j = \int dm d\mathbf{p} = \frac{g_j V_h}{(2\pi\hbar)^3 m_0} \cdot \frac{6\pi(\pi-1)}{c^5} T^4 e^{\mu_h n_j/T} dn_j$$

$$N_h = \sum_{j=s} N_j \simeq \frac{g_h V_h}{(2\pi\hbar)^3 m_0} \cdot \frac{6\pi(\pi-1)}{c^5} T^4 \cdot e^{\mu_h s/T}$$

$$N_q = \sum_{j=s} n_j N_j \simeq \frac{s g_h V_h}{(2\pi\hbar)^3 m_0} \cdot \frac{6\pi(\pi-1)}{c^5} T^4 \cdot e^{\mu_h s/T}$$

Dominated by the smallest hadrons $s = 2$ (heavier species, exponential)

Energy $E_h = 4N_h T$, pressure $\Omega_h = -p_h V_h = -N_h T$

24

Quark-Gluon Transition

Equilibrium $p_q = p_h$, same concentration $n_q = n_h$

$N_h = N_q/s$; therefore $V_h = V_q/s$ (condensation!)

Condensation - First-order Phase Transition

$$\mu_h \simeq -(1/s) T \ln[3 g_h (\pi - 1) T^4 / 4\pi^2 \hbar^3 c^5 m_0 n_h]$$

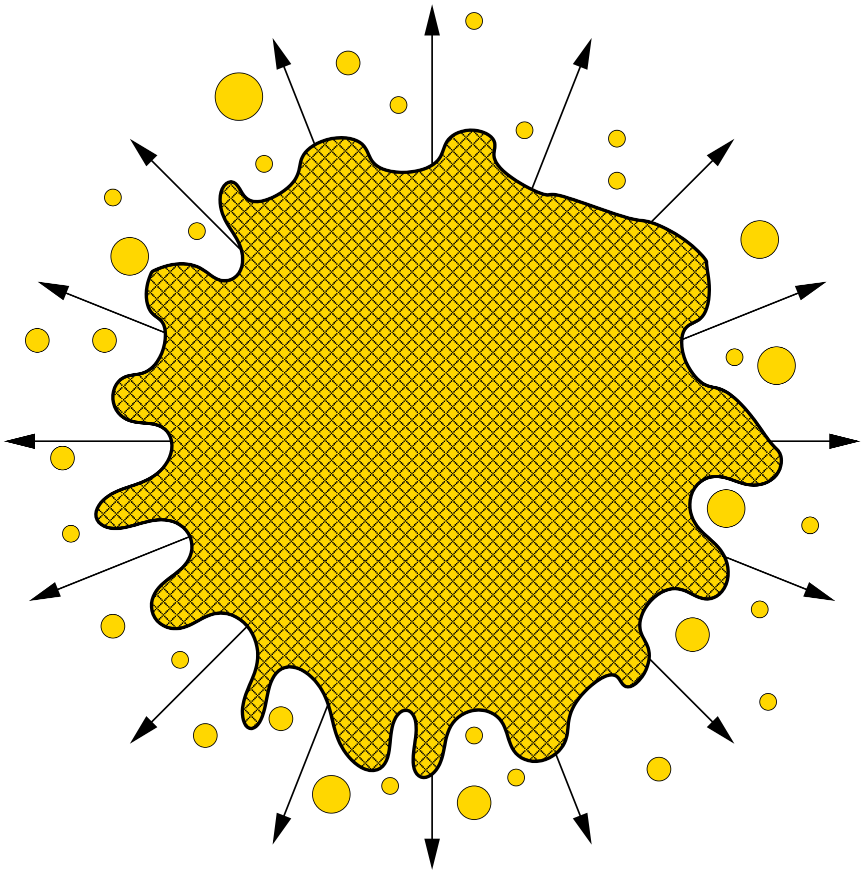
$\mu_h \simeq -(1/s) T \ln(T^4/T_q^3 T_m)$, $T_m = m_0 c^2$; $\mu_q \simeq -3T \ln(T/T_q)$

Transition $\mu_q = \mu_h$, $T_c = T_q (T_q/T_m)^{1/(3s-4)} = T_m / f^{s-1} \gg T_m$ ($T_q = f^{1/3} T_c$)

Latent heat $W_h(T_c) - W_q(T_c) = (5/s - 4) E_q / 3 < 0$ ($W = E + pV$)

25

Quark-Gluon Plasma eaten up by Dragons



26

Feel the Figures

$N \sim 50$; $f = 1/10N^{1/3} \sim 0.03$ (for $10^3 GeV/fm^3$)

$T_c \sim 30T_m$ for $s = 2$ ($T_c \gg T_m$)

$T_m \sim 4MeV$ (the lightest quark)?

$T_c \sim 120MeV = T_0(= 1GeV)(1 + ct/aN^{1/3})^{-3/4}$

$t \sim 10^{-22}s$ ($ct/aN^{1/3} \sim 1/20$)

Plasma Expansion $R_0 \longrightarrow R \sim 20R_0$

27

Hadronized energy at this stage $E'_q = 3N'_qT_c \sim 3fE_p \sim 10\%\ E_p$

(efficiency quotient !)

Latent heat $(5/s - 4)E'_q/3 \sim -50\% E'_q$; great remanent entropy, disorder; weak yield of more massive hadrons in each species!

How many quarks hadronize? At first stage

$N'_q = fN_q = fN_{q0}(1 + ct/aN^{1/3})^{3/4} = f \cdot 10^3 N \cdot 10 \sim 300N \sim sN_h \sim 2N_h$

28

Whats next?

The first stage, N_q'

N_q' leave as hadrons

N_q, R diminish, slightly increase of temperature (latent heat!)

Quick recovery of critical temperature, **the process's going on! Successively, slice by slice, like onion shells!**

Rarefied plasma, quark-gluon "decoupling", more massive hadrons...

29

And The End!

Look for hadrons distributions, discrete mass spectrum, sparse quark constituency (n_j!), fit them with classical (or quantal) statistics, T (actually T_c!) and μ parameters, compute the number of quarks N_q, check the conservation of the numbers $N_q = \sum n_j N_j$, check energy conservation, etc, etc

Prove the condensation!

(Note: no restriction to classical statistics!)

30

And What?

Nothing!

If everything goes all right, we learn nothing!

If something goes wrong, we have something more to think about!