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Two-dimensional charged bosons in magnetic field

M. Apostol
Department of Theoretical Physics,
Institute of Atomic Physics, Magurele-Bucharest MG-6,
POBox MG-35, Romania
e-mail:apoma@theor1.ifa.ro

Abstract

It is shown that a hypothetical gas of charged bosons in two dimensions condenses when placed in a transverse magnetic field.

We consider a hypothetical gas of N identical bosons in two-dimensions, each of mass m, charge q and spin S, placed in a transverse magnetic field H directed along the z-axis. We assume that the effects of the boson-boson interaction are compensated by their interaction with the substrate, and by the boundaries of the two-dimensional gas, so that we may limit ourselves to the single-particle energy, given by

$$E_{ns} = \hbar\omega\,(n+1/2) - \alpha_s \ \ , \ \ n = 0, 1, 2, ... \ \ , \tag{1}$$

where $\omega = qH/mc$ is the cyclotron frequency and $\alpha_s = g_m \mu_B sH$ is the Zeeman energy; g_m is the gyromagnetic factor, μ_B is the Bohr magneton, and $s = -S, -S+1, ...S$ is the spin projection along the z-axis. As it is well-known, an orbital degeneracy $N_{xy} = A(qH/ch) = (A/2\pi)(m\omega/\hbar)$ occurs, where A is the area of the surface covered by the gas, so that the number of particles may be written as

$$N = \frac{A}{2\pi}\frac{m\omega}{\hbar}\sum_s \sum_n \frac{1}{\exp\left[\hbar\omega\,(n+1/2) - \alpha_s - \mu\right]\beta - 1} \ \ , \tag{2}$$

where μ is the chemical potential, and $\beta = 1/T$ is the inverse of the temperature. Denoting by a the inter-particle distance, $A = Na^2$, we introduce the characteristic energy $\varepsilon_0 = 2\pi\hbar^2/gma^2$, where $g = 2S+1$ is the spin degeneracy, and re-express (1) as

$$\varepsilon_0/\hbar\omega = \frac{1}{g}\sum_s \sum_n \frac{1}{\exp\left[\hbar\omega\,(n+1/2) - \alpha_s - \mu\right]\beta - 1} \ \ . \tag{3}$$

The energy $\hbar\omega$ is, usually, very small in comparison with the temperature, $\hbar\omega\beta \ll 1$, and we might be tempted to replace the summation over n in (3) by an integral, according to the formula

$$\sum_a^{b-1} f(n+1/2) = \int_a^b dn \cdot f(n) - \frac{1}{24} f'(n)\,|_a^b \ \ . \tag{4}$$

However, this formula is valid for $|f(n+1/2) - f(n-1/2)| \ll |f(n)|$, and the Bose distribution in (3) fulfils this requirement only for very large values of n, or for very small values of the fugacity $z = \exp{(\mu\beta)} \ll 1$. The ratio $\varepsilon_0/\hbar\omega$ is usually much larger than unity, so that z must be close to unity; it follows that (4) is inapplicable, and the main contribution to (3) comes from $n = 0$. This means that the bosons are condensed on the zero-point energy level, i.e. the two-dimensional gas of charged bosons in magnetic field exhibits a Bose-Einstein condensation (at any temperature), in contrast with the two-dimensional ideal gas of bosons without magnetic field.

Since $\hbar\omega\beta \ll 1$ and $\alpha_s\beta \ll 1$ as well, we may neglect their contributions to (3), and retaining only the $n = 0$ term, we obtain $z = (\varepsilon_0/\hbar\omega) / (1 + \varepsilon_0/\hbar\omega) \cong 1$, the energy $E = (\hbar\omega/2)N$, as expected, and the grand-canonical potential $\Omega \cong -N (\hbar\omega/\varepsilon_0) \ln (\varepsilon_0/\hbar\omega) = -pA$, where p is the pressure. In the present limit $\hbar\omega/\varepsilon_0 \ll 1$ the entropy of the ensemble vanishes, as expected.