Journal of Theoretical Physics

Founded and Edited by M. Apostol **175** (2009)

ISSN 1453-4428

Reflected and refracted waves in a semi-infinite plasma

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Abstract

We compute the reflected and refracted electromagnetic fields for an ideal semi-infinite (half-space) plasma, as well as the reflection coefficient, by using a general procedure based on equations of motion, Maxwell's equations and suitable boundary conditions. The approach consists of representing the charge disturbances by a displacement field in the positions of the moving particles (electrons). The propagation of an electromagnetic wave in plasma is treated by means of the retarded electromagnetic potentials, and the resulting integral equations are solved. Generalized Fresnel's relations are thereby obtained for any incidence angle and polarization and the angles of total polarization and total reflection are derived. Bulk and surface plasmon-polariton modes are identified. As it is well known, the field inside the plasma is either damped (evanescent) or propagating (transparency regime), and the reflection coefficient exhibits an abrupt enhancement on passing from the propagating regime to the damped one (total reflection).

PACS: 41.20.Jb; 42.25.Bs; 42.25.Gy; 71.36.+c; 73.20.Mf; 78.20.Ci

Keywords: Semi-infinite Plasma; Reflected and Refracted Waves; Polaritons; Plasmon-Polaritons; Reflection Coefficient

Fresnel's theory of the propagation of the eletromagnetic waves in matter has proved to be very successful for describing reflection and refraction.[1, 2] This is the more so remarkable as the theory has been formulated before the discovery of Maxwell's equations. Fresnel's theory is based on a few assumptions, like the transversality of the electromagnetic waves, general continuity conditions at the boundary of two adjoining media, and, especially, the representation of the matter polarization and response by the dielectric function (and, in a more general form, by the magnetic permeability and electrical conductivity). Among these general hypotheses, it is especially the latter that raises some querries in applying the theory to particular cases, the more so as the dielectric functions are either introduced by various ansatzen or are model dependent. In addition, there are difficulties with a proper definition of the dielectric function in structures with special, restricted geometries. It would be desirable, therefore, of having both Fresnel's theory and the dielectric function derived in a consistent manner, at least for reasonably realistic models.[3]-[16] This is particularly relevant for recent investigations of the electromagnetic waves in structures with special geometries, where a possible enhancement of the electromagnetic radiation has been reported.[17]-[19]

We present here a computation of the reflected and refracted electromagnetic fields for a semi-infinite (half-space) plasma. We represent the charge disturbances as $\delta n = -ndiv\mathbf{u}$, where n is the (constant, uniform) charge concentration and $\mathbf{u}$ is a displacement field of the mobile charges (electrons). This representation is valid for $\mathbf{Ku(K)} \ll 1$, where $\mathbf{K}$ is the wavevector and $\mathbf{u(K)}$ is the Fourier component of the displacement field. We assume a rigid neutralizing background of positive charge, as in the well-known jellium model.

We assume a plane wave incident on the plasma surface under angle α. Its frequency is given by $\omega = cK$, where c is the velocity of light and the wavevector $\mathbf{K} = (\mathbf{k}, \kappa)$ has the in-plane component $\mathbf{k}$ and the perpendicular-to-plane component κ, such as $k = K \sin\alpha$ and $\kappa = K \cos\alpha$. In addition, $\mathbf{k} = k(\cos\varphi, \sin\varphi)$. The electric field is taken as $\mathbf{E}_0 = E_0(\cos\beta, 0, -\sin\beta)e^{i\mathbf{kr}}e^{i\kappa z}e^{-i\omega t}$, and we impose the condition $\cos\beta\sin\alpha\cos\varphi - \sin\beta\cos\alpha = 0$ (transversality condition $\mathbf{KE}_0 = 0$). The angle β defines the direction of the polarization of the incident field.

In the presence of an electromagnetic field $\mathbf{E}_0$ we use the equation of motion

$$\ddot{\mathbf{u}} = -\frac{e}{m}\mathbf{E} - \frac{e}{m}\mathbf{E}_0 \ , \tag{1}$$

for the displacement field $\mathbf{u}$, where $-e$ is the electron charge, m is the electron mass and $\mathbf{E}$ is the polarizing field. We leave aside the dissipation effects (which can easily be included in equation (1)). We consider an ideal semi-infinite plasma extending over the half-space $z > 0$ (and bounded by the vacuum for $z < 0$). The displacement field $\mathbf{u}$ is then represented as $(\mathbf{v}, u_3)\theta(z)$, where $\mathbf{v}$ is the displacement component in the (x, y)-plane, u_3 is the displacement component along the z-direction and $\theta(z) = 1$ for $z > 0$ and $\theta(z) = 0$ for $z < 0$ is the step function. We use Fourier transforms of the type

$$\mathbf{u}(r, z; t) = \sum_{\mathbf{k}} \int d\omega \mathbf{u}(\mathbf{k}, z; \omega)e^{i\mathbf{kr}}e^{-i\omega t} \tag{2}$$

where $\mathbf{r}$ is the (x, y)-in-plane position vector. Equation (1) becomes

$$\omega^2\mathbf{u} = \frac{e}{m}\mathbf{E} + \frac{e}{m}\mathbf{E}_0 e^{i\kappa z} \ , \tag{3}$$

for $z > 0$. In equation (3) we have preseved explicitly only the z-dependence (*i.e.* we leave aside the factors $e^{i\mathbf{kr}}e^{-i\omega t}$). We find it convenient to employ the vector potential

$$\mathbf{A}(\mathbf{r}, z; t) = \frac{1}{c}\int d\mathbf{r}' \int dz' \frac{\mathbf{j}(\mathbf{r}', z'; t - R/c)}{R} \tag{4}$$

and the scalar potential

$$\Phi(\mathbf{r}, z; t) = \int d\mathbf{r}' \int dz' \frac{\rho(\mathbf{r}', z'; t - R/c)}{R} \ , \tag{5}$$

where $\mathbf{j} = -ne\dot{\mathbf{u}}\theta(z)e^{i\mathbf{kr}}e^{-i\omega t}$ is the current density, $\rho = nediv\mathbf{u} = ne\left(i\mathbf{kv} + \frac{\partial u_3}{\partial z}\right)\theta(z)e^{i\mathbf{kr}}e^{-i\omega t} + neu_3(0)\delta(z)e^{i\mathbf{kr}}e^{-i\omega t}$ is the charge density and $R = \sqrt{(\mathbf{r} - \mathbf{r}')^2 + (z - z')^2}$. The integrals in equations (4) and (5) implies the known integral[20]

$$\int_{|z|}^{\infty} dx J_0\left(k\sqrt{x^2 - z^2}\right)e^{i\omega x/c} = \frac{i}{\kappa}e^{i\kappa|z|} \ , \tag{6}$$

where J_0 is the zeroth-order Bessel function of the first kind (and $\kappa^2 = \omega^2/c^2 - k^2$). It is convenient to use the projections of the in-plane displacement field $\mathbf{v}$ on the vector $\mathbf{k}$ and on the vector $\mathbf{k}_\perp = k(-\sin\varphi, \cos\varphi)$, $\mathbf{k}_\perp\mathbf{k} = 0$. We denote these components by $v_1 = \mathbf{kv}/k$ and $v_2 = \mathbf{k}_\perp\mathbf{v}/k$, and use also the components $E_1 = \mathbf{kE}/k$, $E_2 = \mathbf{k}_\perp\mathbf{E}/k$ and similar ones for the external field $\mathbf{E}_0$. We give here the components of the external field

$$E_{01} = E_0\cos\beta\cos\varphi \ , \ E_{02} = -E_0\cos\beta\sin\varphi \ , \ E_{03} = -E_0\sin\beta \ . \tag{7}$$

One can check immediately the transversality condition $E_{01}k + E_{03}\kappa = 0$. Making use of $\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - grad\Phi$, equations (4) and (5) give the electric field

$$E_1 = -2\pi ine\kappa \int_0 dz' v_1(z') e^{i\kappa|z-z'|} - 2\pi ne\frac{k}{\kappa}\int_0 dz' u_3(z')\frac{\partial}{\partial z'}e^{i\kappa|z-z'|}$$

$$E_2 = -2\pi ine\frac{\omega^2}{c^2\kappa}\int_0 dz' v_2(z') e^{i\kappa|z-z'|} \tag{8}$$

$$E_3 = 2\pi ne\frac{k}{\kappa}\int_0 dz' v_1(z')\frac{\partial}{\partial z}e^{i\kappa|z-z'|} - 2\pi ine\frac{k^2}{\kappa}\int_0 dz' u_3(z') e^{i\kappa|z-z'|} + 4\pi neu_3$$

for $z > 0$. It is worth observing in deriving these equations the non-intervertibility of the derivatives and the integrals, according to the identity

$$\frac{\partial}{\partial z}\int_0 dz^{'} f(z^{'})\frac{\partial}{\partial z^{'}}e^{i\kappa\left|z-z^{'}\right|} = \kappa^2\int_0 dz^{'} f(z^{'})e^{i\kappa\left|z-z^{'}\right|} - 2i\kappa f(z) \tag{9}$$

for any function $f(z)$, $z > 0$; it is due to the discontinuity in the derivative of the function $e^{i\kappa\left|z-z^{'}\right|}$ for $z = z^{'}$. Now, we employ equation of motion (3) in equations (8) and get the integral equations

$$\omega^2 v_1 = -\frac{i\omega_p^2\kappa}{2}\int_0 dz' v_1(z')e^{i\kappa|z-z'|} - \frac{\omega_p^2 k}{2\kappa}\int_0 dz' u_3(z')\frac{\partial}{\partial z'}e^{i\kappa|z-z'|} + \frac{e}{m}E_{01}e^{i\kappa z}$$

$$\omega^2 v_2 = -\frac{i\omega_p^2\omega^2}{2c^2\kappa}\int_0 dz' v_2(z')e^{i\kappa|z-z'|} + \frac{e}{m}E_{02}e^{i\kappa z} \tag{10}$$

$$\omega^2 u_3 = \frac{\omega_p^2 k}{2\kappa}\int_0 dz' v_1(z')\frac{\partial}{\partial z}e^{i\kappa|z-z'|} - \frac{i\omega_p^2 k^2}{2\kappa}\int_0 dz' u_3(z')e^{i\kappa|z-z'|} + \omega_p^2 u_3 + \frac{e}{m}E_{03}e^{i\kappa z}$$

for the coordinates $v_{1,2}$ and u_3 in the region $z > 0$, where $\omega_p = \sqrt{4\pi ne^2/m}$ is the plasma frequency. The second equation (10) can be solved straightforwardly by noticing that

$$\frac{\partial^2}{\partial z^2}\int_0 dz' v_2(z')e^{i\kappa|z-z'|} = -\kappa^2\int_0 dz' v_2(z')e^{i\kappa|z-z'|} + 2i\kappa v_2 \ . \tag{11}$$

We get

$$\frac{\partial^2 v_2}{\partial z^2} + (\kappa^2 - \omega_p^2/c^2)v_2 = 0 \ . \tag{12}$$

The solution of this equation is

$$v_2 = \frac{2eE_{02}}{m\omega_p^2}\cdot\frac{\kappa\left(\kappa-\kappa^{'}\right)}{K^2}e^{i\kappa^{'}z} \ , \tag{13}$$

where

$$\kappa^{'} = \sqrt{\kappa^2 - \omega_p^2/c^2} = \frac{1}{c}\sqrt{\omega^2\cos^2\alpha - \omega_p^2} \ . \tag{14}$$

The wavevector $\kappa^{'}$can also be written in a more familiar form $\kappa^{'} = (\omega/c)\sqrt{\varepsilon - \sin^2\alpha}$, where $\varepsilon = 1 - \omega_p^2/\omega^2$ is the dielectric function. The corresponding component of the (total) electric field (the refracted field) can be obtained from equation (3); it is given by $(m\omega^2/e)\,v_2$. For $\kappa^2 < \omega_p^2/c^2$ ($\omega\cos\alpha < \omega_p$) this field does not propagate. For $\kappa^2 > \omega_p^2/c^2$ (ω greater than the transparency edge $\omega_p/\cos\alpha$) it represents a refracted wave (transparency regime) with the refraction angle $\alpha^{'}$ given by Snell's law

$$\frac{\sin\alpha^{'}}{\sin\alpha} = \frac{1}{\sqrt{1-\omega_p^2/\omega^2}} = 1/\sqrt{\varepsilon} \ . \tag{15}$$

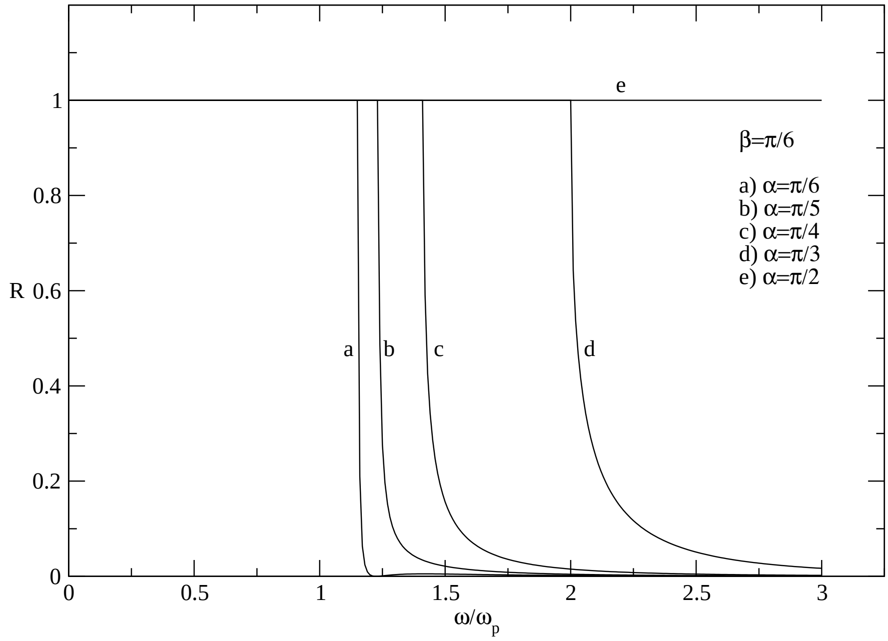

Figure 1: Reflection coefficient for a semi-infinite plasma for $\beta = \pi/6$ and various incidence angles α. One can see the shoulder occurring at the transparency edge $\omega_p/\cos\alpha$ and the zero occurring at $\omega^2 = \omega_p^2/\left(1-\tan^2\alpha\right)$ for $\alpha = \beta = \pi/6$ ($R_2 = 0$,$\varphi = 0$).

The polariton frequency is given by

$$\omega^2 = c^2K^2 = \omega_p^2 + c^2K'^2 \ , \tag{16}$$

as it is well known, where $K'^2 = \kappa'^2 + k^2$.

The first and the third equations (10) can be solved by using an equation similar with equation (11) and by noticing that they imply

$$\kappa'^2 u_3 = ik\frac{\partial v_1}{\partial z} \ . \tag{17}$$

We get

$$v_1 = \frac{2eE_{01}}{m\omega_p^2}\cdot\frac{\kappa'\left(\kappa-\kappa'\right)}{\kappa\kappa'+k^2}e^{i\kappa' z} \tag{18}$$

and

$$u_3 = \frac{2eE_{03}}{m\omega_p^2}\cdot\frac{\kappa\left(\kappa-\kappa'\right)}{\kappa\kappa'+k^2}e^{i\kappa' z} \ . \tag{19}$$

Similarly, the corresponding components of the refracted field are given by equation (3). It is easy to check the transversality condition $v_1k + u_3\kappa' = 0$.

We can see that the polarization field $\mathbf{E}$ in equation (1) cancels out the original, incident field $\mathbf{E}_0$ and gives the total, refracted field $m\omega^2\mathbf{u}/e$ inside the plasma. This is an illustration of the so-called Ewald-Oseen extinction theorem.[8, 21] We note that a possible treatment of the propagation of the electromagnetic waves in matter by means of integral equations was suggested previously.[21]

In order to get the reflected wave (region $z < 0$) we turn to equations (8) and use therein the solutions given above for $v_{1,2}$ and u_3. It is worth noting here that the discontinuity term $\omega_p^2 u_3$ does not appear anymore in these equations (because $z' > 0$ and $z < 0$ and we cannot have $z = z'$). The integrations in equations (8) are straightforward and we get the field

$$E_1 = E_{01}\frac{\kappa - \kappa'}{\kappa + \kappa'} \cdot \frac{\kappa\kappa' - k^2}{\kappa\kappa' + k^2} e^{-i\kappa z} \ , \tag{20}$$

$$E_2 = E_{02}\frac{\kappa - \kappa'}{\kappa + \kappa'} e^{-i\kappa z} \tag{21}$$

and

$$E_3 = -E_{03}\frac{\kappa - \kappa'}{\kappa + \kappa'} \cdot \frac{\kappa\kappa' - k^2}{\kappa\kappa' + k^2} e^{-i\kappa z} \ . \tag{22}$$

We can see that this field represents the reflected wave ($\kappa \to -\kappa$), and we can check its transversality to the propagation wavevector. Making use of the reflected field $\mathbf{E}_{refl}$ given by equations (20)-(22) and the refracted field $\mathbf{E}_{refr}$ obtained from equations (3) and (8) ($\mathbf{E}_{refr} = \mathbf{E} + \mathbf{E}_0 = m\omega^2\mathbf{u}/e$) one can check the continuity of the electric field and electric displacement at the surface ($z = 0$) in the form $E_{1,2refl} + E_{01,2} = E_{1,2refr}$, $E_{3refl} + E_{03} = \varepsilon E_{3refr}$, where $\varepsilon = 1 - \omega_p^2/\omega^2$. The angle of total polarization (Brewster's angle) is given by $\kappa\kappa' - k^2 = 0$, or $\tan^2\alpha = 1 - \omega_p^2/\omega^2 = \varepsilon$ (for $\alpha < \pi/4$). The above equations provide generalized Fresnel's relations between the amplitudes of the reflected, refracted and incident waves at the surface for any incidence angle and polarization. They can also be written by using $\omega^2 = \omega_p^2/(1 - \varepsilon)$, where ε is the dielectric function.

The reflection coefficient $R = |\mathbf{E}_{refl}|^2 / |\mathbf{E}_0|^2$ can be obtained straightforwardly from the reflected fields given by equations (20)-(22). It can be written as

$$R = R_1\left[\cos^2\beta\sin^2\varphi + R_2\left(\cos^2\beta\cos^2\varphi + \sin^2\beta\right)\right] \ , \tag{23}$$

where

$$R_1 = \left|\frac{\sqrt{\omega^2\cos^2\alpha - \omega_p^2} - \omega\cos\alpha}{\sqrt{\omega^2\cos^2\alpha - \omega_p^2} + \omega\cos\alpha}\right|^2 \tag{24}$$

and

$$R_2 = \left|\frac{\cos\alpha\sqrt{\omega^2\cos^2\alpha - \omega_p^2} - \omega\sin^2\alpha}{\cos\alpha\sqrt{\omega^2\cos^2\alpha - \omega_p^2} + \omega\sin^2\alpha}\right|^2 \ . \tag{25}$$

The first term in the *rhs* of equation (23) corresponds to $\beta = 0$ ($\varphi = \pi/2$; s-wave, electric field perpendicular to the plane of incidence), while the second term corresponds to $\beta = \alpha$ ($\varphi = 0$; p-wave, electric field in the plane of incidence). It is easy to see that there exists a cusp (shoulder) in the behaviour of the function $R(\omega)$, occurring at the transparency edge $\omega = \omega_p/\cos\alpha$, where the reflection coefficient exhibits a sudden enhancement on passing from the propagating regime to the damped one, as expected (total reflection). The condition for total reflection can also be written as $\sin\alpha = \sqrt{\varepsilon}$, where $R = 1$ ($R_{1,2} = 1$), as it is well known. For illustration, the reflection coefficient is shown in Fig. 1 for $\beta = \pi/6$ and various incidence angles. The reflection coefficient is vanishing for $\omega^2 = \omega_p^2/(1 - \tan^2\alpha)$ for $\alpha = \beta < \pi/4$ ($R_2 = 0$, $\varphi = 0$).

The present approach can be extended to a plasma slab of finite thickness d, $0 < z < d$, where the displacement field $\mathbf{u}$ can be represented as $(\mathbf{v}, u_3)\left[\theta(z) - \theta(z - d)\right]$. We have computed the electromagnetic field inside the slab, the reflected and transmitted fields and the reflection and transmission coefficients. The field inside the slab consists of a superposition of two plane waves $e^{\pm i\kappa' z}$, where κ' is given by the same equation (14). The transparency edge is given by the same equation $\omega\cos\alpha = \omega_p$ as for a semi-infinite plasma. Generalized Fresnel's relations have thereby been obtained, for both surfaces of the slab, any incidence angle and polarization. Apart from characteristic oscillations, the reflection and transmission coefficients exhibit an appreciable enhancement on passing from the propagating regime to the damped regime. The method can also be applied to other structures with more particular geometries.

The same method can be used for treating the plasmons in structures with special geometries. Indeed, the electric force in equation of motion (1) must then be replaced by the Coulomb (non-retarded) force. By using this procedure we have obtained for a semi-infinite plasma the well-known bulk plasmons with frequency ω_p and surface plasmons with frequency $\omega_p/\sqrt{2}$. Similarly, for a plasma slab we have derived the plasmon frequencies given by $\omega_p^2\left(1\pm e^{-kd}\right)/2$.[22]-[29] We have also computed the energy loss for these plasmas and the dielectric response. It is shown that the surface terms do not change the bulk dielectric function as usually defined (*i.e.* for a plane wave), since the surface contributions to the dielectric response are localized. The surface contribution to the energy loss exhibits characteristic oscillations in the transient regime near the surfaces.

It is worth investigating the eigenvalues of the homogeneous system of integral equations (10), for parameter κ given by $\kappa=\sqrt{\omega^2/c^2-k^2}$. Such eigenvalues are given by the roots of the vanishing denominator in equations (18) and (19), *i.e.* by equation $\kappa\kappa^{'}+k^2=0$. This equation has real roots for ω only for the damped regime, *i.e.* for $\kappa=i\left|\kappa\right|$ and $\kappa^{'}=i\left|\kappa^{'}\right|$. Providing these conditions are satisfied, there is only one acceptable branch of excitations, given by

$$\omega^2=\frac{2\omega_p^2c^2k^2}{\omega_p^2+2c^2k^2+\sqrt{\omega_p^4+4c^4k^4}}\ . \tag{26}$$

We can see that $\omega\sim ck$ in the long wavelength limit and it approaches the surface-plasmon frequency $\omega\sim\omega_p/\sqrt{2}$ in the non-retarded limit ($ck\rightarrow\infty$). These excitations are surface plasmon-polariton modes. They imply $v_2=0$ and $v_1,u_3\sim e^{-\left|\kappa^{'}\right|z}$. In addition, a careful analysis of the homogeneous system of equations (10) reveals another branch of excitations, given by $\omega=\omega_p$, which, occurring in this context, may be termed the bulk plasmon-polariton modes. They are characterized by $v_2=0$ and $v_1(\mathbf{k},0)=0$. For all these modes we have $u_3=\left[ic^2k/\left(\omega^2-c^2k^2-\omega_p^2\right)\right](\partial v_1/\partial z)$.

Other effects related to the dynamics of plasmons and polaritons for a semi-infinite electron plasma, or, in general, various plasmas with rectangular geometries, as well as structures with more particular geometries, can be computed similarly by using the method presented here. The dissipation can be included in this treatment (as for metals) and a model can be formulated for dielectrics, amenable to the method presented above. This will allow the treatment of more realistic cases as well as various interfaces, in particular plasmas (or metals) bounded by dielectrics. These investigations are left for forthcoming publications.

Acknowledgments. The authors are indebted to the members of the Laboratory of Theoretical Physics at Magurele-Bucharest for many useful discusssions, and to dr. L. C. Cune for his help in various stages of this work.

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