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Coherent X- and gamma rays from Compton (Thomson) backscattering by a polaritonic pulse

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Abstract

It is shown that Compton (Thomson) backscattering by polaritonic pulses of electrons accelerated with relativistic velocities in a rarefied plasma may produce coherent X- and gamma rays, as a consequence of the quasi-rigidity of the electrons inside the polaritonic pulses and their relatively large number. The classical results of the Compton scattering are re-examined in this context, the energy of the scattered photons and their cross-section are analyzed, especially for backscattering, the great enhancement of the scattered flux of X- or gamma rays due to the coherence effect is highlighted and numerical estimates are given for some typical situations.

Key words: laser accelerated electrons; plasma polaritonic pulses; Compton backscattering; coherent X- and gamma rays

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It is well known that laser pulses focused in a rarefied plasma can accelerate electrons up to considerable relativistic energies in the range of MeV's or even GeV's .[1]-[13] Various models, both analytical and numerical, in particular the particle-in-cell simulations, point toward the basic role played by plasmons and polaritons in laser-driven electron acceleration. It is widely agreed that the propagation of the laser radiation in plasma is governed by polaritonic excitations, arising from electrons interacting with the electromagnetic radiation. The well-known polaritonic dispersion equation is given by $\Omega = \sqrt{\omega_p^2 + \omega^2}$, where $\omega_p = 4\pi n e^2/m$ is the plasma frequency (n being the plasma density, $-e$ - the electron charge and m - the electron mass) and $\omega = ck$ is the frequency of the laser electromagnetic wave ($\mathbf{k}$ being the wavevector and c- the light velocity). Polaritonic pulses propagating with the group velocity $\mathbf{v} = c^2\mathbf{k}/\Omega$ can be formed by a superposition of plane waves with wavevectors $\mathbf{k} = \mathbf{k}_0 + \mathbf{q}$, where $\mathbf{k}_0$ is the wavevector of the laser radiation (frequency $\omega_0 = ck_0$, wavelength $\lambda_0 = 2\pi/k_0$) and the $\mathbf{q}$'s are restricted to $q < q_c \ll k_0$. A wavepacket of linear size $d \simeq 1/q_c \gg \lambda_0$ is then obtained, propagating with the group velocity $v = c\omega_0/\sqrt{\omega_p^2 + \omega_0^2}$. In the particular case of a sufficiently rarefied plasma $\omega_p \ll \omega_0$ this group velocity can be written as $v \simeq c(1 - \omega_p^2/2\omega_0^2)$ and the mobile electrons are transported with the energy

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \simeq mc^2\frac{\omega_0}{\omega_p} \gg mc^2 \ , \tag{1}$$

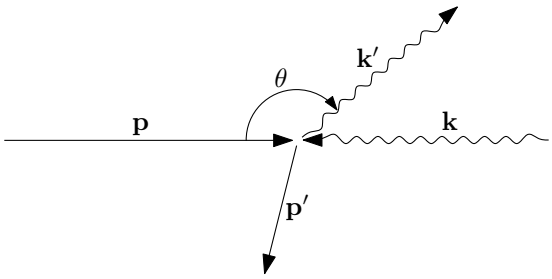

Figure 1: Head-on electron-photon collision.

which may acquire values much higher than the electron rest energy $mc^2 = 0.5MeV$.[14] For typical values $\hbar\omega_0 = 1eV$ ($\lambda_0 = 2\pi c/\omega_0 \simeq 1\mu m$ and $\hbar$ is Planck's constant) and an electron density $n = 10^{18}cm^{-3}$ we get $\hbar\omega_p = 3 \times 10^{-2}eV$ and $E \simeq 17MeV$.

It was shown recently[14] that the propagating polaritonic pulse is polarized, in the sense that the mobile electrons in the propagating pulse are displaced from their equilibrium positions with respect to the quasi-rigid background of positive ions, such that the polarization field compensates, practically, the laser field. The electrons inside the pulse accumulate on the surface of the pulse, along a direction which is transverse to the direction of pulse propagation (laser radiation is transverse), such as a new equibrium is reached, in the presence of the laser field. The number of polarized electrons in the polaritonic pulse, as estimated in Ref. [14], is given

$$N = nd^2\lambda_0 \frac{\varepsilon_p^2}{4mc^2\varepsilon_0^2}\sqrt{\pi\varepsilon_{el}W_0} \ , \tag{2}$$

where $\varepsilon_p = \hbar\omega_p$, $\varepsilon_0 = \hbar\omega_0$ ($\hbar$ is Planck's constant), $\varepsilon_{el} = e^2/d$ and W_0 is the total amount of field energy in the pulse ($W_0 = I_0 d^3/c$, where I_0 is the laser intensity). For typical values $I_0 = 10^{18}w/cm^2$, $d = 1mm$ ($W_0 = 10^{23}eV$ and $\varepsilon_{el} = 10^{-6}eV$), $n = 10^{18}cm^{-3}$ ($\varepsilon_p = 3 \times 10^{-2}eV$), $\varepsilon_0 = 1eV$ ($\lambda_0 \simeq 1\mu$) and $mc^2 = 0.5MeV$ we get $N \simeq 10^{11}$ electrons in the pulse (transported with the energy $\simeq 17MeV$), wich is a relatively high flux of electrons. Their total energy is $W \simeq 10^{18}eV$, the remaining energy (up to $W_0 = 10^{23}eV$) being left in the polarized laser pulse. Numerical data from recent experimental measurements[11]-[13] seem to be in fair agreement with equations (1) and (2) given here.

It is worth emphasizing that the polarized electrons in the polaritonic pulse are practically quasi-rigid (though subjected to very slow density oscillations). They are carried along by the pulse in an inertial motion, while the quasi-rigid ions are depolarized by a wake field and an electron backflow, which give rise to plasma oscillations outside the pulse. This is the well-known picture of wakefield accelerated electrons, and the related bubble models, supported by many theoretical models and numerical simulations.[1], [15]-[19]

The quasi-rigid electrons in the polaritonic pulse moving with relativistic velocities may offer a unique opportunity of coherent Compton backscattering, which may produce coherent high-energetic X- or even gamma rays, *i.e.* an X-ray or gamma-ray laser.

The Compton scattering of gamma rays by a moving electron is shown schematically in Fig. 1. We assume a head-on (unpolarized) collision. From the momentum-energy conservation $p+k = p'+k'$ (where, with usual notations, $p = (E, \mathbf{p})$, $k = (\omega, \mathbf{k})$, etc, $c = \hbar = 1$), written as $p' = p + k - k'$, we get $pk - pk' - kk' = 0$, or, making use of $p^2 = p'^2 = m^2$, $k^2 = k'^2 = 0$,

$$\omega' = \omega \frac{E + |\mathbf{p}|}{E + |\mathbf{p}|\cos\theta + \omega(1 - \cos\theta)} \ . \tag{3}$$

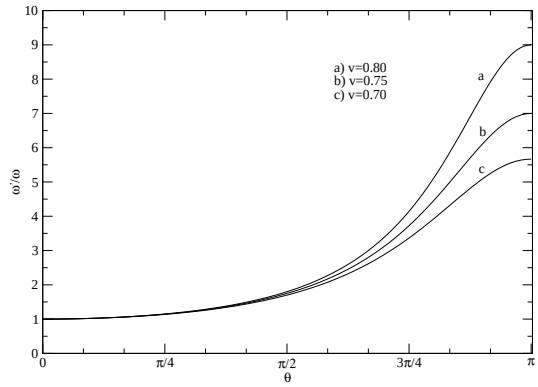

Figure 2: The ratio of the energy of the scattered photon to the energy of the incident photon *vs* scattering angle for a few values of the polariton (electron) velocity v (equation (4) for $1-v \gg 4\gamma^2(1+v)$).

Since $|\mathbf{p}| = vE = mv/\sqrt{1-v^2}$, this equation can also be written as

$$\omega' = \omega \frac{1+v}{1+v\cos\theta + \gamma\sqrt{1-v^2}(1-\cos\theta)} \ , \tag{4}$$

where $\gamma = \omega/m$ and v is the velociy of the electron (velocity of the polaritonic pulse). For all relevant situations (except ultrarelativistic limit) the inequality $2\gamma\sqrt{1+v} \ll \sqrt{1-v}$ is satisfied (Thomson scattering). The ratio ω'/ω given by equation (4) *vs* angle θ is shown in Fig. 2 in this case (for $4\gamma^2(1+v) \ll 1-v$) for a few values of the parameter v. The maximum value of the frequency ω' of the scattered photon is obtained for the scattering angle $\theta \simeq \pi$ (backscatering). This increase is sometimes assigned to a Doppler effect, which would introduce a relativistic factor $4/(1-v^2) \simeq (1+v)/(1-v)$ for $v \simeq 1$. For the typical parameter values used in this paper $1-v \simeq \omega_p^2/2\omega_0^2 \simeq 4.5\times 10^{-4}$, which is much greater than $2\gamma\sqrt{1-v^2} \simeq 10^{-7}$ (we take the frequency of the incident photon $\omega = 1eV$, $\gamma \simeq 2\times 10^{-6}$). Therefore, we may neglet the γ-term in equation (4), and get a maximum scattered frequency

$$\omega' \simeq \omega \frac{1+v}{1-v} \simeq 10keV \tag{5}$$

for the backscattering angle $\theta = \pi$. It is easy to see that an increase by an order of magnitude in the energy of the accelerated electrons ($E_{el} \simeq m\omega_0/\omega_p$) means a decrease by two orders of magnitude in $1-v$ ($1-v \simeq \omega_p^2/2\omega_0^2$), such that, by equation (5), we may get $\omega' \simeq 1MeV$ for the frequency of the backscattered gamma rays. Such high backscattering frequencies are concentrated around $\theta = \pi$ within a range $\Delta\theta \simeq \sqrt{2(1-v)/3v}$.

The well-known Compton cross-section can be written as[20]

$$\begin{aligned} d\sigma = 8\pi r_e^2 \tfrac{m^2 d(-t)}{(s-m^2)^2}[\left(\tfrac{m^2}{s-m^2} + \tfrac{m^2}{u-m^2}\right)^2 + \\ + \tfrac{m^2}{s-m^2} + \tfrac{m^2}{u-m^2} - \tfrac{1}{4}\left(\tfrac{s-m^2}{u-m^2} + \tfrac{u-m^2}{s-m^2}\right)] \ , \end{aligned} \tag{6}$$

where $r_e = e^2/m$ is the classical electron radius and

$$\begin{aligned} s = (p+k)^2 = m^2 + 2pk \ , \ u = (p-k')^2 = m^2 - 2pk' \ , \\ t = (k'-k)^2 = -2kk' \end{aligned} \tag{7}$$

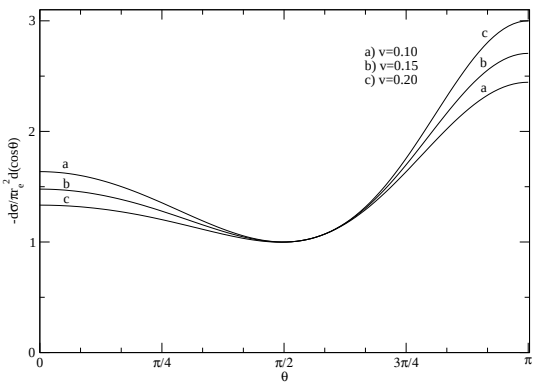

Figure 3: Compton cross-section *vs* scattering angle for a few values of the polariton (electron) velocity v (equation (9), $1-v \gg 4\gamma^2(1+v)$).

are the invariant kinematical variables. By straightforward calculations this expression can be put in the form

$$d\sigma = \pi r_e^2 \frac{(1-v^2)\sin\theta d\theta}{\left[1+v\cos\theta+\gamma\sqrt{1-v^2}(1-\cos\theta)\right]^2} \times$$
$$\times \left[\left(\frac{v+\cos\theta}{1+v\cos\theta}\right)^2 + \gamma\sqrt{1-v^2}\frac{1-\cos\theta}{1+v\cos\theta} + \frac{1+v\cos\theta}{1+v\cos\theta+\gamma\sqrt{1-v^2}(1-\cos\theta)}\right] , \tag{8}$$

where the transport velociy v is shown explicitly. Similarly, for the parameter values used here we may neglect the γ-terms in equation (8) (Thomson scattering), and get

$$d\sigma \simeq \pi r_e^2 \frac{(1-v^2)}{(1+v\cos\theta)^2}\left[\left(\frac{v+\cos\theta}{1+v\cos\theta}\right)^2 + 1\right]\sin\theta d\theta \ . \tag{9}$$

This cross-section is shown in Fig. 3 for a few values of the parameter v. The total backscattering cross-section is given by

$$\sigma_b \simeq \pi r_e^2 \frac{(1-v^2)}{(1+v\cos\theta)^2}\left[\left(\frac{v+\cos\theta}{1+v\cos\theta}\right)^2 + 1\right]\Bigg|_{\theta=\pi} \Delta(-\cos\theta) = \pi r_e^2 \frac{1+v}{1-v}(\Delta\theta)^2 \simeq 4\pi r_e^2/3 \tag{10}$$

and the rate of the bakscattered photons is $dN_{ph}/dt = c\sigma_b n_{ph}$, where n_{ph} is the photon density in the incident flux. The energy loss of the scattered (recoil) electron for backscatering is $\Delta E = \omega' - \omega \simeq 2\omega v/(1-v)$ ($\Delta E/E \simeq 2\gamma v\sqrt{(1+v)/(1-v)} \ll 1$), which is approximately equal with the energy of the scattered photon $\omega' \simeq \omega(1+v)/(1-v)$ given above for $v \simeq 1$ (since $\omega \ll \omega'$). The momentum transferred to the electron in the scattering process is very small, in comparison with the initial momentum of the electron. It is important to note that for a polaritonic pulse this momentum is transferred to the whole ensemble of electrons, as a consequence of the rigidity of the electrons in the polaritonic pulse. For the sake of the comparison, we note that the total cross-section is $8\pi r_e^2/3 \simeq 2\sigma_b$, as it is well known.

The cross-section computed above refers to one electron (and one photon). The field bi-spinors in the interaction matrix element (the scattering amplitude) between the initial state and the final state are normalized to unity. If we have N electrons, then each of them contributes individually to the cross-section, which is multiplied by N (*i.e.* $\sigma_b \to N\sigma_b$). This is an incoherent scattering. For the electrons in the polaritonic pulse the situation is different. These electrons are not independent anymore (because of their rigidity inside the pulse), and they suffer the scattering collectively. This amounts to normalize the bi-spinors to N, such that each bi-spinor carries now a factor

$\sqrt{N}$. Consequently, the scattering amplitude acquires an additional factor N and the cross-section acquires an additional factor N^2. In comparison with the incoherent scattering we get an additional factor N in the coherent scattering, which increases considerably the cross-section for large values of N.[21]-[25]

From the above estimations we can see that the energy of the backscattered photons is much higher than the energy of the incident photons. Therefore, in the following estimations we can neglect the energy of the incident photons. The energy of the scattered photons is produced at the expense of the energy of the electrons. By successive Compton scattering we may expect a certain limitation on the duration of the scattering process for electron pulses (beside the limitations caused by the pulse duration, both for the electrons and the incident photons). Such a limitation is more stringent for the coherent scattering (due to the occurrence of the factor N^2).

Making use of the rate $d^2N_{ph}/d\theta dt = c(d\sigma/d\theta)n_{ph}$ of the scattered photons we can write down the rate of the energy produced by Compton (Thomson) scattering

$$dE^{coh} = \left(\int d\theta\omega' dN_{ph}/d\theta\right) dt = N^2 c n_{ph} \left(\int \omega' d\sigma\right) dt \ . \tag{11}$$

The integral in equation (11) can be computed by using ω' given by equation (4) (with $\gamma = 0$) and the differential cross-section given by equation (9). The result is

$$dE^{coh} = \frac{8\pi}{3} N^2 \omega c r_e^2 n_{ph} \frac{1}{1-v} dt \ . \tag{12}$$

This energy must be compared with the energy loss of the electrons in the polaritonic pulse,

$$-NdE = -Nmd\frac{1}{\sqrt{1-v^2}} \ . \tag{13}$$

Integrating the equation $dE^{coh} = -NdE$ with the new variable $x = m/E$, we get easily

$$\frac{8\pi}{3} N\omega c n_{ph} \Delta t = m \int_{x_0}^{1} dx \frac{1}{1+\sqrt{1-x^2}} \ , \tag{14}$$

where $x_0 = m/E_0 \ll 1$ corresponds to the initial energy of the polaritonic pulse. The integral in equation (14) can easily be estimated ($\simeq \pi/2 - 1$), so we get the duration Δt of the scattering

$$\Delta t \simeq (\pi/2 - 1)\frac{3mc}{8\pi N\hbar\omega r_e^2 n_{ph}} \ , \tag{15}$$

where we have re-established in full the universal constants.

We assume an incident flow of photons with intensity $I = 10^{14} w/cm^2$ focused on a spatial region of size $d = 1mm$ (picosecond pulses); the energy is $W = Id^3/c \simeq 3J$ and, for photon energy $\omega = 1eV$, we get a photon density $n_{ph} \simeq 5 \times 10^{22} cm^{-3}$. For $N = 10^{11}$ given before for the polaritonic pulse (and $r_e = 2.8 \times 10^{-13} cm$) we get $\Delta t \simeq 10^{-15} s$ (femtoseconds). This time is an estimate for the duration of the collision, and for the duration of emission of the backscattered photons. As we can see, it does not depend, practically, on the electron energy in the polaritonic pulse (E_0), for high, relativistic energies. It is expected that the polaritonic pulse is "stopped", and, in fact, destroyed, after the lapse of this time.

The total energy of the backscatterd photons can be estimated similarly, by using equation (11) and dt/dE from $dE^{coh} = -NdE$, where dE^{coh} is given by equation (12). Let us assume that we

are interested in the photon bakscattering within an angle $\Delta\theta = \alpha\sqrt{2(1-v)/3v}$, with $\alpha \ll 1$. Then, we get easily

$$E_b^{coh} = \frac{1}{4}N\alpha^2 \int_m^{E_0} \frac{(1+v)^2}{v} dE \ , \tag{16}$$

and, folowing the same technique as above, we get $E_b^{coh} \simeq \alpha^2 N E_0$, where we can recognize the total energy of the polaritonic pulse $W = NE_0$. This result is valid for $\alpha \ll 1$. For high, relativistic velocities $\alpha \simeq 1$, and practically the whole polaritonic energy is recovered in the backscattering photons.

In conclusion, we may say that the polaritonic pulses of electrons transported by laser radiation focused in a rarefied plasma may serve as targets for coherent Compton backscattering in the X-rays or gamma rays energy range, therefore as a means for obtaining an X-ray or gamma ray laser. The coherent scattering, which enhances considerably the photon output and ensure its coherence, is due to the quasi-rigidity of the electrons in the propagating polaritonic pulse, which ensures (within certain limits) the stability of this interacting formation of matter and electromagnetic radiation. The energy and cross-section of the Compton (Thomson) backscattering was re-examined in this paper in the context of the coherent scattering by polaritonic pulses, and the (pulse) duration of the backscattering emission was also estimated. Similar ideas have been advanced recently, especially for laser-driven accelerated electron mirrors.[26]-[32]

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