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Non-inertial electromagnetic effects in matter. Gyromagnetic effect.

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Abstract

Non-inertial electromagnetic effects in matter, *i.e.* electromagnetic fields created by a non-inertial motion of material bodies, are discussed within the Drude-Lorentz (plasma) model of matter polarization. It is shown that an oscillatory motion of a point-like body, or wavelike motion in an extended body give rise to electromagnetic fields with the same frequency as the frequency of the original motion, while shock-like movements of a point-like body generate electromagnetic fields with the characteristic (atomic scale) frequency of the bodies. Based on these facts, a possible, qualitative explanation is put forward for sonoluminscence. The polarization of a rigid body induced by rotations is discussed in various circumstances. A uniform rotation produces a static electric field in a dielectric and a stationary current (and a static magnetic field) in a conductor. The latter corresponds to the gyromagnetic effect (while the former may be called the gyroelectric effect). Both fields are computed for a sphere and the gyromagnetic coefficient is derived. A non-uniform rotation induces emission of electromagnetic fields. The equations of motion for the polarization are linearized for slight non-uniformites of the angular velocity and solved both for a dielectric and a conducting sphere. The electromagetic field emitted by a dielectric spherically-shaped body in (a slightly) non-uniform rotation has the characteristic (atomic scale) frequency of the body (slightly shifted by the uniform part of the angular frequency). In the same conditions, a conducting sphere emits an electromagnetic field whose frequency is double the uniform part of the angular fequency.

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Introduction

The magnetization of a rotating body (Barnett effect),[1] or the rotation of a magnetized body (Einstein-de Haas effect)[2] are both known as the gyromagnetic effect(s). By extension, the rotation of a spin (magnetic dipole) in a magnetic field, or the rotation of an electric dipole in an electric field are also called gyromagnetic (respectively gyroelectric) effects (phenomena). Such effects are attractive experimental tools, since they provide access to magnetic susceptibility, orbital magnetic effects, magnetic properties of matter, including powders and granular matter, etc.[3]-[11] Although these effects are known for a long time, the gyromagnetic coefficient for macroscopic bodies remained a phenomenological parameter. Here, we derive the gyromagnetic coefficient for a sphere, by using the well-known Drude-Lorentz (plasma) model of matter polarization. Historically, the Drude-Lorentz model has proven useful in describing the electric conduction or the optical properties of matter.[12]-[17] Recently, it was used for describing the reflection and refraction, as well as plasmons, polaritons and van der Waals-London and Casimir forces in matter interacting with the electromagnetic field.[18, 19]

Here, we put the problem in more general terms. Since the rotation associated with the gyromagnetic effect is a non-inertial motion, we can extend the resulting electromagnetic properties to other non-inertial motions, like translations, for instance. We show here that a point-like body in oscillatory motion, or a wave propagating in an extended body generate electromagnetic fields with the same frequency as the frequency of the oscillatory motion or the wave frequency. A shock-like movement of a point-like body generate electromagnetic fields with the characteristic (atomic scale) frequency of the body. A possible, qualitative explanation of the sonoluminiscence[20, 21] is advanced on this basis. It is also shown that a uniformly rotating dielectric sphere develops a static (quadrupolar) electric field (which may be called the gyroelectric effect), while a uniformly rotating conducting sphere sustains stationary electric currents which generate a magnetic field. This is the gyromagnetic effect, and the gyromagnetic coefficient is computed here for a spherically-shaped body. Slightly non-uniform rotations are also discussed in the context of the linearized equations of motion for the polarization. It is shown that a dielectric spherically-shaped body in a slightly non-uniform rotation generates electromagnetic fields with the characteristic (atomic scale) frequency of the body (slightly shifted by the uniform part of the angular frequency), while a conducting sphere in the same conditions emits electromagnetic fields with the frequency equal to the double of the uniform part of the angular velocity.

Non-inertial translations

We assume a simple model of homogeneous matter consisting of identical, mobile charges q moving in a neutralizing background of charges $-q$. A local relative displacement $\mathbf{u}$ generates a polarization charge density ρ and a polarization current density $\mathbf{j}$ given by

$$\rho = -nq div\mathbf{u} \ , \ \mathbf{j} = nq\dot{\mathbf{u}} \ , \tag{1}$$

where n is the charge density. The polarization (dipole momentum of the unit volume) is given by $\mathbf{P} = qn\mathbf{u}$, so the charge and current densities can also be written in the usual form $\rho = -div\mathbf{P}$ and $\mathbf{j} = \dot{\mathbf{P}}$. Let the background moves as a rigid body with velocity $\mathbf{V}$ and let $m\omega_c^2\mathbf{u}$ be an elastic force acting locally upon the charges, where m is the (reduced) mass of the charges and ω_c is a characteristic frequency. The frequency ω_c is an atomic-scale frequency, corresponding to a model of dielectrics. For conductors, where the electrons are quasi-free, $\omega_c = 0$. The equation of motion for the charge displacement reads

$$m\ddot{\mathbf{u}} = -m\dot{\mathbf{V}} - m\omega_c^2\mathbf{u} - m\gamma\dot{\mathbf{u}} \ , \tag{2}$$

where γ is a damping coefficient ($\gamma \ll \omega_c$, for $\omega_c \neq 0$). Usually, we set $\gamma = 0$, as for an ideal body (plasma). Similarly, we neglect the collision processes of the charges in the body. Making use of the Fourier transforms we get

$$\mathbf{u}(\omega) = \frac{-i\omega\mathbf{V}(\omega)}{\omega^2 - \omega_c^2 + i\omega\gamma} \tag{3}$$

and

$$\mathbf{u}(t) = \frac{1}{2\pi}\int d\omega \frac{-i\omega\mathbf{V}(\omega)}{\omega^2 - \omega_c^2 + i\omega\gamma} e^{-i\omega t} \tag{4}$$

(where the integration must be performed in the lower half-plane).

For an oscillatory motion with amplitude B and frequency Ω the velocity is given by $V(t) = B\Omega\cos\Omega t$ and

$$u(t) = -\frac{B\Omega^2}{\Omega^2-\omega_c^2}\sin\Omega t \tag{5}$$

(where we dropped out the damping factor γ). For a wave with a wavevector k, $V(t) = B\Omega\cos(\Omega t - kx)$ and

$$u(t) = -\frac{B\Omega^2}{\Omega^2-\omega_c^2}\sin(\Omega t - kx)\ . \tag{6}$$

For a shock of duration T ($T \ll t$) and velocity V_0, $V(t) = TV_0\delta(t)$ and

$$u(t) = TV_0\cos\omega_c t\ . \tag{7}$$

The polarization charges and currents corresponding to the polarization $\mathbf{P} = qn\mathbf{u}$ give rise to electromagnetic fields described by the Kirchhoff's retarded potentials

$$\begin{aligned} \Phi(\mathbf{R},t) &= \int d\mathbf{R}'\frac{\rho(\mathbf{R}',t-|\mathbf{R}-\mathbf{R}'|/c)}{|\mathbf{R}-\mathbf{R}'|}\ , \\ \mathbf{A}(\mathbf{R},t) &= \frac{1}{c}\int d\mathbf{R}'\frac{\mathbf{j}(\mathbf{R}',t-|\mathbf{R}-\mathbf{R}'|/c)}{|\mathbf{R}-\mathbf{R}'|}\ , \end{aligned} \tag{8}$$

where the integration is performed over the volume of the body.

For a point-like body located at the origin we take $\mathbf{u} = v\mathbf{u}_0(t)\delta(\mathbf{R})$, where v is the volume of the body. The polarization charge and current are given by

$$\rho = -nqv(\mathbf{u}_0 grad)\delta(\mathbf{R})\ ,\ \mathbf{j} = nqv\frac{\partial\mathbf{u}_0}{\partial t}\delta(\mathbf{R})\ . \tag{9}$$

The potentials can easily be computed:

$$\begin{aligned} \Phi(\mathbf{R},t) &= nqv\left[\frac{\mathbf{R}}{cR^2}\frac{\partial\mathbf{u}_0(t-R/c)}{\partial t} + \frac{\mathbf{R}\mathbf{u}_0(t-R/c)}{R^3}\right]\ , \\ \mathbf{A}(\mathbf{R},t) &= nqv\frac{1}{cR}\frac{\partial\mathbf{u}_0(t-R/c)}{\partial t}\ . \end{aligned} \tag{10}$$

(We can check the Lorenz gauge $div\mathbf{A} + (1/c)\partial\Phi/\partial t = 0$). Making use of equation (5) for an oscillatory motion we get the potentials

$$\begin{aligned} \Phi(\mathbf{R},t) &= -\frac{Bnqv\Omega^2}{\Omega^2-\omega_c^2}\left[\frac{\mathbf{en}}{cR^2}\Omega\cos\Omega(t-R/c) + \frac{\mathbf{Re}}{R^3}\sin\Omega(t-R/c)\right]\ , \\ \mathbf{A}(\mathbf{R},t) &= -\frac{Bnqv\Omega^2}{\Omega^2-\omega_c^2}\frac{\mathbf{e}}{cR}\Omega\cos\Omega(t-R/c)\ , \end{aligned} \tag{11}$$

where $\mathbf{e}$ is the unit vector along the direction of the motion. We can see that the body in oscillatory motion radiates electromagnetic waves with the motion frequency Ω. Making use of equation (7) for a shock, we get

$$\begin{aligned} \Phi(\mathbf{R},t) &= nqvTV_0\left[-\frac{\mathbf{Re}}{cR^2}\omega_c\sin\omega_c(t-R/c) + \frac{\mathbf{Re}}{R^3}\cos\omega_c(t-R/c)\right]\ , \\ \mathbf{A}(\mathbf{R},t) &= -nqvTV_0\frac{\mathbf{e}}{cR}\omega_c\sin\omega_c(t-R/c) \end{aligned} \tag{12}$$

and the radiation emitted has the characteristic frequency ω_c of the body.

Let us assume now that we have an infinitely-extending body subjected to a wave-like motion, as given by equation (6). The potentials are given by

$$\begin{aligned}\Phi(\mathbf{R},t) &= -\tfrac{8\pi \mathbf{Bk}nq\Omega^2}{\Omega^2-\omega_c^2}\tfrac{\cos(\Omega t-\mathbf{kR})}{k^2-\Omega^2/c^2}\ ,\\ \mathbf{A}(\mathbf{R},t) &= -\tfrac{8\pi \mathbf{B}nq\Omega^3}{c(\Omega^2-\omega_c^2)}\tfrac{\cos(\Omega t-\mathbf{kR})}{k^2-\Omega^2/c^2}\ ,\end{aligned} \tag{13}$$

where we can introduce the velocity $v = \Omega/k$ of the propagating wavelike motion of the background. We can see that the emitted radiation has the frequency Ω of the wave propagating in the body. However, an ultrasound wave propagating in a liquid containing small gas bubbles can squeeze these bubbles off (according to the current ideas in the field[20, 21]), such as to create small, localized shocks, which ionize the gas and the liquid. Then, the moving charges can give rise to an electromagnetic field with high frequencies (ω_c), as described by equations (12) for shocks. This might be the origin of the sonoluminiscence.

We can see from the above discussion that a non-uniform translation of a body can generate electromagnetic fields. This is a non-inertial electromagnetic effect.

Rotations. Static electric field

Let us assume that the rigid background is moving with an angular velocity $\overrightarrow{\Omega}$. The motion of the displacement $\mathbf{u}$ is described by

$$\ddot{\mathbf{u}} = \mathbf{r}\times\dot{\overrightarrow{\Omega}} + 2\dot{\mathbf{u}}\times\overrightarrow{\Omega} + \overrightarrow{\Omega}\times(\mathbf{r}\times\overrightarrow{\Omega}) - \omega_c^2\mathbf{u} - \gamma\dot{\mathbf{u}}\ , \tag{14}$$

where the first term in the *rhs* comes from the non-uniformity of the angular velocity, the second represents the Coriolis force and the third is the centrifugal force. We chose $\overrightarrow{\Omega}$ and $\dot{\overrightarrow{\Omega}}$ oriented along the z-axis, *i.e.* $\overrightarrow{\Omega} = \Omega\mathbf{e}_z$ and $\dot{\overrightarrow{\Omega}} = \dot{\Omega}\mathbf{e}_z$, and see immediately that $u_z = 0$. We get two coupled equations

$$\begin{aligned}\ddot{u}_x + \omega_c^2 u_x + \gamma\dot{u}_x - 2\Omega\dot{u}_y &= y\dot{\Omega} + x\Omega^2\ ,\\ \ddot{u}_y + \omega_c^2 u_y + \gamma\dot{u}_y + 2\Omega\dot{u}_x &= -x\dot{\Omega} + y\Omega^2\end{aligned} \tag{15}$$

for the other two components of the displacement.

We consider first $\Omega = const$ (a uniform rotation). Usually, $\Omega \ll \omega_c$. Averaging out the high-frequency oscillations of the solution of the homogeneous system of equations (15), we are left with the (particular) solution ($\gamma = 0$)

$$u_x = x\frac{\Omega^2}{\omega_c^2}\ ,\ u_y = y\frac{\Omega^2}{\omega_c^2}\ . \tag{16}$$

We can see that the centrifugal force pushes the charges towards the surface of the rotating body. Since $\mathbf{u}$ does not depend on the time, the current, the vector potential and the magnetic field are vanishing. We are left with a static scalar potential as given by equation (8). We compute this scalar potential for a spherically-shaped body of radius a, for which the displacement field $\mathbf{u}$ reads

$$\mathbf{u} = \frac{\Omega^2}{\omega_c^2}(x,\, y,\, 0)\theta(a-R)\ , \tag{17}$$

where $\theta(x) = 1$ for $x > 0$ and $\theta(x) = 0$ for $x < 0$ is the step function. The calculations are straightforward. Making use of equation (1) we compute the polarization charge density (paying attention to the derivatives of the step function), expand the Coulomb potential in equation (8) in Legendre polynomials and use the well-known addition theorem for the two vectors $\mathbf{R}$ and $\mathbf{R}'$. We get easily the potential

$$\Phi(\mathbf{R}) = -4\pi nq\frac{\Omega^2}{\omega_c^2}\begin{cases} \frac{1}{3}a^2 - \frac{1}{5}(2R^2 - z^2) \ , \ R < a \ , \\ \\ \frac{a^2}{15R^3}\left(\frac{3z^2}{R^2} - 1\right) \ , \ R > a \ . \end{cases} \tag{18}$$

We can see that this is a quadrupole potential, with a charge contribution inside the sphere. The corresponding electric field $\mathbf{E} = -grad\Phi$ can be easily computed from equation (18). We give here the electric field inside the sphere,

$$\mathbf{E}(\mathbf{R}) = -\frac{16\pi}{5}nq\frac{\Omega^2}{\omega_c^2}\left(x, \, y, \, \frac{1}{2}z\right) \ . \tag{19}$$

We can see that the x, y-components of the electric field are proportional to the corresponding components of the polarization ($P = nq\mathbf{u}$, where $\mathbf{u}$ is given by equation (17)). In addition, there appears also a z-component of the electric field, due to the non-uniform polarization along this direction. It is also worth noting that the average of the electric field (and polarization) over the volume of the sphere is vanishing. We can call this effect the gyroelectric effect.

Rotations. Gyromagnetic effect

For conductors, the situation is different. For $\omega_c = 0$ (and $\dot{\Omega} = 0$, $\gamma = 0$), equations (15) become

$$\ddot{u}_x - 2\Omega\dot{u}_y = x\Omega^2 \ , \ \ddot{u}_y + 2\Omega\dot{u}_x = y\Omega^2 \ . \tag{20}$$

Again, averaging out the oscillating terms, we are left with the solution

$$\dot{u}_x = \frac{1}{2}\Omega y \ , \ \dot{u}_y = -\frac{1}{2}\Omega x \ . \tag{21}$$

We can see that the combined effect of the Coriolis and the centrifugal forces leads to the occurrence of circular polarization currents in the plane transverse to the rotation axis. These currents (given by $\mathbf{j} = nq\dot{\mathbf{u}}$) are stationary and divergence-free ($div\mathbf{j} = 0$). They generate a magnetic field $\mathbf{H} = curl\mathbf{A}$, where $\mathbf{A}$ is given by equations (8). Again, the calculations for a spherically-shaped body are straightforward. This time, the contributions are dipolar (arising from the associated Legendre function P_1^1). We get the vector potential

$$\mathbf{A}(\mathbf{R}) = \frac{\pi nq\Omega}{3c}(y, \, -x, \, 0)\begin{cases} a^2 - \frac{3}{5}R^2 \ , \ R < a \ , \\ \\ \frac{2a^5}{5R^2} \ , \ R > a \ . \end{cases} \tag{22}$$

and the magnetic field inside the sphere

$$\mathbf{H} = -\frac{2\pi nq\Omega}{5c}\left(xz, \, yz, \, \frac{5}{3}a^2 - (2R^2 - z^2)\right) \ . \tag{23}$$

We can see that, both inside and outside the sphere, the magnetic field has the specific two-poles pattern of a magnetic dipole.

The average of the magnetic field over the volume of the sphere gives the only non-vanishing contribution

$$\overline{H}_z = -\frac{16\pi nqa^2}{25c}\Omega \ . \tag{24}$$

The coefficient in front of the angular velocity Ω in equation (24) is the coeficient of the gyromagnetic effect. For non-magnetic matter $\overline{H}_z$ is the magnetization, and $\mu = V\overline{H}_z$ is the magnetic moment, where V is the volume of the body. Equation (24) can then be written as

$$\mu = -\frac{8\pi}{5}\frac{Q}{Mc}L \ , \tag{25}$$

where Q is the total (mobile) charge of the body, M is the mass of the body and L is the angular momentum (with respect to the rotation axis). From the equation of motion $d\mathbf{L}/dt = \overrightarrow{\mu} \times \mathbf{H}_0$ in an external magnetic field $\mathbf{H}_0$, we get $d\overrightarrow{\mu}/dt = \gamma\mathbf{H}_0 \times \overrightarrow{\mu}$, where $\gamma = (8\pi/5)(Q/Mc)$ is the gyromagnetic ratio. By analogy with the quantum particles, the numerical factor $-16\pi/5$ in equation (25) can be viewed as the g-factor in the relationship between the magnetic moment and the angular momentum.

Similar calculations can be done for other shapes of the bodies, however, with appreciable technical dificulties in many cases.

Time dependence. Dielectrics

In order to include the time dependence we linearize equations (15) by writing

$$\Omega = \Omega_0 + \Omega_1(t) \tag{26}$$

and

$$u_{x,y} = u_{x,y0} + u_{x,y1}(t) \ , \tag{27}$$

where $\Omega_0 = const$, $\Omega_1(t) \ll \Omega_0$ and $u_{x,y0} = const$ are given by equations (16) (where Ω is replaced by Ω_0). It is not necessary to have $u_{x,y1}(t) \ll u_{x,y0}$. Upon linearization equations (15) become

$$\begin{aligned} &\ddot{u}_{x1} + \omega_c^2 u_{x1} + \gamma\dot{u}_{x1} - 2\Omega_0\dot{u}_{y1} = y\dot{\Omega}_1 + 2x\Omega_0\Omega_1 \ , \\ &\ddot{u}_{y1} + \omega_c^2 u_{y1} + \gamma\dot{u}_{y1} + 2\Omega_0\dot{u}_{x1} = -x\dot{\Omega}_1 + 2y\Omega_0\Omega_1 \ . \end{aligned} \tag{28}$$

We take the Fourier transform of these equations and omit for the moment the argument ω in $u_{x,y1}(\omega)$ and $\Omega_1(\omega)$. Equations (28) read

$$\begin{aligned} &(\omega^2 - \omega_c^2 + i\omega\gamma)u_{x1} - 2i\Omega_0\omega u_{y1} = C \ , \\ &(\omega^2 - \omega_c^2 + i\omega\gamma)u_{y1} + 2i\Omega_0\omega u_{x1} = D \ , \end{aligned} \tag{29}$$

where

$$C = (iy\omega - 2x\Omega_0)\Omega_1 \ , \ D = -(ix\omega + 2y\Omega_0)\Omega_1 \ . \tag{30}$$

We note the symmetry of the system of equations (29) for $x \longleftrightarrow y$ and $\Omega_{0,1} \longleftrightarrow -\Omega_{0,1}$. The solutions of equations (29) can readily be obtained. For $\Omega_0 \ll \omega_c$ (which is the realistic condition) they have four poles $\pm\omega_c \pm \Omega_0 - i\gamma/2$ in the lower half-plane. We take the inverse Fourier transform (the integration must be performed in the lower half-plane) and get

$$\begin{aligned} &u_{x1}(t) = \Omega_1(\omega_c)\cos\omega_c t\,(y\cos\Omega_0 t - x\sin\Omega_0 t) \ , \\ &u_{y1}(t) = -\Omega_1(\omega_c)\cos\omega_c t\,(y\sin\Omega_0 t + x\cos\Omega_0 t) \ . \end{aligned} \tag{31}$$

In the amplitudes of the oscillating functions in equations (31) we used the approximation $\Omega_0 \ll \omega_c$, and assumed also $\Omega_1(\omega_c) = \Omega_1^*(\omega_c)$ (and put $\gamma = 0$). As a consequence of this approximation, the displacement components given by equations (31) are determined solely by the non-uniform angular velocity (the $\dot{\Omega}$- term in equation (14)), the role of the Coriolis force being that of shifting the frequency ω_c to $\omega_c \pm \Omega_0$. Within this linear approximation the centrifugal force does not contribute (it determines the constant displacements $u_{x,y0}$). Comparing equations (16) with equations (31), we can see that, although $\Omega_1 \ll \Omega_0 \ll \omega_c$, it is possible to have $u_{xy1} \gg u_{x,y0}$, if the Fourier component $\Omega_1(\omega_c)$ is sufficiently large (as, for instance, for shocks). Further on, we leave aside the displacements $u_{x,y0}$ and focus ourselves on the effect of the time-dependent components $u_{x,y1}$. It is easy to see from equations (31) that the displacement $\mathbf{u}$ performs a wobbling motion, including a rotation with frequency Ω_0 and a radial oscillation with freuency ω_c ($u_{x1}^2+u_{y1}^2 = \Omega_1^2(\omega_c)r^2\cos^2\omega_c t$).

We simplify further equations (31) by noticing that they imply oscillations with the frequencies $\omega_c \pm \Omega_0$ where we may neglect Ω_0 in comparison with ω_c. We get

$$u_{x1}(t) = \Omega_1 y \cos\omega_c t \ , \ u_{y1}(t) = -\Omega_1 x \cos\omega_c t \ . \tag{32}$$

Making use of equations (1), we can see that the polarization charge density is vanishing for a sphere ($div\,[\mathbf{u}_1\theta(a-R)] = 0$, where a is the radius of the sphere); we are left with the polarization current density

$$\mathbf{j}_1 = nq\Omega_1\omega_c(-y,\, x)\theta(a-R)\sin\omega_c t \ , \tag{33}$$

which we use to compute the vector potential from equations (8). The vector potential $\mathbf{A}$ given by equations (8) contains the "retarded" Coulomb potential $e^{i\omega_c|\mathbf{R}-\mathbf{R}'|/c}/\,|\mathbf{R}-\mathbf{R}'|$. This potential has an expansion in Legendre polynomials P_n, whose coefficients are given in Ref. [20]. It reads

$$\begin{gathered}\frac{e^{i\lambda|\mathbf{R}-\mathbf{R}'|}}{|\mathbf{R}-\mathbf{R}'|} = \\ = \frac{i\pi}{2}(RR')^{-1/2}\sum_{n=0}(2n+1)J_{n+1/2}(\lambda R_<)H_{n+1/2}(\lambda R_>)P_n(\cos\Theta) \ ,\end{gathered} \tag{34}$$

where $J_{n+1/2}$ and $H_{n+1/2}$ are the Bessel and, respectively, Hankel functions of the first rank, Θ is the angle between the two vectors $\mathbf{R}$ and $\mathbf{R}'$, $R_< = \min(R, R')$, $R_> = \max(R, R')$ and λ is any real parameter ($\lambda = \omega_c/c$). We limit ourselves to the radiation zone $R > a$, $\lambda R \gg 1$ and a macroscopic body for which $\lambda a \gg 1$. We get the vector potential

$$\mathbf{A}(\mathbf{R}) = \frac{4\pi nqa^2c\Omega_1}{\omega_c R^2}\sin\frac{\omega_c a}{c}(-y, x)\sin\omega_c(t - R/c) \ . \tag{35}$$

This vector potential has a dipolar character and satisfies $div\mathbf{A} = 0$. We can see that a dielectric spherically-shaped body which rotates about an axis with a (slightly) non-uniform angular velocity ($\Omega = \Omega_0 + \Omega_1$, $\Omega_0 = const$, $\dot{\Omega}_1 \neq 0$, $\Omega_1 \ll \Omega_0$) emits radiation with the frequency $\simeq \omega_c$, where ω_c is the characteristic (atomic scale) frequency of the body. The amplitude of the emitted field is governed by the Fourier transform $\Omega_1(\omega_c)$ of the angular velocity.

Time dependence. Conductors

In order to cary out the linearization procedure for conductors ($\omega_c = 0$) we introduce the new variables $v_{x,y} = \dot{u}_{x,y}$ and write equations (15) as ($\gamma = 0$)

$$\begin{gathered}\dot{v}_x - 2\Omega v_y = y\dot{\Omega} + x\Omega^2 \ , \\ \dot{v}_y + 2\Omega v_x = -x\dot{\Omega} + y\Omega^2 \ .\end{gathered} \tag{36}$$

Here we set $\Omega = \Omega_0 + \Omega_1(t)$ and $v_{x,y} = v_{x,y0} + v_{x,y1}$, where $\Omega_1 \ll \Omega_0$, $\Omega_0 = const$ and $v_{x0} = \Omega_0 y/2$, $v_{y0} = -\Omega_0 x/2$. The constant components $v_{x,y0}$ are given by eqaution (21) and causes the gyromagnetic effect. We focus here on the time dependent components. The Fourier transforms of the solution of the linearized system of equations (36) have two poles in the lower half-plane at $\pm 2\Omega_0$. The corresponding displacement field is given by

$$\begin{aligned} u_{x1} &= -\tfrac{1}{2}\Omega_1(2\Omega_0)(x\sin 2\Omega_0 t - y\cos 2\Omega_0 t) \ , \\ u_{y1} &= -\tfrac{1}{2}\Omega_1(2\Omega_0)(x\cos 2\Omega_0 t + y\sin 2\Omega_0 t) \ . \end{aligned} \tag{37}$$

In equations (37) $\Omega_1(2\Omega_0)$ is the Fourier transform $\Omega_1(\omega)$ for $\omega = 2\Omega_0$ (we have assumed $\Omega_1^*(\omega) = \Omega_1(\omega)$). In the subsequent calculations we omit the argument $2\Omega_0$ and write simply Ω_1 for $\Omega_1(2\Omega_0)$.

Now, it is easy to compute the electromagnetic potentials given by equations (8). Acording to equations (1) the polarization charge and curent densities for a sphere of radius a are given by

$$\begin{aligned} \rho &= \tfrac{1}{2}nq\Omega_1\left[2\theta(a-R) - \tfrac{a^2-z^2}{a}\delta(a-R)\right]\sin 2\Omega_0 t \ , \\ \mathbf{j} &= -nq\Omega_0\Omega_1\left[(x,\, y)\cos 2\Omega_0 t + (y,\, -x)\sin 2\Omega_0 t\right]\theta(a-R) \ . \end{aligned} \tag{38}$$

The calculations are straightforward and go in the same manner as for a dielectric sphere described above. We give here the leading terms for the realistic conditions $\Omega_0 a/c \ll 1$ and $\Omega_0 R/c \ll 1$, *i.e.* for wavelengths c/Ω_0 much longer than the radius of the sphere and the distances of interest (this is the opposite to the approximation used for a dielectric sphere, where the relevant frequency ω_c corresponds to very short wavelengths). Under these conditions we get the scalar potential

$$\Phi(\mathbf{R}) = \frac{4\pi nqa^5\Omega_1}{15R^3}P_2(\cos\theta)\sin 2\Omega_0(t - R/c) \tag{39}$$

and the vector potential

$$\mathbf{A}(\mathbf{R}) = -\frac{4\pi nqa^5\Omega_0\Omega_1}{15cR^3}\left[(x,\, y)\cos 2\Omega_0(t-R/c) + (y,\, -x)\sin 2\Omega_0(t-R/c)\right] \tag{40}$$

(where $\cos\theta = z/R$). We can check immediately the Lorentz gauge $div\mathbf{A} + (1/c)\partial\Phi/\partial t = 0$ (within our approximation). The retarded contribution to equations (39) and (40) ($2\Omega_0 R/c$ in the oscillatory functions) can be omitted, and the field assumes, in fact, the aspect of a stationary field. We can see that a conducting sphere which rotates about an axis with a (slightly) non-uniform angular velocity emits (quadrupolar) radiation with the frequency $2\Omega_0$, where Ω_0 is the uniform part of the angular velocity. The amplitude of this radiation is governed by the non-uniform part Ω_1 of the angular velocity.

Concluding remarks

Electromagnetic phenomena arising from the non-inertial motion of matter have been investigated in this paper by using the well-known Drude-Lorentz (plasma) model of polarizable matter and the corresponding equation of motion for the electric polarization. It was shown that a point-like body subjected to an oscillatory motion emits an electromagnetic field with the same frequency as the frequency of the oscillatory motion; while the same body subjected to a shock-like movement emits an electromagnetic field with the characteristic (atomic-scale) frequency of the body. A wave propagating in an infinitely extended body generates an electromagnetic field with the same frequency as the wave frequency. If there are inhomogeneities in the body which can be squeezed off by the wave (as, for instance, gas bubbles in a liquid subjected to an ultrasound wave), then the locally ionized charges may undergo shock-like movements, and the emission has the frequency of the characteristic (atomic scale) frequency of the body. This may explain qualitatively the origin of the sonoluminiscence.[20, 21]

It was also shown that a dielectric sphere in uniform rotation develops a static polarization (and electric field), which contanis a quadrupolar term. This may be called a gyroelectric effect. The average of this electric field over the volume of the sphere is vanishing. Similarly, a conducting sphere in uniform rotation sustains circular, static polarization currents, which lead to an (average) axial magnetic field (*i.e.*, oriented along the rotation axis) proportional to the magnitude of the angular velocity. The corresponding vector potential has a dipolar character. This is the well-known gyromagnetic effect, and the gyromagnetic coefficient was computed here for a spherically-shaped body.

Slightly non-uniform rotations have also been investigated here by means of a linearized equation of motion for the electric polarization. A dielectric spherically-shaped body rotating with a slightly non-uniform angular velocity emits an electromagnetic field with the characteristic (atomic scale) frequency (slightly shifted by the uniform part of the angular velocity). A conducting sphere in similar conditions emits an electromagnetic field whose frequency is double the uniform part of the angular frequency.

In conclusion, we may say that a variety of electromagnetic phenomena appear as a result of the non-inertial motion of matter, including static electric or magnetic polarization, as well as emission of electromagnetic field.

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