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Elastic waves in a solid with a rough surface

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Abstract

We derive the elastic waves propagating in a semi-infinite isotropic solid body (half-space) with a rough (corrugated) surface. The model assumes a surface roughness with distinct elastic characteristics than the bulk (inhomogeneous roughness). A perturbation-theoretical scheme is devised for a small roughness (in comparison with the relevant elastic disturbances propagating in the body), and the elastic waves equations are solved in the first-order approximation. It is shown that elastic waves propagating in the bulk generate a localized force acting on the surface, as a consequence of the surface roughness. This force causes both scattered waves localized (and propagating only) on the surface (two-dimensional waves) and scattered waves reflected back in the body. In general, the waves scattered back in the body are both trasverse and longitudinal, irrespective of the original wave acting upon the body surface. The waves localized on the surface, as well as the transverse and longitudinal waves scattered back in the body by the surface roughness are derived, and the effect of both the geometric and elastic characteristics of the surface roughness is discussed. Directional effects, wave slowness and attenuation by diffusive scattering, or possible resonance effects are also discussed. For an enhanced roughness damped waves confined to the surface are identified (rough-surface waves).

Key words: *surface roughness, elastic waves scattering, surface localized waves*

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Highlights: Analytical model for including the surface roughness in the analysis of elastic waves scattering. New method for the propagation of elastic waves in semi-infinite solids, based on coupled integral equations. Solution of these equations for a semi-infinite isotropic solid with a rough surface. Scattered waves reflected back in the body by the surface roughness. Waves scattered by the roughness and localized (and propagating only) on the surface (two-dimensional waves). Damped waves, confined to the surface, generated by the surfece roughness (rough-surface waves).

1 Introduction

Recently, there is a great deal of interest in the role played by the surface roughness (corrugation) in a large variety of physical phenomena, ranging from mechanical properties of the elastic bodies,[1, 2] to hydrodynamical flow of microfluids,[3] dispersive properties of surface plasmon-polariton in nanoplasmonics,[4] terahertz-waves generation[5] or electronic microstructures.[6, 7] Giant corrugations have been found on the graphite surface by scanning tunneling microscopy, due to the elastic deformations induced by atomic forces between tip and surface.[10] Periodic surface corrugation plays a central role in enhanced, or suppreseed, optical transmission in the subwavelength regime,[9] or in highly-directional optical emission.[10] An appreciable reduction in the thermal conductance has been assigned to the phonon scattering by the surface roughness.[11] Stick-slip instability responsible for earthquakes has been studied, as well as the associated radiation of seismic surface waves.[12] It has been recognized that elastic waves propagation effects may play a central role in the surface roughness of the cracks propagating in heterogeneous media, like aluminium alloys, ceramics or rock.[13, 14] The main difficulty in getting more definite results in this problem resides in modelling conveniently the surface roughness such as to arrive at mathematically operational approaches.[15]

We introduce here a model of inhomogeneous surface roughness, whose elastic characteristics are, in general, distinct than the ones of the underlying (isotropic) elastic half-space (semi-infinite solid). It is shown that the elastic waves propagating in the semi-infinite body (incident on and reflected specularly by the surface) generate a force localized on the surface roughness, which is responsible for the scattered waves. The scattered waves are of two kinds: localized (and propagating only) on the surface (two-dimensional waves), and waves scattered back in the body. For an enhanced roughness the waves scattered back in the body may get confined to the surface (damped, rough-surface waves). Light diffusely scattered by a randomly rough surface has been studied both experimentally and theoretically, with emphasis on multiple scattering.[16] The method employed in the present paper is based on a perturbation-theoretical scheme, and the resulting coupled integral equations are solved in the first approximation with respect to the roughness magnitude. Multiple scattering is expected to occur in higher-order approximations. Forward and backward scattering of elastic waves have also been reported in corrugated waveguides.[17] Great insight has been obtained previously in the coupling of the surface (Rayleigh) waves to periodic corrugation (grating),[18]-[20] especialy as regards the wave attenuation, slowing and leaking (outgoing increasing wave), corroborated with band gaps and stop bands, by using non-perturbational techniques. In addition to such results, we show here that the surface roughness may cause localized waves, propagating only on the surface, which may store a certain amount of energy, due to the localization effects. Attenuation of crustal waves across the Alpine range has been reported, which might be associated with the localization of energy in the surface-roughness region.[21] The method presented here can be extended to electromagnetic waves, or fluid waves, propagating in a semi-infinite body with a rough surface. It was employed recently to analyze the elastic waves produced by localized forces in semi-infinite solids.[22]

2 Elastic body with a rough surface

We consider an isotropic elastic body extended boundlessly along the directions $\mathbf{r} = (x, y)$ and limited along the z-direction by a free surface $z = h(\mathbf{r})$, where $h(\mathbf{r}) > 0$ is a function to be further specified (roughness function). The body, which may also be termed a sem-infinite solid (with a rough, corrugated surface), occupies the region $z < h(\mathbf{r})$ (*i.e.* an elastic half-space with a rough surface). It is convenient to write the well-known equation for free elastic waves in an isotropic body[23] as

$$\frac{1}{v_t^2}\ddot{\mathbf{u}} - \Delta\mathbf{u} = m \cdot grad \cdot div\mathbf{u} \ , \tag{1}$$

where $\mathbf{u}(\mathbf{r}, z, t)$ is the displacement field, t denotes the time, v_t is the velocity of the transverse waves, $m = v_l^2/v_t^2 - 1 > 1/3$ (actually 1)[23] and v_l is the velocity of the longitudinal waves. Indeed, equation (1) gives the free transverse waves ($div\mathbf{u} = 0$) propagating with velocity v_t and the free longitudinal waves ($curl\mathbf{u} = 0$) propagating with velocity v_l.

For a semi-infinite body with a surface described by equation $z = h(\mathbf{r})$ and extending in the region $z < h(\mathbf{r})$ the displacement field can be written as

$$\mathbf{u} = (\mathbf{v},\, w)\theta[h(\mathbf{r}) - z)] \ , \tag{2}$$

where $\mathbf{v}$ lies in the (x, y)-plane, w is directed along the z-axis and θ is the step function ($\theta(z) = 0$ for $z < 0$, $\theta(z) = 1$ for $z > 0$). The roughness of the surface (deviation from a plane) is given by the magnitude of the function $h(\mathbf{r})$, which we assume to be very small in comparison with the relevant wavelengths along the z-directions of the elastic disturbances propagating in the body. Consequently, we may use the first-order approximation

$$\mathbf{u} = (\mathbf{v},\, w)[\theta(-z) + h(\mathbf{r})\delta(z)] \tag{3}$$

for equation (2), where $\delta(z)$ is the Dirac function. This is the usual approximation employed in the perturbation-theoretical approaches.[24]-[26] The specific conditions of validity for this approximation will be discussed on the final results.

We write such a displacement field as

$$\mathbf{u} = \mathbf{u}_0 + \delta\mathbf{u}_0 \ , \tag{4}$$

where

$$\mathbf{u}_0 = (\mathbf{v}_0,\, w_0)\theta(-z)\,,\ \ \delta\mathbf{u}_0 = (\mathbf{v}_0,\, w_0)|_{z=0}\, h\delta(z) \ , \tag{5}$$

and assume that $\mathbf{u}_0$ satisfies the wave equation (1)

$$\frac{1}{v_t^2}\ddot{\mathbf{u}}_0 - \Delta\mathbf{u}_0 = m \cdot grad \cdot div\mathbf{u}_0 \tag{6}$$

with specific boundary conditions at $z = 0$. This equation describes incident and (specularly) reflected waves propagating in a semi-infinite solid with a plane surface $z = 0$. We can see that $\delta\mathbf{u}_0$ generates a source-term localized on the surface (a force), which can produce scattered waves. We denote the displacement field associated with these scattered waves by $\mathbf{u}_1$; it satisfies the wave equation

$$\frac{1}{v_t^2}\ddot{\mathbf{u}}_1 - \Delta\mathbf{u}_1 = m \cdot grad \cdot div\mathbf{u}_1 + \frac{\mathbf{f}}{v_t^2} \ , \tag{7}$$

where the force is given by

$$\frac{\mathbf{f}}{v_t^2} = \frac{1}{v_t^2}\delta\ddot{\mathbf{u}}_0 - \Delta\delta\mathbf{u}_0 - m \cdot grad \cdot div\delta\mathbf{u}_0 \ . \tag{8}$$

Equations (7) and (8) represent merely a different way of re-writing the wave equation for a semi-infinite solid with a surface roughness. For waves localized on the surface the solution of equation (7) is $\mathbf{u}_1 = \delta\mathbf{u}_0$. Another solutions are given by the waves scattered back in the body by the surface roughness, *i.e.* waves generated in equation (12) by the source term $\mathbf{f}$ (a particular solution of equation (7)). We generalize this model of surface roughness by assuming that the roughness is inhomogeneous, *i.e.* it is a homogeneous elastic medium with different elastic characteristics than the plane-surface half-space bulk (for instance, different density and elastic constants). Therefore, we introduce distinct velocities $\overline{v}_{t,l}$ and denote all the changed parameters with an overbar (for instance, $\overline{m}=\overline{v}_l^2/\overline{v}_t^2-1$). The force is given in this case by

$$\frac{\overline{\mathbf{f}}}{v_t^2}=\frac{1}{\overline{v}_t^2}\delta\ddot{\mathbf{u}}_0-\Delta\delta\mathbf{u}_0-\overline{m}\cdot grad\cdot div\delta\mathbf{u}_0\ , \tag{9}$$

The results are expressed conveniently by using the relative differences $\eta_{t,l}=1-v_{t,l}^2/\overline{v}_{t,l}^2$. The displacement field $\mathbf{u}_1$ given by equation (7) can be written as $\mathbf{u}_1=(\mathbf{v},\,w)\theta(-z)$.

We might say that, in the presence of a displacement field $\mathbf{u}_0$, the roughness of the surface generates a force $\overline{\mathbf{f}}$, localized on the surface and of the same order of magnitude as the roughness h ($\delta u_0\sim h\delta(z)$). This force is the difference between the inertial force $\delta\ddot{\mathbf{u}}_0/\overline{v}_t^2$ and the elastic force $\Delta\delta\mathbf{u}_0+\overline{m}\cdot grad\cdot div\delta\mathbf{u}_0$; it represents the distinct way the surface follows the elastic motion in comparison with the bulk. Equation (6) gives the free incident and reflected waves propagating in a half-space with a plane surface, while equation (7) gives the scattered waves produced by the roughness of the surface, as a consequence of the source term $\overline{\mathbf{f}}/v_t^2$.

It is worth noting that such a model of inhomogeneous surface may correspond either to a surface whose physical properties have been changed, or to a solid which is homogeneous everywhere, including its rough surface. Indeed, in the latter case, it is precisely the spatial variations of the rough surface which affect its elastic properties, viewed as a homogeneous medium, and render it, in fact, a rough surface which is inhomogeneous with respect to the bulk.

The above perturbation-theoretical scheme can also be writen in a different way, by recasting equation (1) into an equation involving the velocity v_l of the longitudial waves and the parameter $n=1-v_t^2/v_l^2=m/(1+m)$. Then, equations (6) - (8) become

$$\begin{aligned}\tfrac{1}{v_l^2}\ddot{\mathbf{u}}_0-\Delta\mathbf{u}_0&=n(-\Delta\mathbf{u}_0+grad\cdot div\mathbf{u}_0)\ ,\\ \tfrac{1}{v_l^2}\ddot{\mathbf{u}}_1-\Delta\mathbf{u}_1&=n(-\Delta\mathbf{u}_1+grad\cdot div\mathbf{u}_1)+\tfrac{\overline{\mathbf{f}}}{v_l^2}\ ,\end{aligned} \tag{10}$$

where

$$\frac{\overline{\mathbf{f}}}{v_l^2}=\frac{1}{\overline{v}_l^2}\delta\ddot{\mathbf{u}}_0-(1-\overline{n})\Delta\delta\mathbf{u}_0-\overline{n}\cdot grad\cdot div\delta\mathbf{u}_0\ . \tag{11}$$

We solve equation (7) and the second equation (10) for the scattered transverse and, respectively, longitudinal waves by using the Green function method.

3 Plane surface

As it is well known, the elementary solutions of equation (6), or the first equation (10), (homogeneous elastic waves equation) for a half-space with a plane surface are transverse and longitudinal plane waves of the form

$$\mathbf{u}_0=(e^{\pm i\kappa_0 z},\,e^{\pm i\kappa_0^{'} z})e^{-i\omega t+i\mathbf{k}_0\mathbf{r}}\ , \tag{12}$$

where both incident ($+\kappa_0,\,+\kappa_0^{'}$) and reflected ($-\kappa_0,\,-\kappa_0^{'}$) waves are included, ω is the frequency and $\mathbf{k}_0$ is the in-plane wavevector. For $div\mathbf{u}_0=0$ we get the transverse waves, propagating with the velocity v_t ($\omega=v_tK_0$, where $\mathbf{K}_0=(\mathbf{k}_0,\kappa_0)$), with the z-component of the wavevector $\kappa_0=\sqrt{\omega^2/v_t^2-k_0^2}$. For $curl\mathbf{u}_0=0$ we get the longitudinal waves (through $curl\cdot curl\mathbf{u}_0=-\Delta\mathbf{u}_0+grad\cdot div\mathbf{u}_0=0$), propagating with the velocity v_l and the z-component of the wavevector $\kappa_0^{'}=\sqrt{\omega^2/v_l^2-k_0^2}$ ($\omega=v_lK_0^{'}$ and $\mathbf{K}_0^{'}=(\mathbf{k}_0,\,\kappa_0^{'})$). The transverse waves have two polarizations, one in the propagating plane (the $(\mathbf{k}_0, \kappa_0)$-plane), called the p-wave (parallel wave), another perpendicular to the propagating plane, called the s-wave (from the German "senkrecht", which means "perpendicular"). Linear combinations of the plane waves given by equation (12) are subject to conditions imposed on the surface (*e.g.*, free or fixed surface).

We derive here these free waves propagating in a half-space with a plane surface by a different method, which will be used subsequently in deriving the solutions for the scattered waves (equation (7) and the second equation (10)). In order to simplify the notations we omit here the subscript 0.

The solution of equation (6) is written as

$$\mathbf{u} = [\mathbf{v}(z),\, w(z)]\,\theta(-z)e^{-i\omega t+i\mathbf{kr}} \ . \tag{13}$$

Introducing this $\mathbf{u}$ in equation (6) and leaving aside the exponential factor $e^{-i\omega t+i\mathbf{kr}}$ we get

$$\frac{\partial^2\mathbf{u}}{\partial z^2} + \kappa^2\mathbf{u} = \mathbf{S} \ , \tag{14}$$

where $\kappa^2 = \omega^2/v_t^2 - k^2$ and the source $\mathbf{S}$ has the components

$$\begin{gathered}\mathbf{S}_{(x,y)} = -im\mathbf{k}\left(i\mathbf{kv} + \tfrac{\partial w}{\partial z}\right)\theta(-z)+ \\ +\left(\tfrac{\partial \mathbf{v}}{\partial z}\Big|_{z=0} + imk\, w|_{z=0}\right)\delta(z) + \mathbf{v}|_{z=0}\,\delta'(z) \ , \\ S_z = -m\left[i\mathbf{k}\tfrac{\partial\mathbf{v}}{\partial z} + \tfrac{\partial^2 w}{\partial z^2}\right]\theta(-z) + im\,\mathbf{kv}|_{z=0}\,\delta(z)+ \\ +(1+m)\left[\tfrac{\partial w}{\partial z}\Big|_{z=0}\delta(z) + w|_{z=0}\,\delta'(z)\right] \ .\end{gathered} \tag{15}$$

We can see that the source $\mathbf{S}$, which collects all the contributions from $m\cdot grad\mathbf{u}$ and the derivatives of $\theta(-z)$ in $\Delta\mathbf{u}$, acts as an "external force" in equation (14). As it is well known, the particular solution of equation (14) is given by

$$\mathbf{u}(z) = \int dz' G(z-z')\mathbf{S}(z') \ , \tag{16}$$

where

$$G(z) = \frac{1}{2i\kappa}e^{i\kappa|z|} \tag{17}$$

is the Green function for equation (14) (Green function of the one-dimensional Helmholtz equation). Making use of the notations $v_1 = \mathbf{vk}/k$ and $v_2 = \mathbf{vk}_\perp/k$, where $\mathbf{k}_\perp$is a vector perpendicular to $\mathbf{k}$ and of the same magnitude k, equations (15)-(17) lead to

$$v_2 = -\frac{i}{2\kappa}\frac{\partial v_2}{\partial z}\Big|_{z=0} e^{-i\kappa z} - \frac{1}{2}\, v_2|_{z=0}\, e^{-i\kappa z} \tag{18}$$

and

$$\begin{gathered}v_1 = -\tfrac{imk^2}{2\kappa}\int^0 dz' v_1(z')e^{i\kappa|z-z'|} - \tfrac{mk}{2\kappa}\tfrac{\partial}{\partial z}\int^0 dz' w(z')e^{i\kappa|z-z'|}- \\ -\tfrac{i}{2\kappa}\tfrac{\partial v_1}{\partial z}\Big|_{z=0} e^{-i\kappa z} - \tfrac{1}{2}\, v_1|_{z=0}\, e^{-i\kappa z} \ , \\ (1+m)w = -\tfrac{mk}{2\kappa}\tfrac{\partial}{\partial z}\int^0 dz' v_1(z')e^{i\kappa|z-z'|} + \tfrac{im\kappa}{2}\int^0 dz' w(z')e^{i\kappa|z-z'|}- \\ -\tfrac{i}{2\kappa}\tfrac{\partial w}{\partial z}\Big|_{z=0} e^{-i\kappa z} - \tfrac{1}{2}\, w|_{z=0}\, e^{-i\kappa z} \ .\end{gathered} \tag{19}$$

Equation (18) corresponds to the s-wave. It is easy to see that the particular solution given by equation (18) is identically vanishing. Therefore, we are left with the free s-waves given by equation (12), as expected ($\sim e^{\pm i\kappa z}e^{-i\omega t+i\mathbf{kr}}$).

Let us take the second derivative of equations (19) with respect to z and use the identity

$$\frac{\partial^2}{\partial z^2}\int dz' f(z')e^{i\kappa|z-z'|} = -\kappa^2\int dz' f(z')e^{i\kappa|z-z'|} + 2i\kappa f(z) \tag{20}$$

for any arbitrary function $f(z)$. We get

$$\begin{aligned} &\frac{\partial^2 v_1}{\partial z^2} + \kappa^2 v_1 = -imk\left(ikv_1 + \frac{\partial w}{\partial z}\right) \ , \\ &\frac{\partial^2 w}{\partial z^2} + \kappa^2 w = -m\frac{\partial}{\partial z}\left(ikv_1 + \frac{\partial w}{\partial z}\right) \ . \end{aligned} \tag{21}$$

We can see that for $div(v_1,\, w) = 0$, *i.e.* for $ikv_1 + \partial w/\partial z = 0$, we get the free p-waves ($\kappa = \sqrt{\omega^2/v_t^2 - k^2}$), according to equation (12) ($\sim e^{\pm i\kappa z}e^{-i\omega t+i\mathbf{kr}}$). Similarly, for $curl\mathbf{u} = 0$, *i.e.* for $ikw - \partial v_1/\partial z = 0$, equations (21) become

$$(1+m)\frac{\partial^2(v_1,\, w)}{\partial z^2} + (\kappa^2 - mk^2)(v_1,\, w) = 0 \ , \tag{22}$$

or, making use of $m = v_l^2/v_t^2 - 1$,

$$\frac{\partial^2(v_1,\, w)}{\partial z^2} + \kappa'^2(v_1,\, w) = 0 \ \ , \tag{23}$$

where $\kappa' = \sqrt{\omega^2/v_l^2 - k^2}$, *i.e.* free longitudinal waves $\sim e^{\pm i\kappa' z}e^{-i\omega t+i\mathbf{kr}}$.

The longitudinal waves can also be obtained by noting that the coupled equations (19) imply the relationship

$$\frac{\partial v_1}{\partial z} - ikw = Ce^{-i\kappa z} \ \ , \tag{24}$$

where

$$C = -\frac{1}{2}\left(\frac{\partial v_1}{\partial z} - ikw\right)\Big|_{z=0} + \frac{1}{2}\left(i\kappa v_1 - \frac{k}{\kappa}\frac{\partial w}{\partial z}\right)\Big|_{z=0} \ . \tag{25}$$

We use this relationship in one of equations (21), and get

$$\frac{\partial^2 v_1}{\partial z^2} + \kappa'^2 v_1 = -\frac{im\kappa}{1+m}Ce^{-i\kappa z} \ . \tag{26}$$

The particular solution of this equation is vanishing identically, and we are left with free longitudinal waves. Indeed, equation (24) with $C = 0$ corresponds to $curl(v_1,\, w) = 0$.

The p-waves are obtained in a similar way, by starting with the first equation (10). Using $\mathbf{u}$ given by an equation similar with equation (13) we get

$$(1-n)v_2 = \frac{in(\kappa'^2 + k^2)}{2\kappa'}\int^0 dz' v_2(z')e^{i\kappa'|z-z'|} - \frac{i}{2\kappa'}\frac{\partial v_2}{\partial z}\Big|_{z=0} e^{-i\kappa' z} - \frac{1}{2}\,v_2\big|_{z=0}\,e^{-i\kappa' z} \tag{27}$$

and

$$\begin{aligned} (1-n)v_1 = &\frac{in\kappa'}{2}\int^0 dz' v_1(z')e^{i\kappa'|z-z'|} - \frac{nk}{2\kappa'}\frac{\partial}{\partial z}\int^0 dz' w(z')e^{i\kappa'|z-z'|} - \\ &-\frac{i}{2\kappa'}\frac{\partial v_1}{\partial z}\Big|_{z=0} e^{-i\kappa' z} - \frac{1}{2}\,v_1\big|_{z=0}\,e^{-i\kappa' z} \ , \\ w = -&\frac{nk}{2\kappa'}\frac{\partial}{\partial z}\int^0 dz' v_1(z')e^{i\kappa'|z-z'|} + \frac{ink^2}{2\kappa'}\int^0 dz' w(z')e^{i\kappa|z-z'|} - \\ &-\frac{i}{2\kappa'}\frac{\partial w}{\partial z}\Big|_{z=0} e^{-i\kappa' z} - \frac{1}{2}\,w\big|_{z=0}\,e^{-i\kappa' z} \ . \end{aligned} \tag{28}$$

It is easy to see, by taking the second derivative with respect to z, that equation (27) gives the free s-waves. Similarly, by taking the second derivative with respect to z, equations (28) become

$$\frac{\partial^2 v_1}{\partial z^2} + \frac{\kappa'^2}{1-n} v_1 = -ink\frac{\partial w}{\partial z} \ , \qquad \frac{\partial^2 w}{\partial z^2} + (1-n)\kappa^2 w = -ink\frac{\partial v_1}{\partial z} \tag{29}$$

(where we have used the identity $\kappa'^2 + nk^2 = (1-n)\kappa^2$). On the other hand, from equations (28), we get easily the relationship

$$\frac{\partial v_1}{\partial z} + i\frac{\kappa^2}{k} w = \frac{C'}{1-n} e^{-i\kappa' z} \ , \tag{30}$$

where

$$C' = -\frac{1}{2}\left(\frac{\partial v_1}{\partial z} + \frac{i\kappa'^2}{k} w\right)\Bigg|_{z=0} + \frac{1}{2}\left(i\kappa' v_1 + \frac{\kappa'}{k}\frac{\partial w}{\partial z}\right)\Bigg|_{z=0} \ . \tag{31}$$

Making use of this relationship in equations (29) we get

$$\frac{\partial^2 w}{\partial z^2} + \kappa^2 w = -\frac{ink}{1-n} C' e^{-i\kappa' z} \tag{32}$$

and a similar equation for v_1. It is easy to see that the particular solution of equation (32) is identically vanishing, so we are left with the free p-waves. Indeed, equation (30) with $C' = 0$ corresponds to $div(v_1,\, w) = 0$.

4 Scattered waves

We consider now a bulk incident transverse wave and reflected transverse and longtudinal waves given by

$$\mathbf{u}_0 = \left(\mathbf{u}_0^{(1)} e^{i\kappa_0 z} + \mathbf{u}_0^{(2)} e^{-i\kappa_0 z} + \mathbf{u}_0^{(3)} e^{-i\kappa_0' z}\right) e^{-i\omega t + i\mathbf{k}_0\mathbf{r}} \tag{33}$$

(for $z < 0$), where the amplitudes $\mathbf{u}_0^{(1,2,3)}$ satisfy the corresponding conditions of transverse and, respectively, longitudinal waves. For instance, in the representation $\mathbf{u}_0 = (\mathbf{v}_0,\, w_0)$ we have $\mathbf{k}_0\mathbf{v}_0^{(1,2)} \pm \kappa_0 w_0^{(1,2)} = 0$ (including $w_0^{(1,2)} = 0$ for the s-waves) and $\kappa_0\mathbf{v}_0^{(3)}\mathbf{k}_0/k_0 + k_0 w_0^{(3)} = 0$. In addition, the wave given by equation (33) must satisfy the conditions at the surface. For instance, for a fixed surface we have $\mathbf{u}_0|_{z=0} = 0$, while for a free surface, we impose the condition $\sigma_{iz} = 0$, where σ_{ij} is the stress tensor ($i = x,\, y,\, z$). All these conditions fix the amplitudes $\mathbf{u}_0^{(1,2,3)}$, up to the incidence angle and the amplitude of the incident wave, in terms of the reflection coefficients and reflection angles, ultimately in terms of the wave velocities $v_{t,l}$.[23]For an incident s-wave we have only a reflected s-wave ($\mathbf{u}_0^{(3)} = 0$), while for an incident p-wave we have both p- and longitudinal waves. A similar situation occurs for an incident longitudinal wave, with κ_0 and κ_0' interchanged in equation (33). The displacement $\delta\mathbf{u}_0$ given by equation (5) implies $\mathbf{u}_0$ for $z = 0$, so that we may represent this localized contribution of the $\mathbf{u}_0$-wave as

$$\mathbf{u}_0|_{z=0} = (\mathbf{v}_0,\, w_0) e^{-i\omega t + i\mathbf{k}_0\mathbf{r}} \ , \tag{34}$$

where $\mathbf{v}_0$, w_0 include contributions corresponding to various polarizations.

First, we are interested in solving equation (7) for the scattered waves, with the force $\overline{\mathbf{f}}/v_t^2$ generated by the free waves $\mathbf{u}_0$, as given by equation (9). We consider a Fourier component of the form

$$h(\mathbf{r}) = he^{i\mathbf{qr}} \tag{35}$$

for the roughness function, where h is an amplitude (depending on $\mathbf{q}$) and $\mathbf{q}$ denotes a characteristic wavevector (in final results the contribution $\mathbf{q} \rightarrow -\mathbf{q}$ must be included). The localized displacement $\delta\mathbf{u}_0$ given by equation (5) can be written as

$$\delta\mathbf{u}_0 = h(\mathbf{v}_0,\, w_0)e^{-i\omega t+i\mathbf{kr}}\delta(z)\ , \tag{36}$$

where $\mathbf{k} = \mathbf{k}_0 + \mathbf{q}$. Making use of this displacement $\delta\mathbf{u}_0$, the force $\overline{\mathbf{f}}/v_t^2$ given by equation (9) can be computed straightforwardly. Leaving aside the exponential factor $e^{-i\omega t+i\mathbf{kr}}$, it is given by

$$\begin{aligned}\frac{\overline{\mathbf{f}}_{(x,y)}}{v_t^2} &= -h\left[\overline{\kappa}^2\mathbf{v}_0\delta(z) + \mathbf{v}_0\delta^{''}(z) - \overline{m}\mathbf{k}(\mathbf{k}\mathbf{v}_0)\delta(z) + i\overline{m}\mathbf{k}w_0\delta^{'}(z)\right]\ ,\\ \frac{\overline{f}_z}{v_t^2} &= -h\left[\overline{\kappa}^2 w_0\delta(z) + w_0\delta^{''}(z) + i\overline{m}\mathbf{k}\mathbf{v}_0\delta^{'}(z) + \overline{m}w_0\delta^{''}(z)\right]\ ,\end{aligned} \tag{37}$$

where

$$\overline{\kappa} = \sqrt{\omega^2/\overline{v}_t^2 - k^2} \tag{38}$$

and

$$\kappa = \sqrt{\omega^2/v_t^2 - k^2} = \sqrt{\kappa_0^2 - 2\mathbf{k}_0\mathbf{q} - q^2}\ . \tag{39}$$

We add the contributions arising from this force (via the Green function of equation (14)) to the *rhs* of equations (18) and (19) and solve these equations by the procedure described in the previous section. For instance, equation (18) becomes

$$v_2 = -\frac{i}{2\kappa}\left.\frac{\partial v_2}{\partial z}\right|_{z=0} e^{-i\kappa z} - \frac{1}{2}\left.v_2\right|_{z=0} e^{-i\kappa z} - \frac{ih}{2\kappa}(\overline{\kappa}^2 - \kappa^2)v_{02}e^{-i\kappa z} + hv_{02}\delta(z)\ . \tag{40}$$

The displacement v_2 given above includes the localized wave

$$v_{2l} = hv_{02}\delta(z)e^{-i\omega t+i\mathbf{kr}}\ , \tag{41}$$

which is a scattered wave propagating only on the surface (two-dimensional wave). The remaining contribution to equation (40) (terms without $\delta(z)$) represents scattered waves reflected back in the body. We denote this contribution by v_{2r}. Taking the second derivative with respect to z in equation (40) and using the self-consistency condition imposed by this equation on the displacement on the surface, we get immediately the solution

$$v_{2r} = -\frac{ih}{4\kappa}(\overline{\kappa}^2 - \kappa^2)v_{02}e^{-i\omega t+i\mathbf{kr}-i\kappa z}\ . \tag{42}$$

This is an s-wave, scattered back in the body by the surface roughness. We can see that it is the distinct elastic parameters of the surface roughness that ensure this scattering (through $\overline{\kappa}^2 - \kappa^2 = -\omega^2\eta_t/v_t^2 \neq 0$). The occurrence of the wavevector $\mathbf{k} = \mathbf{k}_0 + \mathbf{q}$ in equation (42) is indicative of the selective reflection phenomenon, associated with corrugated surfaces, and in general, of directional effects.

In likewise manner we get the equations for v_1 and w with the force terms given by equation (37). We get the amplitudes for localized waves

$$v_{1l} = hv_{01}\delta(z)\ ,\ w_l = h\frac{1+\overline{m}}{1+m}w_0\delta(z)\ . \tag{43}$$

Equations (21) and (24) remain the same, but the constant C given by equation (25) (entering the relationship (24)) becomes now

$$C = -\frac{1}{2}\left.\left(\frac{\partial v_1}{\partial z} - ikw\right)\right|_{z=0} + \frac{1}{2}\left.\left(i\kappa v_1 - \frac{k}{\kappa}\frac{\partial w}{\partial z}\right)\right|_{z=0} - \frac{h}{2\kappa}(\overline{\kappa}^2 - \kappa^2)(\kappa v_{01} + kw_0)\ . \tag{44}$$

Following the same procedure as described in the previous section we get the scattered waves

$$v_{1r}=-ih\frac{v_t^2}{4\omega^2}(\overline{\kappa}^2-\kappa^2)(\kappa v_{01}+kw_0)e^{-i\omega t+i\mathbf{kr}-i\kappa z}=\frac{i}{4}h\eta_t(\kappa v_{01}+kw_0)e^{-i\omega t+i\mathbf{kr}-i\kappa z} \tag{45}$$

and $w_r=kv_{1r}/\kappa$. We can see that this represent a p-wave ($div(v_{1r},\, w_r)=0$, *i.e.* $kv_{1r}-\kappa w_r=0$). We turn now to the second equation (10) with the force given by

$$\begin{aligned}\frac{\overline{\mathbf{f}}_{(x,y)}}{v_l^2}&=-h\left[(1-\overline{n})\overline{\kappa}^2\mathbf{v}_0\delta(z)+(1-\overline{n})\mathbf{v}_0\delta^{''}(z)-\overline{n}\mathbf{k}(\mathbf{kv}_0)\delta(z)+i\overline{n}\mathbf{k}w_0\delta^{'}(z)\right]\ ,\\ \frac{\overline{f}_z}{v_l^2}&=-h\left[(1-\overline{n})\overline{\kappa}^2w_0\delta(z)+(1-\overline{n})w_0\delta^{''}(z)+i\overline{n}\mathbf{kv}_0\delta^{'}(z)+\overline{n}w_0\delta^{''}(z)\right]\ .\end{aligned} \tag{46}$$

By using the procedure described in the previous section we get a localized displacement

$$\mathbf{v}_l=h\frac{1-\overline{n}}{1-n}\mathbf{v}_0\delta(z)\ ,\ w_l=hw_0\delta(z)\ . \tag{47}$$

We can see, by comparing equations (41), (43) and (47) that the inhomogeneous roughness affects the localized waves in different ways. For the scattered waves reflected back in the body, equations (29) and (30) from the previous section remain unchanged, but the constant $C^{'}$ given by equation (31) (entering the relationship (30)) becomes

$$C^{'}=-\frac{1}{2}\left(\frac{\partial v_1}{\partial z}+\frac{i\kappa^{'2}}{k}w\right)\Big|_{z=0}+\frac{1}{2}\left(i\kappa^{'}v_1+\frac{\kappa^{'}}{k}\frac{\partial w}{\partial z}\right)\Big|_{z=0}-\frac{h}{2k}(\overline{\kappa'}^2-\kappa'^2)(kv_{01}-\kappa' w_0)\ . \tag{48}$$

We get straightforwardly the reflected waves

$$v_{1r}=-ih\frac{v_l^2k}{4\omega^2\kappa'}(\overline{\kappa'}^2-\kappa'^2)(kv_{01}-\kappa' w_0)e^{-i\omega t+i\mathbf{kr}-i\kappa' z}=\frac{i}{4}h\eta_l(kv_{01}-\kappa' w_0)e^{-i\omega t+i\mathbf{kr}-i\kappa' z} \tag{49}$$

and $w_r=-\kappa' v_{1r}/k$. We can see that this scattered wave is a longitudinal wave ($curl(v_{1r},\, w_r)=0$, *i.e.* $-\kappa' v_{1r}=kw_r$).

According to equations (42), (45) and (49), within the present model of surface roughness we get waves scattered back in the body only for a rough surface with elastic characteristics different from those of the body (inhomogeneous roughness, $\eta_{t,l}\neq 0$). For a homogeneous roughness, *i.e.* for $\eta_{t,l}=0$, we get only scattered waves localized on the surface, given by

$$\mathbf{u}_l=\delta\mathbf{u}_0=h(\mathbf{r})(\mathbf{v}_0,\, w_0)e^{-i\omega t+i\mathbf{k}_0\mathbf{r}}\delta(z)\ , \tag{50}$$

as expected.

5 Discussion

The localized waves have the general form of the incoming wave $e^{-i\omega t+i\mathbf{k}_0\mathbf{r}}$modulated by the roughness function $h(\mathbf{r})$. If $\mathbf{q}$ is a characteristic wavevector of this roughness function and $\mathbf{k}=\mathbf{k}_0+\mathbf{q}$, the velocity of the localized waves is given by $v_s=\omega/k=v_{t,l}k_0/k\sin\theta$, where θ is the incidence angle of the incoming (transverse or longitudinal) wave. The directional effects are clearly seen from the presence of $k==\sqrt{k_0^2+2\mathbf{k}_0\mathbf{q}+q^2}$ in the denominator of this relation. It is worth noting that for $\mathbf{q}=\pm\mathbf{k}_0$, *i.e.* for a surface roughness modulated with the same wavelength as the original $\mathbf{u}_0$-wave, there appear scattered waves with half the wavelength of the original $\mathbf{u}_0$-waves (wavevector $2\mathbf{k}_0$) and the whole surface suffers a vibration (independent of the coordinate $\mathbf{r}$), a characteristic resonance phenomenon ($\mathbf{k}=0$). The waves corresponding to the wavevector $2\mathbf{k}_0$ have a velocity $\omega/2k_0$, which is twice as small as the original velocity on the surface. This is indicative of the slowness phenomenon, associated with rough surfaces.

The $\mathbf{q}=\pm\mathbf{k}_0$ resonance phenomenon is exhibited also by the waves scattererd back in the body. Another resonance phenomenon may appear for $\pm 2\mathbf{k}_0\mathbf{q}+q^2=0$, which is the well-known Laue-Bragg condition for the X-rays diffraction in crystalline bodies.[27] In this case, $k=k_0$, $\kappa=\kappa_0$ and $\kappa^{'}=\kappa_0^{'}$ and we can see that the scattered transverse (longitudinal) waves are generated only by the transverse (longitudinal) part in the original $\mathbf{u}_0$-waves, as expected, due to the presence of the factors $\kappa v_{01}+kw_0$ and $kv_{01}-\kappa' w_0$ in equations (45) and, respectively, (49). For $\mathbf{k}_0$ and $\mathbf{q}$ antiparallel the scattered wave propagates in opposite direction with respect to the incident wave.

The results given above hold also for purely imaginary values of the wavevectors κ or $\kappa^{'}$, when the scattered waves become confined to the surface (surface waves), a situation which may occur especially for high values of the magnitude q of the characteristic wavevectors $\mathbf{q}$ ($q\gg k_0$). According to equations (42), (45) and (49), the scattered waves are now damped ($\sim e^{qz}$) and their amplitudes are proportional to the roughness function $h(\mathbf{r})$. It is worth noting that these rough-surface waves are generated by the surface roughness.

As it is well known, the energy of the incident wave is transferred to the reflected waves. In the present case, it is transferred both to the reflected waves as well as to the scattered waves, including the waves localized on the surface and the waves scattered back in the body. According to equations (42), (45) and (49) the energy density of the scattered waves reflected back in the body is proportional to $(h/\overline{\lambda})^2$, where $\overline{\lambda}$ is a characteristic "wavelength" of these waves (projection of the wavelength λ on the surface, or on the direction perpendicular to the surface, or combinations of these). It folows that the validity criterion for our perturbation-theretical scheme is $h\ll\overline{\lambda}$. In the limit of small roughness ($h\rightarrow 0$), the energy of the scattering waves (their amplitude) is vanishing. It is worth estimating the energy of the waves localized on the surface. For simplicity, we consider a homogeneous roughness, with the localized waves given by

$$(\mathbf{v}_l,\,w_l)=h(\mathbf{v}_0,\,w_0)\delta(z)e^{-i\omega t+i\mathbf{k}\mathbf{r}} \tag{51}$$

(according to equation (50)) and choose the wavevector $\mathbf{k}$ directed along the x-axis. The validity condition for these waves is obtained by assuming that the surface roughness extends over a distance of the order of $h_m=\max h(\mathbf{r})$ and use the repreesntation $\delta(z)\simeq 1/h_m$ for the δ-function. Then, the perturbation calculations are valid for $\overline{h}\ll h_m$, where $\overline{h}$ is the average (mean value) of the roughness function $h(\mathbf{r})$. This means that the surface roughness should has but only a few spikes. As it is well known, the (elastic) energy density (per unit mass) can be expressed as

$$\mathcal{E}/\rho=v_t^2(u_{ij}^2-u_{ii}^2)+\frac{1}{2}v_l^2u_{ii}^2\ , \tag{52}$$

where $u_{ij}=(1/2)(\partial u_i/\partial x_j+\partial u_j/\partial x_i)$ is the strain tensor. In our case, we use for computing this strain tensor the displacement given by equation (51). The strain tensor includes factors proportional to $\delta(z)$ and $\delta^{'}(z)$, and the energy density includes factors proportional to $\delta^2(z)$ and $\delta^{'2}(z)$. The leading contribution come from $\delta^{'2}(z)$ -terms:

$$\mathcal{E}/\rho=\frac{h^2}{2}(v_t^2\mathbf{v}_0^2+v_l^2w_0^2)\delta^{'2}(z)\ , \tag{53}$$

giving a surface energy (per unit mass)$\sim h_m\mathcal{E}/\rho$. Making use of the representation $\delta^{'2}(z)\simeq 1/h_m^4$, this surface energy is proportional to h^2/h_m^3, while the corresponding energy of the incident wave goes like h_m/λ^2; the ratio of the two quantities is of the order of $h^2\lambda^2/h_m^4$. We can see that that this ratio may acquire large values, even for $h \ll h_m$ (perturbation criterion satisfied), for $\lambda \gg h_m$. Therefore, the surface waves may store an appreciable amount of energy, as a result of their localization. This phenomenon is related to the discontinuites experienced by the strain tensor along the direction perpendicular to the surface.

6 Particular cases and concluding remarks

From equations (42), (45) and (49) we can get the reflection coefficients, related to the energy, of the waves scattered back in the body. Their general characteristic is the directionality effects. The derivation of these coefficients is complicated in the general case, where we should fix the amplitudes of the original $\mathbf{u}_0$-waves according to the nature of these waves and the boundary conditions. Another complication arises from the fact that we should "renormalize" the amplitudes of the reflected original $\mathbf{u}_0$-waves such as to include (accomodate) the scattered waves in the boundary conditions (a procedure specific to theoretical-perturbation calculations). We limit ourselves here to give the reflection coefficients for a few particular cases.

First, one of the simplest case is an original s-wave, described by

$$\mathbf{u}_0 = 2(0,\, u_0,\, 0)\cos\kappa_0 z \cdot e^{-i\omega t + i\mathbf{k}_0\mathbf{r}} \ , \tag{54}$$

where $\mathbf{k}_0$ is directed along the x-axis. Making use of equation (52), the energy density (per unit mass) of the incident wave in equation (54) is $\mathcal{E}_0/\rho = \omega^2 u_0^2$. We must compute the projections $v_{01,2}$ of the amplitude of this wave on $\mathbf{k} = \mathbf{k}_0 + \mathbf{q}$ and $\mathbf{k}_\perp$. Introducing the angle α between $\mathbf{q}$ and $\mathbf{k}_0$, we get $v_{01} = 2u_0 q\sin\alpha/k$ and $v_{02} = 2u_0(k_0 + q\cos\alpha)/k$ (and, of course, $w_0 = 0$). We can see, from equations (42), (45) and (49), that an incident s-wave produce both s- and p- scattered transverse waves as well as a scattered longidudinal wave, due to the surface roughness. Making use of these equations we compute easily the amplitudes of these waves and get the reflection coefficients

$$R_s = \eta_t \frac{h\omega^2}{4v_t^2\kappa k}(k_0 + q\cos\alpha) \ , \ R_p = \eta_t \frac{h\omega q}{4v_t k}\sin\alpha \ , \ R_l = \eta_l \frac{h\omega q}{4v_l k}\sin\alpha \ . \tag{55}$$

The energy density carried on by these waves is given by $\mathcal{E}_{s,p,l}/\mathcal{E}_0 = R_{s,p,l}^2$. We stress upon the complicated direction-dependence (angle α) of these reflection coefficients, included both in κ and k. The formulae given by equations (55) become more simple for normal incidence ($\mathbf{k}_0 = 0$).

For normal incidence there is another simple case concerning longitudinal waves described by

$$\mathbf{u}_0 = 2(0,\, 0,\, u_0)\cos\kappa_0^{'} z \cdot e^{-i\omega t} \ , \tag{56}$$

where $\kappa_0^{'} = \omega/v_l$. The energy density per unit mass of this incident wave is $E_0/\rho = \omega^2 u_0^2$. According to equations (42), (45) and (49), the scattered waves in this case area p-wave and a longidudinal wave. Their reflection coefficients are much more simple now,

$$R_p = \eta_t \frac{h\omega q}{4v_t\kappa} \ , \ R_l = \eta_l \frac{h\omega\kappa^{'}}{4v_l q} \ . \tag{57}$$

The squares of these coefficients give the fraction of energy carried on by these waves.

It is worth stressing that all the above formulae are valid only for κ, k, $q \neq 0$ (non-vanishing denominators).

We can see from the above particular cases, as well as from the general equations (42), (45) and (49)), that the total amount of energy carried on diffusively by the waves scattered by the surface roughness implies sums of the form $\sum_{\mathbf{q}} |h(\mathbf{q})|^2 f(\mathbf{q})$, where $h(\mathbf{q})$ is the Fourier transform of the roughness function $h(\mathbf{r})$ and $f(\mathbf{q})$ are specific functions corresponding to the waves' nature (factors implying k, κ, $\kappa^{'}$, etc). Qualitatively, in order to maximize this energy, it is necessary, apart from particular cases of gratings (one, or a few wavevectors $\mathbf{q}$), to include as many Fourier components as possible, *i.e.* the surface should be as rough as possible in order to have a good attenuation, a reasonably expected result.

In conclusion, we may say that we have introduced a model of inhomogeneous surface roughness for a semi-infinite isotropic elastic body and solved the wave equations for the elastic waves scattered by this surface roughness in the first-order approximation with respect to the roughness magnitude. The scattered waves are of two kinds: waves localized (and propagating only) on the surface, given by equations (43) and (47), and scattered waves reflected back in the body by the surface roughness, both transverse, as given by equations (42) and (45), and longitudinal, as given by equation (49). The latter may become confined to the surface (damped, rough-surface waves) for an enhanced roughness (large wavevectors q). The reflected waves are absent for a homogeneous roughness ($\eta_{t,l} = 0$), where only the localized waves survive.

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