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Scattering of longitudinal waves by a rough surface

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Abstract

A model of inhomogeneous rough surface is introduced for a semi-infinite ideal fluid and a perturbation-theoretical scheme is devised, with the roughness function as a peturbation parameter, for computing the waves scattered by the surface roughness. The waves scattered by the rough surface are both waves localized (and propagating only) on the surface (two-dimensional waves) and waves reflected back in the fluid. They exhibit directional effects, slowness, attenuation or resonance phenomena, depending on the spatial characteristics of the roughness function. The reflection coefficients and the energy carried on by these waves are calculated both for fixed and free surfaces. In some cases, the surface roughness may generate waves confined to the surface (damped, rough-surface waves).

Key words: *ideal fluids, rough surface, scattered waves, localized waves*

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Highlights: A model of inhomogeneous rough surface is introduced for a semi-infinite ideal fluid. A theoretical-perturbation scheme with the roughness function as a perturbation parameter is put forward in order to compute the waves (sound) scattered by the rough surface. The amplitude of the scattered waves differs for a fixed or free surface. Scattered waves localized (and propagating only) on the surface are identified (two-dimensional waves), as well as (damped) surface waves created by the surface roughness. Directional character, slowness and attenuation are discussed, as well as possible resonance phenomena.

1 Introduction

The effect of a rough, solid surface on the fluid dynamics, in particular the waves (sound) scattered by the surface roughness, enjoys a great deal of interest. The interaction between a solid wall and the fluid flow, as well as the action of a solid interface on the fluid dynamics have been emphasized recently.[1, 2] A rough surface shares, to some extent, the properties of a porous medium.[3] The surface roughness was modelled as an inhomogeneous fluid layer on a rigid plate and the scattering of acoustic waves was considered within a radiative regime by means of coupled integral equations.[4] A great deal of insight into the scattering mechanism by rough surfaces has been achieved[5] by means of Biot's theory and its recent developments.[6]-[8] The general characteristics of the waves scattered by a rough surface are directional effects, slowness and attenuation, as well as possible resonances for surface gratings (corrugations). The main difficulty in getting more definite results in this problem resides in modelling conveniently the surface roughness, such as to arrive at mathematically operational approaches.[9]

We present here a model of inhomogeneous surface roughness, *i.e.* a surface whose elastic properties differ from the ones of the semi-infinite (half-space) fluid bulk, in contrast with a homogeneous surface roughness which has the same elastic properties as the bulk. In general, a surface, especially a rough one, acts like a source for scattered waves. We devise here a theoretical-perturbation scheme for treating the wave equation for longitudinal (sound) waves proapagating in a semi-infinite solid with a rough surface. The peturbation parameter is the roughness function, *i.e.* the deviation of the surface from a plane. It is shown that the scattered waves appear in the first-order approximation for a fixed surface, while for a free surface they appear only in the second-order approximation. Two kinds of scattered waves are identified: waves localized (and propagating only) on the surface (two-dimensional waves) and waves reflected back in the fluid. In some cases, the latter waves may get confined to the surface (damped, rough-surface waves). For a homogeneous roughness only the waves localized on the surface survive. The reflection coefficients (and the energy carried on by these waves) are calculated and various characteristics like slowness, attenuation or possible resonance phenomena are discussed.

2 Fluid with a rough surface

We consider a semi-infinite (half-space) homogeneous, isotropic, ideal fluid, extending boundlessly along the $\mathbf{r} = (x, y)$ directions and limited along the z-direction by a surface $z = h(\mathbf{r})$, where $h(\mathbf{r}) > 0$ is a function to be further specified (roughness function). The fluid occupies the region $z < h(\mathbf{r})$. A small displacement field $\mathbf{u}(\mathbf{r}, z, t)$ (where t denotes the time) gives rise to a density imbalance $\delta n = -n div\mathbf{u}$ in the fluid density n, a local change of volume $\delta V = V div\mathbf{u}$ and a local change of pressure δp, depending on the equation of state of the fluid; for an adiabatic change, $\delta p = (\partial p/\partial n)_S \delta n = -n(\partial p/\partial n)_S div\mathbf{u}$, where S denotes the entropy. As it is well known,[10] such a fluid supports longitudinal waves (sound), described by the equation of motion

$$\frac{1}{c^2}\ddot{\mathbf{u}} - grad \cdot div\mathbf{u} = 0 \ , \tag{1}$$

where c is the sound velocity. Indeed, by taking the div in equation (1), we get the wave equation for free waves propagating with velocity c. The displacement field is subjected to the condition $curl\mathbf{u} = 0$. Therefore, it is convenient to introduce the potential function $\Phi = div\mathbf{u}$ (proportional to the pressure) and write equation (1) as

$$\frac{1}{c^2}\ddot{\Phi} - \Delta\Phi = 0 \ . \tag{2}$$

For a semi-infinite fluid with a surface described by equation $z = h(\mathbf{r})$ and extending in the region $z < h(\mathbf{r})$, the potential Φ can be written as

$$\Phi = \varphi(\mathbf{r}, z, t)\theta[h(\mathbf{r}) - z] \ , \tag{3}$$

where $\theta(z) = 1$ for $z > 0$ and $\theta(z) = 0$ for $z < 0$ is the step function. We assume that the magnitude of the roughness function $h(\mathbf{r})$ is small in comparison with the relevant wavelengths of the elastic disturbances propagating in the fluid, so that we may write

$$\Phi \simeq \Phi_0 + \delta\Phi_0 \ , \tag{4}$$

where

$$\Phi_0 = \varphi\theta(-z) \ , \ \delta\Phi_0 = h(\mathbf{r})\varphi\delta(z) + \frac{1}{2}h^2(\mathbf{r})\varphi\delta^{'}(z) + ... \ , \tag{5}$$

where $\delta(z)$ is the Dirac function (and the prime means differentiation with respect to the variable z). The specific conditions of validity for this approximation will be discussed on the final results. We assume that the potential φ satisfies the wave equation

$$\frac{1}{c^2}\ddot{\varphi} - \Delta\varphi = 0 \tag{6}$$

with specific boundary conditions at $z = 0$. This equation describes the incident and (specularly) reflected waves propagating in a fluid with a plane surface $z = 0$. It is easy to see that for a fixed surface $\partial\varphi/\partial z|_{z=0} = 0$, so that we have the plane waves

$$\varphi = 2\varphi_0 \cos\kappa_0 z \cdot e^{-i\omega t + i\mathbf{k}_0\mathbf{r}} \ , \tag{7}$$

where ω is the frequency, $\mathbf{k}_0$ is the in-plane wavevector and $\kappa_0 = \sqrt{\omega^2/c^2 - k_0^2}$. In this case we can limit ourselves to the first order in h in the second equation (5), and get

$$\delta\Phi_0 = 2h(\mathbf{r})\varphi_0\delta(z)e^{-i\omega t + i\mathbf{k}_0\mathbf{r}} \ . \tag{8}$$

For a free surface $\varphi|_{z=0} = 0$, so we have

$$\varphi = 2i\varphi_0 \sin\kappa_0 z \cdot e^{-i\omega t + i\mathbf{k}_0\mathbf{r}} \ ; \tag{9}$$

in this case, the first-order contribution to the second equation (5) is vanishing and we get

$$\delta\Phi_0 = -ih^2(\mathbf{r})\kappa_0\varphi_0\delta(z)e^{-i\omega t + i\mathbf{k}_0\mathbf{r}} \ . \tag{10}$$

We can see that $\delta\Phi_0$ acts as a source-term (a force) localized on the surface, which can generate scattered waves. We denote the potential function associated with these waves byΦ_1; it satisfies the wave equation

$$\frac{1}{c^2}\ddot{\Phi}_1 - \Delta\Phi_1 = f \ , \tag{11}$$

where the force f is given by

$$f = \frac{1}{c^2}\delta\ddot{\Phi}_0 - \Delta\delta\Phi_0 \ . \tag{12}$$

Equation (11) is merely a re-writing of the wave equation for $\delta\Phi_0$. The force f is the difference between the inertial force $\delta\ddot{\Phi}_0/c^2$ and the elastic force $\Delta\delta\Phi_0$; it represents the distinct way the surface follows the wave motion in comparison with the bulk. For localized waves equation (11) has the solution $\Phi_1 = \delta\Phi_0$. Another solutions are given by the waves scattered back in the fluid by the surface roughness, *i.e.* waves generated in equation (11) by the source term f (a particular solution of equation (11)). We generalize this model of surface roughness by introducing a different "sound" velocity $\overline{c}$ in equation (12). The force is then written as

$$f = \frac{1}{\overline{c}^2}\delta\ddot{\Phi}_0 - \Delta\delta\Phi_0 \ . \tag{13}$$

Such a generalization amounts to assuming that the elastic properties of the fluid localized on the rough surface are different than the elastic properties of the fluid bulk, *i.e.* the surface roughness is inhomogeneous in comparison with the bulk. This may correspond either to a surface whose physical properties have been changed, or to a fluid homogeneous everywhere, including its rough surface. Indeed, in the latter case, it is precisely the spatial variations of the rough surface which affect its elastic properties, viewed as a homogeneous medium, and render it, in fact, a rough surface which is inhomogeneous with respect to the bulk. It is convenient to introduce the parameter $\eta = 1 - c^2/\overline{c}^2$ for describing the inhomogeneous roughness. A homogeneous roughness corresponds to $\eta = 0$.

Obviously, according to equations (4) and (5), the scheme of calculation put forwad here is a perturbation-theoretical scheme, with the roughness function $h(\mathbf{r})$ as the perturbation parameter. We limit ourselves here to the first relevant orders of the perturbation theory. We can see that for a fixed surface the first-order approximation is sufficient for getting scattered waves, while for a free surface we have to go to the second-order approximation. This implies already a double scattering by the surface roughness. Higher-orders of the perturbation theory will give multiple scattering.

3 Waves scattered by the rough surface

We use the potential $\delta\Phi_0$ given by equations (8) and (10) to compute the force given by equation (13). The calculations are easily performed for one Fourier component $h(\mathbf{q})e^{i\mathbf{qr}}$of the roughness function $h(\mathbf{r})$, corresponding to the wavevector $\mathbf{q}$ (for simplicity we drop the argument $\mathbf{q}$ in $h(\mathbf{q})$). For a fixed surface, making use of equation (8), we get

$$f = -2h\varphi_0 \left[\overline{\kappa}^2\delta(z) + \delta^{''}(z)\right] e^{-i\omega t+i\mathbf{kr}} \ , \tag{14}$$

where $\mathbf{k} = \mathbf{k}_0 + \mathbf{q}$ and $\overline{\kappa} = \sqrt{\omega^2/\overline{c}^2 - k^2}$. The solution of equation (11) is of the form $\Phi_1 = \varphi_1(z)\theta(-z)e^{-i\omega t+i\mathbf{kr}}$, so that equation (11) becomes

$$\frac{\partial^2\varphi_1}{\partial z^2} + \kappa^2\varphi_1 = \left.\frac{\partial\varphi_1}{\partial z}\right|_{z=0}\delta(z) + \left.\varphi_1\right|_{z=0}\delta^{'}(z) + 2h\varphi_0\left[\overline{\kappa}^2\delta(z) + \delta^{''}(z)\right] \ , \tag{15}$$

where $\kappa = \sqrt{\omega^2/c^2 - k^2}$. We note that $k = \sqrt{k_0^2 + 2\mathbf{k}_0\mathbf{q} + q^2}$ and $\kappa = \sqrt{\kappa_0^2 - 2\mathbf{k}_0\mathbf{q} - q^2}$. The combination of the wavevectors $\mathbf{k}_0$ and $\mathbf{q}$ in $\mathbf{k} = \mathbf{k}_0 + \mathbf{q}$ is the source of directional effects, included both in k and κ. As it is well-known, the Green function of equation (15) (one-dimensional Helmholtz equation) is

$$G(z - z') = \frac{1}{2i\kappa}e^{i\kappa|z-z'|} \ , \tag{16}$$

so that the solution of equation (15) is given by

$$\varphi_1(z) = \int dz' S(z')G(z - z') \ , \tag{17}$$

where the source S denotes the *rhs* of equation (15). The calculations are straightforward. We get a localized solution $\varphi_{1l} = 2h\varphi_0\delta(z)$, which corresponds to $\delta\Phi_0$ given by equation (8), as expected, and a wave reflected back in the fluid, given by

$$\varphi_{1r} = -\frac{ih\varphi_0}{2\kappa}(\overline{\kappa}^2 - \kappa^2)e^{-i\kappa z} \ , \tag{18}$$

or

$$\Phi_{1r} = i\eta\frac{h\varphi_0\omega^2}{2c^2\kappa}e^{-i\omega t+i\mathbf{kr}-i\kappa z} \tag{19}$$

(for $z < 0$).

Likewise, for a free surface, making use of equation (10), we get a localized wave

$$\Phi_{1l} = -ih_2\varphi_0\kappa_0\delta(z)e^{-i\omega t+i\mathbf{kr}} \ , \tag{20}$$

which coincides with $\delta\Phi_0$ given by equation (10), and a reflected wave

$$\Phi_{1r} = \eta\frac{h_2\varphi_0\kappa_0\omega^2}{4c^2\kappa}e^{-i\omega t+i\mathbf{kr}-i\kappa z} \ , \tag{21}$$

where

$$h_2 = \int d\mathbf{r}h^2(\mathbf{r})e^{-i\mathbf{qr}} \tag{22}$$

(which depends on $\mathbf{q}$) is the Fourier transform of the roughness function squared (the integration is performed in equation (22) over the unit area).

From the results derived above we can say, qualitativey, that the perturbation-theoretical scheme of calculation is valid for the magnitude of the roughness function much smaller than the relevant wavelengths. For instance, from equation (19) we have $h\omega^2/c^2\kappa \ll 1$, or $h \ll \lambda\cos\theta_r$, where λ is the wavelength of the scattered wave and θ_r is its reflection angle. From equation (21), we can see that the waves scattered by a free surface is a second-order effect, implying multiple (double) scattering, within this approximation, as expected. For the scattered waves localized on the surface, we may represent the δ-function as extending over a distance of the order of $h_m = \max h(\mathbf{r})$, and the perturbation-theory criterion is satisfied for $\overline{h}(\mathbf{r}) \ll h_m$, where $\overline{h}(\mathbf{r})$ is the average of the roughness function (the roughness function should have a few "spikes" only). For a constant roughness function ($\mathbf{q} = 0$), the criterion of series expansion is not satisfied for localized waves, while the scattered waves are reflected back along the original Φ_0-waves; this particular case should be included in the original formulation of the problem for the Φ_0-waves.

It is also worth noting that we have waves Φ_{1r} scattered back in the fluid only for an inhomogeneous roughness ($\eta \neq 0$); for a homogeneous roughness we have only the waves Φ_{1l} localized on the surface.

4 Discussion and concluding remarks

The localized waves Φ_{1l} have the general form of the incoming wave $e^{-i\omega t+i\mathbf{k}_0\mathbf{r}}$modulated by the roughness function $h(\mathbf{r})$ (for a fixed surface) or $h^2(\mathbf{r})$ (for a free surface). If $\mathbf{q}$ is a characteristic wavevector of these roughness functions and $\mathbf{k} = \mathbf{k}_0 + \mathbf{q}$, the velocity of the localized waves is given by $c_s = \omega/k = ck_0/k\sin\theta$, where θ is the incidence angle of the incoming wave. The directional effects are clearly seen from the presence of $k == \sqrt{k_0^2 + 2\mathbf{k}_0\mathbf{q} + q^2}$ in the denominator of this relation. It is worth noting that for $\mathbf{q} = \pm\mathbf{k}_0$, *i.e.* for roughness functions ($h(\mathbf{r})$ or $h^2(\mathbf{r})$) modulated with the same wavelength as the original Φ_0-wave, there appear scattered waves with half the wavelength of the original Φ_0-waves (wavevector $2\mathbf{k}_0$) and, in addition, the whole surface suffers a vibration (independent of the coordinate $\mathbf{r}$), corresponding to $\mathbf{k} = 0$, a characteristic resonance phenomenon. The waves corresponding to the wavevector $2\mathbf{k}_0$ have a velocity $\omega/2k_0$, which is twice as small as the original velocity on the surface. This is indicative of the slowness phenomenon, associated with rough surfaces.

The $\mathbf{q} = \pm\mathbf{k}_0$ resonance phenomenon is exhibited also by the waves scattered back in the fluid. Another resonance phenomenon may appear for $\pm 2\mathbf{k}_0\mathbf{q} + q^2 = 0$, which is the well-known Laue-Bragg condition for the X-rays diffraction in crystalline bodies (or surface gratings).[11] In this case $k = k_0$, $\kappa = \kappa_0$, and for $\mathbf{k}_0$ and $\mathbf{q}$ antiparallel the scattered waves propagate in opposite direction with respect to the original incident Φ_0-waves.

A worth noting case corresponds to $q \gg k_0$, when the wavevector κ may become purely imaginary ($\kappa \simeq -q$) and the scattered waves are confined to the surface. According to equations (19) and (21), the reflected waves are now damped ($\sim e^{qz}$) and their amplitudes are proportional to the roughness functions $h(\mathbf{r})$ or $h^2(\mathbf{r})$. These surface waves are generated by the rough surface; they may be called rough-surface waves.

As it is well-known,[10] the sound propagation in fluids is also described by means of another potential function Ψ, defined by $\delta p = -\rho\partial\Psi/\partial t$ and $\mathbf{v} = \dot{\mathbf{u}} = grad\Psi$, where ρ is the (mass) density and $\mathbf{v}$ is the fluid velocity. Then, Euler's equation $\rho\partial\mathbf{v}/\partial t + grad\delta p = 0$ is satisfied identically, and the continuity equation $\partial\delta\rho/\partial t + \rho div\mathbf{v} = 0$ becomes the wave equation $\partial^2\Psi/\partial t^2 - c^2\Delta\Psi = 0$, through $\delta p = (\partial p/\partial\rho)_S\delta\rho$, with the sound velocity given by $c^2 = (\partial p/\partial\rho)_S$. The connection between the two potential function Ψ and Φ is simple. It is given by

$$\delta p = -\rho\partial\Psi/\partial t = (\partial p/\partial\rho)_S\delta\rho = -\rho(\partial p/\partial\rho)_S div\mathbf{u} = -\rho c^2\Phi \quad , \tag{23}$$

or

$$\frac{\partial\Psi}{\partial t} = c^2\Phi \ ; \tag{24}$$

for a monochromatic wave $\Psi = (ic^2/\omega)\Phi$. According to equation (1), the energy density (per unit mass) carried on by the longitudinal waves in a fluid is given by

$$e = \frac{1}{2}\dot{\mathbf{u}}^2 + \frac{1}{2}c^2\Phi^2 = \frac{1}{2}\frac{c^4}{\omega^2}(grad\Phi)^2 + \frac{1}{2}c^2\Phi^2 \quad , \tag{25}$$

where equation (24) is used for a monochromatic wave. For a plane wave, equation (25) gives $e = c^2\Phi^2$.

As it is well known, the energy of the incident wave is transferred to the reflected waves. In the present case, it is transferred both to the specularly reflected waves as well as to the scattered waves, including the waves localized on the surface and the waves scattered back in the fluid. Within our approximation, in the limit $h \to 0$, equation (25) gives the main contribution

$$e_l \simeq \frac{2c^4}{\omega^2}h^2\varphi_0^2\delta'^2(z) \tag{26}$$

for waves localized on a fixed surface and

$$e_l \simeq \frac{c^4}{2\omega^2}h_2^2\varphi_0^2\kappa_0^2\delta'^2(z) \tag{27}$$

for waves localized on a free surface. We can see that the localized waves can store an appreciable energy, especially for a fixed surface, arising from the component of the fluid velocity perpendicular to the surface. Indeed, taking approximately $\delta'^2(z) \simeq 1/h_m^4$ (and $\delta(z) \simeq 1/h_m$), we get the ratio of the energy density stored on a fixed surface (equation (26)) to the energy density of the incident wave of the order of $\simeq h^2\lambda^2/h_m^4$, which may achieve large values even for $h/h_m \ll 1$, for wavelengths λ much longer than the extension h_m of the surface roughness. This result reflects the large kinetic energy of the fluid particles acting upon a fixed surface.

Using equations (19) and (21), we can calculate the reflection coefficients of the scattered waves (the ratio of their amplitude to the amplitude φ_0 of the incident wave): $R = i\eta h\omega^2/2c^2\kappa$ for a fixed surface and $R = \eta h_2\kappa_0\omega^2/4c^2\kappa$ for a free surface. It is worth noting the directionality effects exhibited by these reflection coefficients, through κ appearing in the denominator. The energy density carried on by the scattered waves is the square of these reflection coefficients. We can see that the total amount of energy carried on diffusively by the waves scattered by the surface roughness implies sums of the form $\sum_{\mathbf{q}} |h(\mathbf{q})|^2 / \kappa^2(\mathbf{q})$, or $\sum_{\mathbf{q}} |h_2(\mathbf{q})|^2 / \kappa^2(\mathbf{q})$, where $h(\mathbf{q})$ and $h_2(\mathbf{q})$ are the Fourrier transform of the roughness function $h(\mathbf{r})$ and, respecticvely, $h^2(\mathbf{r})$ and $\kappa(\mathbf{q}) = \sqrt{\kappa_0^2 - 2\mathbf{k}_0\mathbf{q} - q^2}$. In order to maximize this energy, it is necessary, apart from particular cases of gratings (one, or a few wavevectors $\mathbf{q}$), to include as many Fourier components as possible, *i.e.* the surface should be as "rough" as possible in order to have a good attenuation, a reasonably expected result.

Finally, it is worth noting that we should "renormalize" the amplitudes of the reflected original Φ_0-waves such as to include (accomodate) the scattered waves in the boundary conditions (a procedure specific to theoretical-perturbation calculations).

In conclusion, we may say that we have introduced a model of inhomogeneous surface roughness for a semi-infinite (half-space) homogeneous, isotropic, ideal fluid and solved the wave equation for the waves scattered by this surface roughness in the first, relevant orders of approximation with respect to the roughness magnitude. For a fixed surface, the scattered waves appear in the first-order aproximation, while for a free surface they appear in the second-order approximation. The scattered waves are of two kinds: waves localized (and propagating only) on the surface (two-dimensional waves) and scattered waves reflected back in the fluid by the surface roughness. In some cases, the latter waves may become confined to the surface (rough-surface waves). The reflected waves are absent for a homogeneous roughness, where there exist only localized waves.

A similar model for surface roughness can also be employed for elastic waves in solid bodies or electromagnetic waves, as well as for small inhomogeneities distributed in the fluid volume (scatterers). These subjects are left for a forthcoming investigation.

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References

[3] D. L. Johnson, T. J. Plona and H. Kojima, "Probing porous media with first and second sound. II. Acoustic properties of watersaturated porous media", J. Appl. Phys. **76** 115-125 (1994).

[2] F. Padilla, M. de Billy and G. Quentin, "Theoretical and experimental studies of surface waves on solid-fluid interfaces when the value of the fluid sound velocity is located between the shear and the longitudinal ones in the solid", J. Acoust. Soc. Am. **106** 666-673 ((1999).

[1] J.-Z. Wu and J.-M. Wu, "Interaction between a solid surface and a viscous compressible flow field", J. Fluid Mech. **254** 183-211 (1993).

[4] B. Zhang and S. N. Chandler-Wilde, "Acoustic scattering by an inhomogeneous layer on a rigid plate", SIAM J. Appl. Math. **58** 1931-1950 (1998).

[5] I. Tolstoy, "Coherent sound scatter from a rough interface between arbitrary fluids with particular reference to roughness element shapes and corrugated surfaces", J. Acoust. Soc. Am. **72** 960-972 (1982).

[6] M. A. Biot and I. Tolstoy, "Formulation of wave propagation in infinite media by normal coordinates with application to diffraction", J. Acoust. Soc. Am. **29** 381-391 (1957).

[7] M. A. Biot, "Reflection on a rough surface from an acoustic point source", J. Acoust. Soc. Am. **29** 1193-1200 (1957).

[8] M. A. Biot, "Generalized boundary conditions for multiple scatter in acoustic reflection", J. Acoust. Soc. Am. **44** 1616-1622 (1968).

[9] See, for instance, A. Meier, T. Arens, S. N. Chandler-Wilde and A. Kirsch, "A Nystrom method for a class of integral equations on the real line with application to scattering by diffraction gratings and rough surfaces", J. Int. Eqs.&Appls. **12** 281-321 (2000).

[10] L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics*, vol. 6, *Fluid Mechanics*, Elsevier, Oxford, 2002.

[11] C. Kittel, *Introduction to Solid State Physics*, Wiley, NJ, 2005.