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Electromagnetic reflection and refraction for a semi-infinite solid with a rough surface

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Abstract

The electromagnetic field scattered by a rough surface of a semi-infinite solid is computed within the first-order of a perturbation scheme with the surface roughness as a perturbation parameter. The calculations are based on the equation of motion of the polarization within the Lorentz-Drude (plasma) model of polarizable, non-magnetic, homogeneous matter. It is shown that the surface roughness contributes its component modulated with half the in-plane wavelength of the incident wave to the scattered field in the first order of the perturbation scheme. Within the first-order approximation, the wave reflected by the surface roughness adds to the main (specularly) reflected wave, while the wave transmitted by the surface roughness into the solid propagates along the original direction of the incident wave (distinct from the main refracted wave). A (two-dimensional) mode, resonant at the longitudinal frequency of the solid, is identified, confined to (and propagating only on) the surface, due to the surface roughness.

Key words: *surface roughness; scattered electromagnetic field; Lorentz-Drude model; polarization motion*

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Highlights: Electromagnetic scattered field by surface roughness; perturbation-theoretical scheme; calculations within the first-order approximation; contribution of the surface roughness component modulated with half the in-plane wavelength of the incident wave; strictly two-dimensional mode, due to the surface roughness, resonant at the longitudinal frequency of the solid.

1 Introduction

Recently, there is a great deal of interest in the role played by the surface roughness (corrugation) in a large variety of physical phenomena, including the dispersive properties of the surface plasmon-polariton in nanoplasmonics,[1] terahertz-waves generation[2] or electronic microstructures.[3, 4] Enhanced, or suppressed, optical transmission in the subwavelength regime is associated with surface corrugation,[5] which induces also a highly-directional optical emission.[6] Multiple scattering has been emphasized, both experimentally and theoretically, in light scattered diffusely by a randomly rough surface.[7] The scattering theory within the Born approximation was applied to the surface roughness modelled by a dispersive (position dependent) dielectric function.[8]

The main difficulty in getting more definite results in such problems resides in modelling conveniently the surface roughness such as to arrive at more mathematically operational approaches.[9]

We present here a perturbation-theoretical scheme, with the surface roughness as a perturbation parameter, which allows the computation of the electromagnetic field scattered by the surface roughness in a semi-infinite solid. The scheme is based on the equation of motion of the polarization, whose degrees of freedom are explicitly introduced, within the well-knowm Lorentz-Drude (plasma) model of polarizable, non-magnetic, homogeneous matter. The scattered field is computed within the first order of the perturbation theory, where the component of the surface roughnes modulated with half the in-plane wavelength of the incident wave contributes to scattering. The wave reflected by the surface roughness adds to the main (specularly) reflected wave, while the wave scattered by the surface roughnesss into the solid propagates along the original direction of the incident wave (distinct from the refracted wave). The surface roughness contributes to the reflection and transmission coefficients in higher orders of the perturbation theory (starting with thesecond order), as expecetd. However, for damped waves in conductors, there exists a first-order contribution to these coefficients, arisingfrom thes urface roughness. As we can see, the main qualitative features of the scattering by surface roughness (directionality, change in the reflection and transmission coefficients, etc) appear even in the first-order of the perturbation theory. In addition, it is shown that the surface roughness generates a surface mode, *i.e.* a mode strictly confined to, and propagating only on the surface (a two-dimensional wave), resonant at the longitudinal frequency of the solid.

2 Semi-infinite solid with a rough surface

We consider a polarizable homogeneous body with a density n of mobile charges q moving in a uniform rigid neutralizing background. A small displacement $\mathbf{u}(\mathbf{R},t)$ of these charges, where $\mathbf{R}=(\mathbf{r},z)$, $\mathbf{r}=(x,y)$ is the position vector and t denotes the time, produces a chage density $\rho=-nq div\mathbf{u}$ and a current desity $\mathbf{j}=nq\dot{\mathbf{u}}$, corresponding to a polarization $\mathbf{P}=nq\mathbf{u}$. The vector potential is given by

$$\mathbf{A}(\mathbf{R},t)=\frac{1}{c}\int d\mathbf{R}'\frac{\mathbf{j}(\mathbf{R}',t-\left|\mathbf{R}-\mathbf{R}'\right|/c)}{|\mathbf{R}-\mathbf{R}'|}\ , \tag{1}$$

or, with the temporal Fourier transform,

$$\mathbf{A}(\mathbf{R},\omega)=\frac{1}{c}\int d\mathbf{R}'\frac{\mathbf{j}(\mathbf{R}',\omega)}{|\mathbf{R}-\mathbf{R}'|}e^{i\lambda\left|\mathbf{R}-\mathbf{R}'\right|}\ , \tag{2}$$

where $\lambda=\omega/c$. The scalar potential Φ is obtained from $div\mathbf{A}=i\lambda\Phi$ (Lorenz gauge) and the fields are given by $\mathbf{E}=i\lambda\mathbf{A}-grad\Phi$ (electric field), $\mathbf{H}=curl\mathbf{A}$ or $curl\mathbf{E}=i\lambda\mathbf{H}$ (magnetic field). We use the well-known decomposition[10]

$$\frac{e^{i\lambda|\mathbf{R}-\mathbf{R}'|}}{|\mathbf{R}-\mathbf{R}'|}=\frac{i}{2\pi}\int d\mathbf{k}\frac{1}{\kappa}e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}')}e^{i\kappa|z-z'|} \tag{3}$$

for the spherical wave (Green function) in equation (2), where $\kappa=\sqrt{\lambda^2-k^2}$, as well as Fourier transforms of the type

$$\mathbf{u}(\mathbf{r},z)=\frac{1}{(2\pi)^2}\int d\mathbf{k}\mathbf{u}(\mathbf{k},z)e^{i\mathbf{k}\mathbf{r}}\ . \tag{4}$$

For simplicity, the argument ω is omitted in such formulae, as well as, ocasionally, the wavevector argument $\mathbf{k}$.

Next, we consider a semi-infinite solid extending over the region $z > h(\mathbf{r})$, where $h(\mathbf{r})$, with $\int d\mathbf{r}h(\mathbf{r}) = 0$, is the surface roughness function, to be further specified. The polarization for this body is taken as

$$\mathbf{P} = nq(\mathbf{u},\, u_z)\theta(z - h(\mathbf{r})) \ , \tag{5}$$

where $\mathbf{u}$ lies in the $\mathbf{r}$-plane, u_z is directed along the z-direction and θ is the step function ($\theta(z) = 1$ for $z > 0$, $\theta(z) = 0$ for $z < 0$). We assume that the magnitude of the roughness function $h(\mathbf{r})$ is much smaller than the relevant wavelengths of the electromagnetic field and use the approximation

$$\begin{gathered} \mathbf{P} = \mathbf{P}^{(0)} + \mathbf{P}^{(1)} \ , \\ \mathbf{P}^{(0)} = nq(\mathbf{u},\, u_z)\theta(z) \ , \ \mathbf{P}^{(1)} = -nqh(\mathbf{r})(\mathbf{u},\, u_z)\delta(z) \ , \end{gathered} \tag{6}$$

where $\delta(z)$ is the Dirac delta-function. The specific conditions of validity for such an approximation, which is a first-order perturbation-theoretical approach, will be discussed below. We can see that the polarization $\mathbf{P}^{(0)}$ corresponds to a half-space extending over the region $z > 0$, while the polarization $\mathbf{P}^{(1)}$ is a surface polarization localized on the surface $z = 0$. We compute the electromagnetic potentials and fields as described above (equation (2)) for the charges and currents corresponding to these polarizations. The calculations are straightforward. Leaving aside the factor nq (it will be restored in the final formulae), we get

$$\begin{gathered} \mathbf{A}^{(0)} = \tfrac{2\pi\lambda}{\kappa} \int_0 dz'(\mathbf{u},\, u_z)e^{i\kappa|z-z'|} \ , \\ \Phi^{0)} = \tfrac{2\pi}{\kappa} \int_d^\infty dz'\mathbf{k}\mathbf{u}e^{i\kappa|z-z'|} - \tfrac{2\pi i}{\kappa}\tfrac{\partial}{\partial z} \int_d^\infty dz' u_z e^{i\kappa|z-z'|} \end{gathered} \tag{7}$$

and

$$\begin{gathered} \mathbf{A}^{(1)}(\mathbf{k}, z) = -\tfrac{2\pi\lambda}{\kappa} \left(\mathbf{g}(\mathbf{k}, z=0),\, g_z(\mathbf{k}, z=0)\right) e^{i\kappa|z|} \ , \\ \Phi^{(1)}(\mathbf{k}, z) = -2\pi \left[\tfrac{1}{\kappa}\mathbf{k}\mathbf{g}(\mathbf{k}, z=0) + g_z(\mathbf{k}, z=0)sgn(z)\right] e^{i\kappa|z|} \ , \end{gathered} \tag{8}$$

where

$$g(\mathbf{k}, z) = \int d\mathbf{r}h(\mathbf{r})\mathbf{u}(\mathbf{r}, z)e^{-i\mathbf{k}\mathbf{r}} = \frac{1}{(2\pi)^2}\int d\mathbf{q}h(\mathbf{q})\mathbf{u}(\mathbf{k} - \mathbf{q}, z) \tag{9}$$

and a similar formula for $g_z(\mathbf{k}, z)$, $h(\mathbf{q})$ being the Fourier transform of the roughness function $h(\mathbf{r})$ ($sgn(z) = +1$ for $z > 0$, $sgn(z) = -1$ for $z < 0$). Further on, we may omit the arguments $\mathbf{k}$, z of these functions and understand $z = 0$ in the functions $\mathbf{g}$, g_z.

In order to compute the electric field it is convenient to refer the in-plane vectors (*i.e.*, vectors parallel with the surface of the half-space) to the vectors $\mathbf{k}$ and $\mathbf{k}_\perp = e_z \times \mathbf{k}$, where e_z is the unit vector along the z-direction; for instance, we write

$$\mathbf{u} = u_1\frac{\mathbf{k}}{k} + u_2\frac{\mathbf{k}_\perp}{k} \tag{10}$$

and a similar representation for the electric field parallel with the surface of the half-space. The components u_1 and u_z correspond to the p-wave (parallel wave), while the component u_2 corresponds to the s-wave (from the German senkrecht which means perpendicular). In performing the calculations, it is worth paying attention to the derivative of the modulus function, according to the equation

$$\frac{\partial^2}{\partial z^2}e^{i\kappa|z-z'|} = -\kappa^2 e^{i\kappa|z-z'|} + 2i\kappa\delta(z - z') \ . \tag{11}$$

We get the electric field

$$E_1^{(0)} = 2\pi i\kappa \int_0^\infty dz' u_1 e^{i\kappa|z-z'|} - \frac{2\pi k}{\kappa}\frac{\partial}{\partial z}\int_0^\infty dz' u_z e^{i\kappa|z-z'|} \ ,$$

$$E_2^{(0)} = \frac{2\pi i\lambda^2}{\kappa}\int_0^\infty dz' u_2 e^{i\kappa|z-z'|} \ , \tag{12}$$

$$E_z^{(0)} = -\frac{2\pi k}{\kappa}\frac{\partial}{\partial z}\int_0^\infty dz' u_1 e^{i\kappa|z-z'|} + \frac{2\pi i k^2}{\kappa}\int_0^\infty dz' u_z e^{i\kappa|z-z'|} - 4\pi u_z\theta(z)$$

and

$$E_1^{(1)} = -2\pi i\left(\kappa g_1 - k g_z sgn(z)\right) e^{i\kappa|z|} \ ,$$

$$E_2^{(1)} = -\frac{2\pi i\lambda^2}{\kappa} g_2 e^{i\kappa|z|} \ , \tag{13}$$

$$E_z^{(1)} = 2\pi i k\left(g_1 sgn(z) - \frac{k}{\kappa}g_z\right) e^{i\kappa|z|} + 4\pi g_z\delta(z) \ ,$$

where $g_{1,2}$ are the projections of $\mathbf{g}$ on the vectors $\mathbf{k}$ and, respectively, $\mathbf{k}_\perp$. It is easy to check the following relations:

$$ikE_1^{(0)} + \frac{\partial E_z^{(0)}}{\partial z} = -4\pi\left(iku_1 + \frac{\partial u_z}{\partial z}\right)\theta(z) - 4\pi u_z(z=0)\delta(z) \ ,$$

$$k\frac{\partial E_1^{(0)}}{\partial z} + i\kappa^2 E_z^{(0)} = -4\pi i\lambda^2 u_z\theta(z) \tag{14}$$

and

$$ikE_1^{(1)} + \frac{\partial E_z^{(1)}}{\partial z} = 4\pi i k g_1\delta(z) + 4\pi g_z\delta'(z) \ ,$$

$$k\frac{\partial E_1^{(1)}}{\partial z} + i\kappa^2 E_z^{(1)} = 4\pi i\lambda^2 g_z\delta(z) \ , \tag{15}$$

which are the expression of Maxwell equations for this geometry.

3 Equations of motion. The perturbation scheme

In polarizable homogeneous (non-magnetic) matter the displacement field $\mathbf{u}(\mathbf{R}, t)$ is subjected to Newton's equation of motion

$$m\ddot{\mathbf{u}} = q(\mathbf{E} + \mathbf{E}_0) - m\omega_c^2\mathbf{u} - m\gamma\dot{\mathbf{u}} \ , \tag{16}$$

where m is the mass of the mobile charges, $\mathbf{E}$ is the electric field of the polarization charges and currents (calculated in the previous section), E_0 is an external electric field, ω_c is a characteristic frequency and γ is a damping coefficient. This is the well-known Lorentz-Drude model of matter polarization.[11]-[13] Taking the temporal Fourier transform of equation, with $\mathbf{E}_t = \mathbf{E} + \mathbf{E}_0$ the total electric field, we get the electric susceptibility $\chi(\omega) = P/E_t$ ($P = nqE$) and the dielectric function

$$\varepsilon(\omega) = 1 + 4\pi\chi(\omega) = \frac{\omega^2 - \omega_c^2 - \omega_p^2}{\omega^2 - \omega_c^2 + i\omega\gamma} \ , \tag{17}$$

where $\omega_p = \sqrt{4\pi nq^2/m}$ is the plasma frequency. For $\omega_c = 0$ in equation (17) we get the dielectric function of a conductor; $\omega_c \neq 0$ corresponds to dielectrics. This is also known as the Lydane-Sachs-Teller dielectric function,[14] with the longitudinal frequency $\omega_L = \sqrt{\omega_c^2 + \omega_p^2}$ and the transverse frequency $\omega_T = \omega_c$. In general, the damping coefficient γ is much smaller than these frequencies, so we limit ourselves to the ideal case $\gamma = 0$. It is worth noting the absence of the magnetic part of the Lorentz force in equation (16), according to the non-relativistic motion of the slight displacement $\mathbf{u}$. It is easy to see that, apart from relativistic contributions, it would introduce non-linearities in equation (16), which are beyond our assumption of a small displacement $\mathbf{u}$. Using spatial Fourier transforms, this approximation can be formulated as $\mathbf{K}\mathbf{u}(\mathbf{K}) \ll 1$, where $\mathbf{K}$ is the wavevector.

For temporal Fourier transforms equation (16) can also be written as ($\gamma = 0$)

$$(\omega^2 - \omega_c^2)\mathbf{u} = -\frac{q}{m}(\mathbf{E} + \mathbf{E}_0) \ . \tag{18}$$

According to the results given in the previous section, the polarization electric field $\mathbf{E}$ depends on the displacement field $\mathbf{u}$. For a plane surface ($h(\mathbf{r}) = 0$) we denote this displacement field by $\mathbf{u}^{(0)}$ and write equation (18) as

$$(\omega^2 - \omega_c^2)\mathbf{u}^{(0)} = -\frac{q}{m}\left[\mathbf{E}(\mathbf{u}^{(0)}) + \mathbf{E}_0\right] \tag{19}$$

(for $z > 0$). The electric field $\mathbf{E}(\mathbf{u}^{(0)})$ is given by equations (12) (with $\mathbf{u}$ replaced by $\mathbf{u}^{(0)}$). In the presence of the surface roughness, according to equation (6), the displacement field acquires a small additional contribution, denoted by $\mathbf{u}^{(1)}$, which corresponds to the field $\mathbf{E}^{(1)}(\mathbf{u})$, where $\mathbf{u} = \mathbf{u}^{(0)} + \mathbf{u}^{(1)}$. This additional contribution is governed by the equation of motion

$$(\omega^2 - \omega_c^2)\mathbf{u}^{(1)} = -\frac{q}{m}\mathbf{E}^{(1)}(\mathbf{u}) \ . \tag{20}$$

In keeping with the first-order of the perturbation theory, we may write this equation as

$$(\omega^2 - \omega_c^2)\mathbf{u}^{(1)} = -\frac{q}{m}\mathbf{E}^{(1)}(\mathbf{u}^{(0)}) \tag{21}$$

(for $z > 0$), where $\mathbf{E}^{(1)}(\mathbf{u}^{(0)})$ is given by equations (13), with $\mathbf{u}$ replaced by $\mathbf{u}^{(0)}$ in the g -functions (equation (9)). We solve equations (19) for $\mathbf{u}^{(0)}$ with an external plane wave field $\mathbf{E}_0$ and compute the field $\mathbf{E}^{(1)}(\mathbf{u}^{(0)})$ by equations (13) (both for $z > 0$ and $z < 0$); it is the field scattered by the surface roughness. The additional contribution $\mathbf{u}^{(1)}$ is obtained from equation (20).

4 Plane surface: zeroth order approximation

We focus now on equations (19) for a plane surface, with the electric field $\mathbf{E}$ given by equations (12). The incident wave is described by $\mathbf{E}_0 e^{-i\omega t + i\mathbf{k}\mathbf{r} + i\kappa z}$. We take the second derivative of equation (19) for $u_2^{(0)}$ with respect to z and use the relation given by equation (11). We get

$$\frac{\partial^2 u_2^{(0)}}{\partial z^2} + \kappa'^2 u_2^{(0)} = 0 \ \ , \tag{22}$$

where

$$\kappa'^2 = \kappa^2 - \frac{\lambda^2\omega_p^2}{\omega^2 - \omega_c^2} = \lambda^2\varepsilon - k^2 \ . \tag{23}$$

Therefore, the solution is

$$u_2^{(0)} = A_2 e^{i\kappa' z} \ \ , \tag{24}$$

where A_2 is a constant amplitude. We can see that the field propagates in the half-space with a modified wavevector κ', according to the Ewald-Oseen extinction theorem.[15] By equation (23) we can check the well-known polaritonic dispersion relation $\varepsilon\omega^2 = c^2K'^2$, where $\mathbf{K}' = (\mathbf{k}, \kappa')$ is the wavevector. Introducing the solution given by equation (24) in equation (19) we get the amplitude A_2 given by

$$A_2 \frac{\omega_p^2 \lambda^2}{2\kappa(\kappa' - \kappa)} = \frac{q}{m} E_{02} \ . \tag{25}$$

Making use of equations (12) for $z < 0$ we get the reflected field

$$E_2^{(0)} = -2\pi nqA_2 \frac{\lambda^2}{\kappa(\kappa' + \kappa)} e^{-i\kappa z} = \frac{\kappa - \kappa'}{\kappa + \kappa'} E_{02} e^{-i\kappa z} \ , \ z < 0 \ . \tag{26}$$

The (total) electric field inside the half-space is obtained from equation (19) as

$$E_2^{(0)} = -\frac{m}{q}(\omega^2 - \omega_c^2) A_2 e^{i\kappa' z} = \frac{2\kappa}{\kappa + \kappa'} E_{02} e^{i\kappa' z} \ , \ z > 0 \ . \tag{27}$$

It is easy to see that equation (26) defines the well-known Fresnel reflection coefficient[16]

$$R_s = \left| \frac{E_2^{(0)}}{E_{02}} \right|^2 = \left| \frac{\cos\theta_0 - \sqrt{\varepsilon}\cos\theta_r}{\cos\theta_0 + \sqrt{\varepsilon}\cos\theta_r} \right|^2 \tag{28}$$

for the s-wave, where $\theta_{0,r}$ denote the incidence and, respectively, refraction angles, $\sin\theta_0 = \sqrt{\varepsilon}\sin\theta_r$. Similarly, equation (27) defines the transmission coefficient for the s-wave.

Equations (19) are solved in a similar way for $u_{1,z}^{(0)}$. It is convenient to form the combinations $iku_1^{(0)} + \partial u_z^{(0)}/\partial z$ and $k\partial u_1^{(0)}/\partial z + i\kappa^2 u_z^{(0)}$, and use the relations given by equations (14). We find immediately that $u_{1,z}^{(0)}$ satisfy the same equation (22), with solutions

$$u_1^{(0)} = A_1 e^{i\kappa' z} \ , \ u_z^{(0)} = -\frac{k}{\kappa'} A_1 e^{i\kappa' z} \ , \tag{29}$$

where the amplitude A_1 is given by

$$A_1 \omega_p^2 \frac{\kappa\kappa' + k^2}{2\kappa'(\kappa' - \kappa)} = \frac{q}{m} E_{01} \ . \tag{30}$$

Inserting the solution given by equation (29) in equations (12) for $z < 0$ we get the reflected field

$$E_1^{(0)} = -2\pi nqA_1 \frac{\kappa\kappa' - k^2}{\kappa'(\kappa' + \kappa)} e^{-i\kappa z} = \frac{\kappa' - \kappa}{\kappa' + \kappa} \cdot \frac{\kappa\kappa' - k^2}{\kappa\kappa' + k^2} E_{01} e^{-i\kappa z} \ , \ z < 0 \tag{31}$$

and $E_z^{(0)} = (k/\kappa) E_1^{(0)}$; hence, the Fresnel reflection coefficient

$$R_p = \left| \frac{\sqrt{\varepsilon}\cos\theta_0 - \cos\theta_r}{\sqrt{\varepsilon}\cos\theta_0 + \cos\theta_r} \right|^2 \tag{32}$$

for the p-wave.[16] The (total) electric field is proportional to $\mathbf{u}^{(0)}$ (equation (19)), so we can get the transmission coefficient for the p-wave. In both cases (s- and p-waves) we can check that the reflection and transmission coefficients add to unity, as expected.

It is worth noting that there appears a resonance in equation (31) for $\kappa\kappa' + k^2 = 0$, provided κ and κ' are both purely imaginary. This resonance is given by

$$\omega^2 = \frac{2c^2k^2(\omega_L^2 + \omega_T^2)}{\omega_L^2 + 2c^2k^2 + \sqrt{(\omega_L^2 - 2c^2k^2)^2 - 4c^2k^2\omega_T^2}} \ . \tag{33}$$

We can see that in the long-wavelength limit $k \to 0$ the frequency given by equation (33) approaches the (surface) polaritonic frequency $\omega \sim ck(1 + \omega_T^2/\omega_L^2)$, while in the opposite limit $k \to \infty$ we get the surface plasmon frequency $\omega \simeq \sqrt{(\omega_l^2 + \omega_T^2)/2}$. We may call this resonance surface plasmon-polariton mode.[18]

5 The scattered field

We compute now the scattered field $\mathbf{E}^{(1)}$, as given by equations (13), in the first-order approximation of the perturbation theory, by making use of the displacement field $\mathbf{u}^{(0)}$ given by equations (24) and (29).

First, it is worth noting that the z-component of the scattered field given by equations (13) has a localized part

$$E_{zl}^{(1)} = 4\pi nqg_z\delta(z) \ . \tag{34}$$

This contribution corresponds to the motion of the localized part

$$P_z^{(1)} = -nqg_z\delta(z) \tag{35}$$

of the polarization given by equations (6). Indeed, the equation of motion (16) can also be written as

$$(\omega^2 - \omega_c^2)P_z^{(1)} = -\frac{\omega_p^2}{4\pi}E_z^{(1)} \ , \tag{36}$$

and using the field given by equation (34) we get

$$(\omega^2 - \omega_c^2)g_z = \omega_p^2 g_z \ . \tag{37}$$

We can see that the polarization, as well as the displacement field and the electric field localized on the surface exhibit a resonance for the longitudinal frequency $\sqrt{\omega_p^2+\omega_c^2}$. This resonant mode is purely two-dimensional, *i.e.* it is confined to the surface ($z=0$) and is propagating only on the surface.

Further on, we consider only the propagating fields in equations (13) (*i.e.* without $E_{zl}^{(1)}$). First, we note that the field scattered by the surface roughness into the solid is propagating along the same direction $\mathbf{K}=(\mathbf{k},\,\kappa)$ as the incident wave (it is proportional to $e^{i\mathbf{kr}+i\kappa z}$), in contrast with the zeroth order field (refracted field) which is transmitted into the solid along the direction $\mathbf{K}^{'}=(\mathbf{k},\,\kappa^{'})$ (it goes like $e^{i\mathbf{kr}+i\kappa' z}$). On the other hand, the field reflected by the surface roughness propagates along the same direction $(\mathbf{k},\,-\kappa)$ as the main reflected field. This is true only within the first-order approximation of the perturbation theory. Higher-order approximations, both in the expansion of the polarization given by equation (6) and in the equation of motion (20) will give a field scattered by the surface roughness along any direction (depending on the roughness function).

According to equations (24) and (29), we assume a zeroth order displacement field of the form

$$u_{1,2}^{(0)}(\mathbf{r},\,z) = A_{1,2}e^{i\mathbf{kr}+i\kappa' z} \tag{38}$$

(and $u_z^{(0)} = -(k/\kappa^{'})u_1$). Then, it is easy to see that equations (9) lead to

$$g_{1,2} = A_{1,2}^* h(2\mathbf{k}) \ , \ g_z = -\frac{k}{\kappa^{'}}A_1^* h(2\mathbf{k}) \ , \tag{39}$$

where $h(2\mathbf{k})$ is the Fourier transform of the roughness function $h(\mathbf{r})$ for twice the in-plane wavevector $\mathbf{k}$ of the incident wave. For instance, if $h(\mathbf{r}) = 2h\cos 2\mathbf{kr}$, then $h(2\mathbf{k}) = h$. We can see that, within the first-order theory of perturbation, only the surface roughness modulated with half the in-plane wavelength of the incident wave contributes to the scattering. Assuming $A_{1,2}$ real, from equations (13), (26) and (31) we can write the reflected field as

$$E_{1,2} = E_{1,2}^{(0)} + E_{1,2}^{(1)} = E_{1,2}^{(0)}\left[1 + i(\kappa^{'}+\kappa)h(2\mathbf{k})\right] \ , \ z<0 \tag{40}$$

(and $E_z = (k/\kappa)E_1$). Hence, we can see the condition of validity $\left|(\kappa' + \kappa)h(2\mathbf{k})\right| \ll 1$ for the perturbation scheme employed here: the magnitude of the roughness function should be much smaller than the relevent wavelengths. This condition can also be put in a more interesting form as

$$(\sqrt{\varepsilon}\cos\theta_r + \cos\theta_0)h(2\mathbf{k}) \ll \lambda \ , \tag{41}$$

where ε is the dielectric function, $\theta_{0,r}$ are the incidence and, respectively refraction angles ($\sin\theta_0 = \sqrt{\varepsilon}\sin\theta_r$) and $\lambda = c/\omega$ is the wavelength of the incident wave. A similar condition holds for the field scattered into the solid, whose amplitudes satisfy a relation similar with equation (40).

We can see from equation (40) that, in general, the surface roughness contributes to the reflection (and transmission) coefficient in the second-order of the perturbation theory, as expected. However, for conductors ($\omega_c = 0$) it may happen that κ' acquires purely imaginary values (especially for large incidence angles). Then, the wave is damped inside the conductor, and the surface roughness changes the reflaction coefficient according to

$$R \rightarrow R\left[1 - 2\left|\kappa'\right| h(2\mathbf{k})\right] \ . \tag{42}$$

A cooresponding increase occurs in the (damped) transmission coefficient.

It is easy to see that the inclusion of the damping factor γ in equation (16) leads to an imaginary part

$$\delta\varepsilon = \frac{i\omega\gamma\omega_p^2}{(\omega^2 - \omega_c^2)^2} \tag{43}$$

in the dielectric function and, correspondingly (by using $\varepsilon\omega^2 = c^2K'^2$), an imaginary part $\delta\kappa' = \omega\delta\varepsilon/2c\cos^3\theta_r$. We can see from equation (40) that the relative contribution of the surface roughness to the absorption is $3\omega\delta\varepsilon h(2\mathbf{k})/c\cos^3\theta_r$ (which should be much less than unity).

6 Concluding remarks

A perturbation-theoretical scheme was devised, with the surface roughness as a perturbation parameter, for the reflection and refraction of the electromagnetic waves for a semi-infinite solid. The polarization degrees of motion has been introduced explicitly, within the Lorentz-Drude (plasma) model of polarizable, non-magnetic, homogeneous matter. The field scattered by the surface roughness has been calculated within the first order of the perturbation scheme. It is shown that the component of the surface roughness modulated with half the in-plane wavelength of the incident wave contributes to the scattered field within the first-order approximation. Within the same approximation, the field reflected by the surface roughness adds to the main (specularly) reflected wave, while the field transmitted by the surface roughness into the solid propagates along the same direction as the incident wave (distinct from the refracted wave). A (strictly two-dimensional) mode, resonant at the longitudinal frequency of the solid, has been identified, confined to the surface, and propagating only on the surface. The contribution of the surface roughness to the reflection (and transmission) coefficient occurs only in the higher-order of the perturbation scheme, starting with the second-order (not calculated here). However, for damping waves in conductors, the first order of the perturbation theory may contribute to the reflection and transmission coefficients, as it was shown here.

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