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Dynamics of laser pulses in plasma

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Abstract

The propagation of electromagnetic pulses in plasma is analyzed, especially in connection with electrons accelerated by laser beams in rarefied gaseous plasmas. The interaction of the electromagnetic field with matter is introduced by using the polarization (displacement) field and supplementing Maxwell's equations with the equation of motion of this displacement field. This treatment amounts to using the Lorentz-Drude (plasma) model of polarizable (non-magnetic) matter and provides a solution of the electromagnetic field equations in matter. The plasmon and polariton eigenmodes of electromagnetic field-matter interaction are obtained. The extinction theorem is discussed and its implications in this context are presented, especially in connection with propagation, diffraction and refraction of electromagnetic waves in matter. The construction of a wavepacket and a pulse of electromagnetic field is described and the propagation of the polaritonic pulse in plasma is derived. The typical characteristics of a pulsed polariton, like velocity, transported charge, intensity, electromagnetic and mechanical energy are estimated for the favourable conditions of a rarefied plasma.

Introduction. The idea of accelerating electrons by focalizing intense laser pulses in plasma appeared as early as 1979.[1] The laser power has increased appreciably, by several orders of magnitude, in the 1980s and 1990s decades, following the introduction of "chirped amplification" procedure[2] (see also Ref. [3]).[1] Thereafter, a series of papers appeared, reporting on electrons accelerated in plasma by laser beams up to energies of the order of MeVs and even GeVs.[4]-[15] The table-top laser is envisaged to provide an alternative to the costly, big particle accelerators.

The typical characteristics of a nowadays laser, which we use here for illustrative purposes, are: radiation wavelength $1\mu m$ (infrared, frequency $2\times10^{15}s^{-1}$, photon energy $1eV$), energy per pulse $50J$, pulse duration $\tau=50fs=5\times10^{-14}s$, corresponding to $10^{15}w$ ($1Pw$) power. (For a $(15\mu m)^2$ pulse cross-section the intensity is $\simeq10^{20}w/cm^2$; repetition rate $\simeq1s$). In air (vacuum), such a pulse has a length $15\mu m$ (speed of light $c=3\times10^{10}cm$), corresponding to cca 15 wavelengths. The electric field in such a pulse is $\simeq10^6 statvolt/cm$ ($10^{10}V/m$, $1V/m=(1/3)\times10^{-4}statvolt/cm$), the magnetic field is $\simeq10^6Gs$ (10^2Ts, $1Ts=10^4Gs$). They are comparable with the atomic fields.[2]

[1] Materials are limited to laser intensities of $Gw's/cm^2$. In order to increase this power, the light pulse is streched by means of various optical devices (gratings, prisms, etc), *i.e.* it is made to last more in time, by taking advantage of the optical dispersion, thus leading to a lower power, which, in turn, can be further amplified. Finally, the pulse is compressed as highly as one desires, in principle, to get a high power. This is the basics of the chirped amplification idea.

[2] As such, they may produce non-linearities, X- and gamma rays, vacuum polarization and electron-positron pair creation, etc (see, for instance, Ref. [16] and references therein); or self-focusing, filamentation, harmonic generation, stimulated Raman and Brillioun scattering.

The convenient way of analyzing the laser pulses propagating in plasma is the pulsed polariton (which may equally well be termed "polaritonic pulse").[17] It is based on the wavepacket and dispersion concepts. Electromagnetic radiation (and fields, in general) propagating in matter interact with matter and get polaritonic (the non-retarded quasi-static limit is the plasmon). By means of experimental devices (collimators, apertures, etc) a wave packet of any shape can be obtained, in principle. Usually, it is preferable to have a focalized one, in order to increase the power.

Laser beam interacting with plasma. We asssume a homogeneous plasma of mobile (free) charges q with mass m (electrons) and concentration n in a rigid neutralizing (ionic) background (at room temperature). We are interested in small disturbances occurring over distances much larger than the inter-particle (inter-atomic) distance and over times much longer than the characteristic atomic periods, *i.e.* we assume macroscopic averages (both quantum mechanical and statistical), appropriate, as usually, for classical electromagnetism. Under the action of an external electromagnetic field the plasma gets polarized, *i.e.* a small charge density $\rho = -nqdiv\mathbf{u}$ and a small current density $\mathbf{j} = nq\dot{\mathbf{u}}$ occur in plasma (in matter, in general), where $\mathbf{u}(\mathbf{r}, t)$ is a local displacement field of the mobile charges, depending on the position $\mathbf{r}$ and the time t.[3] We note that $\mathbf{P} = nq\mathbf{u}$ is the polarization (dipole moment density). We leave aside the magnetic effects, and assume a non-magnetic plasma, since, usually, the plasma magetization is very weak.

With usual notations the internal (polarization) field obey the Maxwell equations

$$\begin{gathered} div\mathbf{E} = -4\pi nqdiv\mathbf{u} \ , \ div\mathbf{H} = 0 \ , \\ curl\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} \ , \ curl\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}nq\dot{\mathbf{u}} \ . \end{gathered} \tag{1}$$

We can see that only two equations (1) are independent, but we have three unknowns ($\mathbf{E}$, $\mathbf{H}$ and $\mathbf{u}$). The third equation is provided by the equation of motion for the field $\mathbf{u}$

$$m\ddot{\mathbf{u}} = q(\mathbf{E}_0 + \mathbf{E}) - m\gamma\dot{\mathbf{u}} \ , \tag{2}$$

where $\mathbf{E}_0$ is the external electric field and γ is a damping parameter. We leave aside the Lorentz force in the above equation (and limit ourselves to the non-relativistic Newton's equation of motion) since the charge velocities in plasma are much smaller than the speed of light c (by at most two or three orders of magnitude).[4] Equation (2) gives the well-known Lorentz-Drude electric susceptibility[18, 19]

$$\chi(\omega) = -\frac{nq^2}{m}\frac{1}{\omega^2 + i\omega\gamma} \tag{3}$$

(for conductors), by $\mathbf{P}(\omega) = \chi(\omega)[\mathbf{E}_0(\omega) + \mathbf{E}(\omega)]$; hence, the dielectric function $\varepsilon(\omega) = 1 + 4\pi\chi(\omega)$, where the plasma frequency $\omega_p = \sqrt{4\pi nq^2/m}$ appears.[5] The equation of motion (2) provides the missing equation (or its suitable extensions) for the solution of the Maxwell equations in (non-magnetic) matter; it is perfectly adequate and compatible to the assumptions of the classical electromagnetism, and defines in fact the classical model of electromagnetic matter.[6]

[3] A slight generalization for non-homogeneous plasma is $\rho = -qdiv(n\mathbf{u})$. We note the continuity equation $\partial\rho/\partial t + div\mathbf{j} = 0$.

[4] In addition, the internal magnetic field produced by the current $nq\dot{\mathbf{u}}$ leads to non-linear terms in the Lorentz force, which may be neglected in view of our small disturbances. Bound charges can be included in our treatment, by assuming a characteristic frequency ω_c (or several), with an equation of motion $m\ddot{\mathbf{u}} = q(\mathbf{E}_0 + \mathbf{E}) - m\omega_c^2\mathbf{u} - m\gamma\dot{\mathbf{u}}$.

[5] From equation (3) and definition $\mathbf{j}(\omega) = \sigma(\omega)[\mathbf{E}_0(\omega) + \mathbf{E}(\omega)]$, $\mathbf{j} = \dot{\mathbf{P}}$ we get also the conductivity $\sigma(\omega) = -i\omega\chi(\omega)$.

[6] Rumours say that Einstein was captivated by Drude model and intended to criticize it seriously; an intention

Our aim now is to solve equations (1) and (2). Since we deal with infinite matter, it is convenient to introduce Fourier decompositions of the type

$$\mathbf{u}(\mathbf{r},t)=\frac{1}{(2\pi)^4}\int d\mathbf{k}d\omega\mathbf{u}(\mathbf{k},\omega)e^{-i\omega t+i\mathbf{k}\mathbf{r}} \tag{4}$$

and use local axes of coordinates denoted by 1, 2, 3, where 1 is directed along $\mathbf{k}$ and 2 and 3 are transverse directions, perpendicular to $\mathbf{k}$ (1) and to one another. Taking the projections on these axes in equations (1) and (2) we get easily

$$u_1(\mathbf{k},\omega)=-\frac{q}{m}\frac{E_{01}(\mathbf{k},\omega)}{\omega^2-\omega_p^2+i\omega\gamma}\ , \tag{5}$$

which defines the longitudinal polarizability ($P_1=nqu_1=\alpha_1E_{01}$)

$$\alpha_1(\omega)=-\frac{q}{m}\frac{1}{\omega^2-\omega_p^2+i\omega\gamma}\ ;\ (\alpha_1=\chi/(1+4\pi\chi))\ ; \tag{6}$$

and the fields

$$E_1=-4\pi nqu_1\ ,\ E_t(\mathbf{k},\omega)=E_{01}(\mathbf{k},\omega)+E_1(\mathbf{k},\omega)=\frac{\omega^2}{\omega^2-\omega_p^2+i\omega\gamma}E_{01}(\mathbf{k},\omega)\ , \tag{7}$$

where E_t denotes the total electric field ($H_1=0$).[7]

Similarly, we get for the transverse fields

$$u_2(\mathbf{k},\omega)=-\frac{q}{m}\frac{(\omega^2-c^2k^2)E_{02}(\mathbf{k},\omega)}{\omega^2\left(\omega^2-\omega_p^2-c^2k^2\right)+isgn\omega\cdot 0^+} \tag{8}$$

(which gives a transverse polarizability) and

$$\begin{aligned}E_2(\mathbf{k},\omega)&=\tfrac{\omega_p^2}{\omega^2-\omega_p^2-c^2k^2+isgn\omega\cdot 0^+}E_{02}(\mathbf{k},\omega)\ ,\\ E_t(\mathbf{k},\omega)&=\tfrac{\omega^2-c^2k^2}{\omega^2-\omega_p^2-c^2k^2+isgn\omega\cdot 0^+}E_{02}(\mathbf{k},\omega)\ ;\end{aligned} \tag{9}$$

and the magnetic field

$$H_3(\mathbf{k},\omega)=\frac{ck}{\omega}E_2(\mathbf{k},\omega)=\frac{\omega_p^2}{\omega^2-\omega_p^2-c^2k^2+isgn\omega\cdot 0^+}H_{03}(\mathbf{k},\omega)\ , \tag{10}$$

never materialized, apparently. The Lorentz-Drude model, basically the equation of motion for the displacement field, was well known in the maturity stages of the electromagnetism (about 1900); for instance, Sommerfeld used it extensively for the theory of dispersion. Subsequently, it became obsolete, and people preferred a semi-empirical and semi-phenomenological theory of electromagnetic field in matter. A reason for such a curious step backward lies perhaps in the misconception that the Lorent-Drude theory is only a model, subjected to ad-hoc hypotheses. In fact, it might be considered a model, but it is the only valid and viable model, so it may be in fact the theory of matter polarization. Not less relevant, it brings complicated problems, while a semi-empirical and semi-phenomenological theory provides ready and easy answers. On the other side, the model has never been used systematically; for instance, there has never been done a Fourier analysis, nor a derivation of the eigenmodes; hence, a series of misconceptions related to the refraction law, the region (line) of anomalous dispersion (absorption), an illegitimate use of the dielectric function (considered a trustworthy solution), leading to velocities higher than the speed of light in vacuum, etc, etc.

[7]Usually, the damping parameter γ is much smaller than any relevant frequency (except, of course, for low frequencies).

where H_{03} is the (transverse) magnetic component of the external field. We can see that the transverse components of the fields exhibit a spatial dispersion (dependence on the wavevector $\mathbf{k}$), as expected;[8] it is introduced through the Green function $-4\pi c^2/(\omega^2 - c^2k^2 + isgn\omega \cdot 0^+)$ of the Helmholtz equation.

According to equations (5)-(7), the longitudinal fields exhibit an eigenmode at the plasmon frequency ω_p (which is dispersionless). We asssume here that the external field is transverse, as for a radiation (wave) field, so we may leave aside the longitudinal solutions.

The transverse fields given by equations (8) - (10) exhibit an eigenmode at the polaritonic frequency

$$\Omega(k) = \sqrt{\omega_p^2 + c^2k^2} \ , \tag{11}$$

which is dispersive. In addition, we can see that both the charge displacement (polarization) and the total electric field for a transverse (free) external wave are vanishing, since $\omega = ck$ for such a wave. This is the famous Ewald-Oseen extinction theorem.[20]-[22] In the region (ray) where the external wave propagates in matter there is no field; the real field is in another ray, the polaritonic ray, which is the refraction phenomenon.[9] Indeed, for the refraction and, respectively, incidence angles we can writte (Snell's law)

$$\frac{\sin r}{\sin i} = \frac{k'}{k} = \frac{\Omega}{\sqrt{\Omega^2 - \omega_p^2}} = \frac{1}{\sqrt{\varepsilon(\Omega)}} \ , \tag{12}$$

where we put $\Omega = ck'$ (in vacuum). We can see, by equation (12), that the polaritonic frequency (dispersion) is given by equation $\Omega^2\varepsilon(\Omega) = c^2k^2$; that the refraction index (n) is $1/\sqrt{\varepsilon}$; and waves with low frequencies (*e.g.*, $\Omega < \omega_p$) do not propagate in matter (plasma; transparency edge); and the incidence angle has limitations (total reflection, total polarization, etc). For high frequencies the refraction index approaches unity.[10]

[8]The transverse poalrizability is given by

$$\alpha_2(\mathbf{k}, \omega) = -\frac{nq^2}{m}\frac{\omega^2 - c^2k^2}{\omega^2\left(\omega^2 - \omega_p^2 - c^2k^2\right) + i\omega\gamma} = \frac{\chi(\omega)}{1 + 4\pi\frac{\omega^2}{\omega^2 - c^2k^2}\chi(\omega)} \ .$$

[9]This amounts to say that free waves with the dispersion relation $\omega = ck$ cannot be propagated in matter. The external field produces an infinitesimal (vanishing) displacement field, which, in turn, since the matter has an infinite extension, produces a non-vanishing internal field which cancels out the external field; leading thus to a vanishing total field. (Actually, the displacement field is vanishing, while the polarization field is undetermined). The situation is different in the presence of a surface, where the internal field has a constructive interference in another direction (refraction direction) and produces a displacement along that direction (for normal incidence the two directions coincide). The polarization eigenmodes of the infinite matter disappear in the presence of a surface (which exhibits a damped, surface-localized plasmon-polariton mode). In general, the polarization eigenmodes are different for a semi-infinite body (half-space), an external uniform magnetic field, etc.

[10]If bound charges are present (dielectrics, $\omega_c \neq 0$ in equation (2)), the polaritonic mode given by equation (11) is different; it corresponds to the dielectric function

$$\varepsilon(\Omega) = \frac{\Omega^2 - \omega_L^2}{\Omega^2 - \omega_c^2} \ ,$$

where $\omega_L = \sqrt{\omega_p^2 + \omega_c^2}$ is the longitudinal (plasmon) frequency (though the dielectric function must be used in the refraction law for solutions Ω of the equation $\varepsilon(\Omega)\Omega^2 = c^2k^2$); for high frequencies we have a less-than-unity refractive index (as for X- or gamma rays), similar with conductors; for low frequencies we have the usual greater-than-unity refractive index, coming from a second polaritonic mode, with an approximate dispersion relation $\Omega(k) = \omega_c ck/(\omega_L + ck)$, which may be termed the atomic polaritonic mode (since it goes to the atomic frequency

The wavelike nature of light, including reflection, refraction, diffraction, interference, was established during the $18th$ and $19th$ centuries (by Snell, Huygens, Young, Fresnel, Faraday, ...), long before the advent of Maxwell's equations $(1861-62)$.

Extinction theorem. We focus now on the fields in plasma, as produced by an external laser beam. We assume that the external field is purely transverse (as for radiation), and leave aside the subscripts 2 and 3 in equations (8)-(10). We take the y, z-axes along the axes 2 and, respectively, 3, while the x-axis is taken along the $\mathbf{k}$-direction (axis 1).

We can check the extinction theorem by direct calculations, starting with a monochromatic external field

$$E_0(\mathbf{r},t)=E_0\cos\omega_0(t-x/c) \tag{13}$$

with frequency ω_0 propagating along the x-direction (and perpendicular to this direction). The Fourier transform of this field is

$$E_0(\mathbf{k},\omega)=\frac{1}{2}E_0(2\pi)^4\delta(\mathbf{k}_t)[\delta(\omega-\omega_0)\delta(k_x-\omega_0/c)+(\omega_0\rightarrow-\omega_0)]\ , \tag{14}$$

where $\mathbf{k}_t=(k_y,\,k_z)$ is the transverse wavevector and k_x is the longitudinal wavevector. Using this Fourier transform in equations (8) and (9) we get straightforwardly

$$u(\mathbf{r},t)=0\ ,\ E_t(\mathbf{r},t)=0\ ,\ E(\mathbf{r},t)=-E_0(\mathbf{r},t)\ , \tag{15}$$

i.e. the extinction theorem. The result is obtained directly by using the properties of the δ-function in equation (14).[11] It shows that a free electromagnetic wave cannot be propagated in matter, as expected.

Wavepackets. A pure monochromatic free electromagnetic wave is a pure abstraction, an ideal approximation, though one extremely useful. It comes from the wave equation

$$(\omega^2/c^2-k^2)\mathbf{E}(\mathbf{k},\omega)=0 \tag{16}$$

for instance, whose solution can be written as $\mathbf{E}(\mathbf{k},\omega)=\mathbf{E}(\mathbf{k})\delta(\omega/c-k)+\mathbf{E}^*(-\mathbf{k})\delta(\omega/c+k)$; we note that we still have a freedom in the direction of the wavevector and the magnitude of the frequency, for instance; this form can be recognized in the field given by equation (14). The real situation is that the electromagnetic field is constructed in the laboratory by interaction with external agents, which, rigourously speaking, make the field non-free. Apertures, collimators, slits, mirrors, etc may give the electromagnetic field finite spatial and temporal extension. Usually, in such cases the field is not free anymore, and the (dispersion) relationship $\omega=ck$ does not hold any longer, rigurously speaking. For instance, a uniform field in the transverse directions subjected to a slit of size d_t has a corresponding Fourier transform[12]

$$\int d\mathbf{r}_t e^{i\mathbf{k}_t\mathbf{r}_t}=\frac{2\sin k_yd_t/2}{k_y}\cdot\frac{2\sin k_zd_t/2}{k_z}\quad , \tag{17}$$

ω_c in the short wavelength limit). The corresponding Snell law reads

$$\frac{\sin r}{\sin i}=\frac{k'}{k}=\frac{\omega_c-\Omega}{\omega_L}\ ;$$

(note that, in general, the $1/\sqrt{\varepsilon}$-law for refraction does not work). We note that all these refractive indices are given by the ratio of the phase velocity (Ω/k) to the speed of light (which may be trespassed by the phase velocity); and the refraction described here corresponds to the Huygens principle.

[11] Similarly, we can use the inverse Fourier transform of the external field and perform the integrations by taking the contributions of the Ω-poles in equations (8) and (9). This route implies more cumbersome calculations, leading to the same result.

[12] The dimensions along the two directions can be different from one another.

which is a function peaked over $\Delta k_t \simeq 1/d_t$; in the limit $d_t \to \infty$ this function approaches the $\delta(\mathbf{k}_t)$-function, as in equation (14). On the contrary, if the slit is very narrow ($d_t \to 0$), the Fourier transform does not depend on $\mathbf{k}_t$, *i.e.* it contains any $\mathbf{k}_t$. In this case, we can write the function as $d_t^2\delta(\mathbf{r}_t)$; we can check that the Fourier transform of this function is d_t^2, *i.e.* precisely the limit of equation (17) for $d_t \to 0$. This is a wavepacket.

Similarly, let us suppose a field

$$E_0(\mathbf{r},t) = E_0\cos(\omega_0 t - k_{0x}x) \tag{18}$$

subjected to a finite extension d along the x-direction and a a finite duration τ (compare with equation (13)). In equation (18) we asssume $\omega_0 = ck_{x0}$ (we note that $\mathbf{k}_{t0} = 0$) and $d \gg c/\omega_0$, $\tau \gg 1/\omega_0$. A factor $d_t^2\delta(\mathbf{r})$ may be included, or a constant transverse factor of extension d_t, such as its Fourier transform be that given by equation (17). We emphasize that there are two limiting cases of constructing a transverse wavepacket: either one allows for a large (infinite) extension, as for a beam, ray, plane wave (and the geometrical optics holds), or one assumes a δ-type localization, *i.e.* a very narrow wavepacket (pulse), which is at the diffraction limit. Usally, we consider a transverse extension of a few wavelengths, and represent it by a δ-function notation. The corresponding Fourier transform in equation (18) is given by

$$\begin{gathered}\int dtdxE_0\cos(\omega_0 t - k_{0x}x)e^{i\omega t}e^{-ik_x x} = \\ = \tfrac{1}{2}E_0\tfrac{2\sin(\omega-\omega_0)\tau/2}{\omega-\omega_0}\cdot\tfrac{2\sin(k_x-k_{0x})d/2}{k_x-k_{0x}} + (\omega_0 \to -\omega_0)\ .\end{gathered} \tag{19}$$

The same discussion holds for the temporal and longitudinal wavepackets in equation (19). We can say that we have an external field

$$E_0(\mathbf{r},t) = E_0\cos\omega_0(t - x/c)\ , \tag{20}$$

which extends over d_t around $\mathbf{r}_t = 0$, over d around $x = 0$, and lasts a time τ around the initial moment $t = 0$; it may also be represented formally as

$$E_0(\mathbf{r},t) = E_0\cos\omega_0(t - x/c)\cdot\tau\delta(t)\cdot d\delta(x)\cdot d_t^2\delta(\mathbf{r})\ , \tag{21}$$

in the sense that the δ-functions are viewed as supports of finite extensions (equally well we may use step functions). The Fourier transform of this field is given by

$$\begin{gathered}E_0(\mathbf{k},\omega) = \tfrac{1}{2}E_0\tfrac{2\sin(\omega-\omega_0)\tau/2}{\omega-\omega_0}\cdot\tfrac{2\sin(k_x-k_{0x})d/2}{k_x-k_{0x}}\cdot \\ \cdot\tfrac{2\sin k_y d_t/2}{k_y}\cdot\tfrac{2\sin k_z d_t/2}{k_z} + (\omega_0 \to -\omega_0)\ .\end{gathered} \tag{22}$$

It is worth noting that, although close to a free field, rigourously this is not a free field. The (longitudinal) extension of the field is larger than its main wavelength c/ω_0 and its duration is longer than the main period $1/\omega_0$. We can view this wavepacket as consisting of a superposition of many frequencies ω, in the vicinity of ω_0, and many wavevectors $\mathbf{k}$, in the vicinity of $k_{x0} = \omega_0/c$ and $\mathbf{k}_{t0} = 0$. In general, this field may not propagate. It is merely a representation of the electromagnetic perturbation produced in a plasma (at the origin and at the initial moment of time). We may call this wavepacket a (general) pulse.[13]

It is worth estimating the associated magnetic field from $curl\mathbf{E}_0 = -(1/c)\partial\mathbf{H}_0/\partial t$. We get $H_0(\mathbf{r},t) = E_0(\mathbf{r},t)$ for the transverse magnetic field perpendicular to the transverse electric field

[13] Electromagnetic peturbations of the type discussed here are produced usually in (closed) resonant cavities There inside, there exist steady waves, with different frequencies and wavelengths, according to the boundary conditions.

(and both perpendicular to the x-axis). The density of electromagnetic energy is easily estimated as $u_0 = E_0^2(\mathbf{r},t)/4\pi$, and the total electromagnetic energy is $U_0 = E_0^2 dd_t^2/8\pi$. There is also an internal flow of energy, given by the Poynting vector $S_x = cE_0^2(\mathbf{r},t)/4\pi$ and an internal momentum flow ($\mathbf{t} = \partial\mathbf{g}/\partial t$, $\mathbf{g} = \mathbf{S}/c^2$) corresponding to the stress force density $t_x = -\partial_x u_0$. These flows of energy and momentum indicate that, after preparation, left free, such an electromagnetic wavepacket (pulse) has the tendency to move.[14]

We can check by direct calculations that the external pulse given by equation (22), introduced into equations (8) and (9), where the contributions of the Ω-(polaritonic) poles are taken into account, leads to vanishing quantities, like the displacement field, etc. This is due to the fact that, although the pulse has the tendency to move, it does not in fact, since it does not satisfy (in general) a real dispersion equation (relationships like $\omega = ck$, or $\omega = \Omega(k)$). A pulse as that given by equation (22) is simply an external electromagnetic perturbation produced by external causes (it is not a free wave), which does not propagate. It is interesting to estimate in this context the admittance (or impedance) of the plasma for such an external perturbation. A real electromagnetic external field is a free wave (even with a finite extension) which would propagate in plasma (to a certain extent as given by a transmission coefficient). The full description of such a real situation would imply the taking into account the plasma-vacuum interface and the reflection and transmission coefficients. Since such matters are well known, we leave them aside here and turn to the description of the polaritonic pulse in plasma. In addition, we do not use general pulses as the ones described above.

Polaritonic pulse. Since, in accordance with the extinction theorem, a free electromagnetic field cannot be propagated in plasma, we put $E_0 = 0$ in equations (8)-(10) and rewrite them as

$$[\omega^2 - \Omega^2(k)]u(\mathbf{k},\omega) = 0 \ , \ E(\mathbf{k},\omega) = -\frac{m}{q}\omega^2 u(\mathbf{k},\omega) \ , \ H(\mathbf{k},\omega) = \frac{ck}{\omega}E(\mathbf{k},\omega) \ ; \qquad (23)$$

the solution is

$$u(\mathbf{k},\omega) = 2\pi u(\mathbf{k})[\delta(\omega - \Omega(k)) + \delta(\omega + \Omega(k)] \qquad (24)$$

(where $u^*(-\mathbf{k}) = u(\mathbf{k})$).[15] We get

$$u(\mathbf{r},t) = \frac{1}{(2\pi)^3}\int d\mathbf{k}u(\mathbf{k})e^{-i\Omega(k)t+i\mathbf{k}\mathbf{r}} + c.c. \qquad (25)$$

and

$$\begin{aligned} E(\mathbf{r},t) &= -\tfrac{m}{q}\tfrac{1}{(2\pi)^3}\int d\mathbf{k}\Omega^2(k)u(\mathbf{k})e^{-i\Omega(k)t+i\mathbf{k}\mathbf{r}} + c.c. \ , \\ H(\mathbf{r},t) &= -\tfrac{m}{q}\tfrac{1}{(2\pi)^3}\int d\mathbf{k}ck\Omega(k)u(\mathbf{k})e^{-i\Omega(k)t+i\mathbf{k}\mathbf{r}} + c.c. \ . \end{aligned} \qquad (26)$$

Such a steady wave consists of two superposed waves travelling in opposite directions. On opening an end of the cavity, part of these waves travel outside, the other part reflect on the opposite end of the cavity and travel outside too. Therefore, we have outside travelling waves with different frequencies (and corresponding wavevectors). For usual boundary conditions they satisfy the dispersion relation of free waves in vacuum. Entering matter (plasma), their wavevectors change according to the polaritonic dispersion relation (the frequencies remain unchanged), leading thus to a polaritonic pulse. We emphasize that a pulse of a finite duration implies a superposition of frequencies.

[14]The pulse constructed here corresponds rather to a collimator; usually, it is focalized in plasma, which means that d_t is smaller in the focus than at the origin.

[15]These are the polaritonic eigenmodes of the plasma. It is worth noting that they imply a "resonance"; while an oscillator at resonance is disrupted, a wave transfers the motion in space and is not disrupted locally.

These integrals are performed by the method of the stationary phase (method of the saddle point or steepest descent).[23]-[27] The phase $-i\Omega(k)t+i\mathbf{kr}$ is developed in powers of $\mathbf{q}=\mathbf{k}-\mathbf{k}_0$,

$$-i\Omega(k)t+i\mathbf{kr}=-i\Omega(k_0)t+i\mathbf{k}_0\mathbf{r}+(-i\left.\frac{\partial\Omega}{\partial\mathbf{k}}\right|_{\mathbf{k}_0}t+i\mathbf{r})\mathbf{q}-\frac{it}{2}\left.\frac{\partial^2\Omega}{\partial k_i\partial k_j}\right|_{\mathbf{k}_0}q_iq_j+..,\tag{27}$$

around a wavevector $\mathbf{k}_0$, which is chosen such as to vanish the linear term in the expansion; which means that the main contribution to the integrals move with the group velocity

$$\mathbf{v}=\left.\frac{\partial\Omega}{\partial\mathbf{k}}\right|_{\mathbf{k}_0}\ ;\tag{28}$$

indeed, around this point the phases add constructively.

In order to simplify the discussion we choose $\mathbf{k}_0=(k_{0x},\,k_{0y}=0,\,k_{0z}=0)$; the integration over $q_t=(q_y,\,q_z)$ can be approximated by a transverse wavepacket of extension d_t,

$$\int d\mathbf{q}_t e^{i\mathbf{q}_t\mathbf{r}_t}=\frac{2\sin y/2d_t}{y}\cdot\frac{2\sin z/2d_t}{z}\ ,\tag{29}$$

where the range of integration is $\Delta q_x=\Delta q_y\simeq 1/d_t$; for sufficiently small values of d_t we may approximate this wavepacket by $(2\pi)^2\delta(\mathbf{r}_t)$, where $\mathbf{r}_t=(y,\,z)$. We are left, for instance, with

$$E(\mathbf{r},t)\simeq-\frac{m}{q}\Omega_0^2u_0\delta(\mathbf{r}_t)\cdot e^{-i\Omega_0t+ik_{0x}x}\frac{1}{2\pi}\int dq_xe^{-i(vt-x)q_x-\frac{it}{2}\Omega_0^{''}q_x^2}+c.c.\ ,\tag{30}$$

where $u_0=u(\mathbf{k}_0)$ and

$$v=\frac{c\omega_0}{\Omega_0}\ ,\ \Omega_0^{''}=\frac{c^2\omega_p^2}{\Omega_0^3}\ ,\ \omega_0=ck_{x0}\ ,\ \Omega_0=\sqrt{\omega_p^2+\omega_0^2}\ .\tag{31}$$

The integral appearing in equation (30) is given by[16]

$$\frac{1}{2\pi}\int dq_xe^{-i(vt-x)q_x-\frac{it}{2}\Omega_0^{''}q_x^2}\simeq\frac{1}{\sqrt{2\pi it\Omega_0^{''}}}e^{i\frac{(x-vt)^2}{2t\Omega_0^{''}}}\rightarrow_{t\Omega_0^{''}\rightarrow0}\delta(x-vt)\ ,\tag{32}$$

which is a representation of the δ-function in the limit $t\Omega_0^{''}\rightarrow 0$ (it looks like an "imaginary" diffusion).[17] This representation gives an estimation of the extension of the pulse and its lifetime: at $t=0$ the pulse is $\delta(x)$ (equation (32)), we can take it as $\delta(x)\simeq 1/d$; after time Δt the pulse has a height $\simeq 1/\sqrt{\Delta t\Omega_0^{''}}$,which should be compared with $1/d$, and a width $\Delta x\simeq\sqrt{\Delta t\Omega_0^{''}}$(equation (32)). It follows that we may define a lifetime by

$$\frac{1}{\sqrt{\Delta t\Omega_0^{''}}}=\frac{1}{df}\ ,\ \Delta x=\sqrt{\Delta t\Omega_0^{''}}=fd\ ,\tag{33}$$

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[16]The contour of integration is deformed in such a way as to ensure a "maximum" for the quadratic form appearing in equation (30): $q_x+(vt-x)/t\Omega^{''}=\sqrt{2/it\Omega^{''}}z$ (steepest descent around a (general) saddle point).

[17]This is a Fresnel integral. Fresnel diffraction gives interference fringes and a vanishing shadow. Our approximation here of passing to a δ-function is equivalent with a Fraunhofer diffraction: it gives fringes in the shadow region. On the pulse wavefront in equation (32) additional contributions occur; we have therefore propagating "precursors" ("forerunners"), which point out the wave non-locality. They are produced by our selection of a main plane wave (denoted by the subscript 0), which, by its nature, is non-local.

where f is an arbitrary, small, higher-than-unity number. For the limiting case $f=1$, we get $\Delta t \simeq d^2/\Omega_0''$, or, since $d=v\tau$, we have $\Delta t = v^2\tau^2/\Omega_0''$; *i.e.*

$$\Delta t = \tau^2 \frac{\omega_0^2\Omega_0}{\omega_p^2} \ ; \tag{34}$$

hence we get a limiting value for the duration of the pulse

$$\tau = \Delta t = \frac{\omega_p^2}{\Omega_0\omega_0^2} \tag{35}$$

and a limiting value of the pulse extension

$$d = v\tau = c\frac{\omega_p^2}{\Omega_0^2\omega_0} \ . \tag{36}$$

We can see that these limitations depend on the plasma frequency and the frequency ω_0; they can also be written as

$$d = \lambda_0 \frac{\omega_p^2}{\Omega_0^2} < 1 \ , \ \tau = t_0 \frac{\omega_p^2}{\Omega_0\omega_0} < t_0 \ , \tag{37}$$

where $\lambda_0 = c/\omega_0$ is the main wavelength and $t_0 = 1/\omega_0$ is the period of the main radiation. During its lifetime Δt given by equation (34) the pulse flies the distance $l = v\Delta t = c\tau^2\omega_0^3/\omega_p^2$, which is a pretty long distance for $\omega_p \ll \omega_0$ ($l \simeq d\tau\omega_0^3/\omega_p^2$).

It follows that we can write the polaritonic pulse as

$$\begin{aligned} u(\mathbf{r},t) &\simeq 2u_0\cos(\omega_p^2 t/\Omega_0)\delta(x-vt)\delta(\mathbf{r}_t) \ , \\ E(\mathbf{r},t) &\simeq -\tfrac{2m}{q}\Omega_0^2 u_0\cos(\omega_p^2 t/\Omega_0)\delta(x-vt)\delta(\mathbf{r}_t) \ , \\ H(\mathbf{r},t) &\simeq -\tfrac{2m}{q}\Omega_0\omega_0 u_0\cos(\omega_p^2 t/\Omega_0)\delta(x-vt)\delta(\mathbf{r}_t) \ . \end{aligned} \tag{38}$$

First, according to the discussion above, we note that such a pulse makes sense only for $\omega_p \ll \Omega_0 \simeq \omega_0$ (otherwise it is too flat and too slow). According to its construction, it consists of a superposition of frequencies $\Omega(k)$ and wavevetors $\mathbf{k}$ around $\Omega_0 = \Omega(k_0)$ and $\mathbf{k}_0$. For instance, the range of q_x around k_{0x} for a pulse of extension d is $\Delta q_x \simeq 1/d$. It follows that the main frequency Ω_0 (practically ω_0) is the laser frequency, the superposition arising by the effect of the external agents (collimators, lenses, etc) as well as by the finite duration $\tau \simeq d/v \simeq 1/v\Delta q_x \simeq 1/\Delta\Omega$. At $t=0$ we have $E(\mathbf{r},t=0) \simeq -(2m/q)\Omega_0^2 u_0\delta(x-vt)\delta(\mathbf{r}_t)$, which we may write as $E(\mathbf{r},t=0) \simeq E_0 \cdot d\delta(x-vt)\cdot d_t^2\delta(\mathbf{r}_t)$, where E_0 may be viewed as the magnitude of the external electric field. It is then more convenient to express the pulse as

$$\begin{aligned} u(\mathbf{r},t) &\simeq -\tfrac{q}{m\Omega_0^2}E_0\cdot d\delta(x-vt)\cdot d_t^2\delta(\mathbf{r}_t) \ , \\ E(\mathbf{r},t) &\simeq H(\mathbf{r},t) \simeq E_0\cdot d\delta(x-vt)\cdot d_t^2\delta(\mathbf{r}_t) \ ; \end{aligned} \tag{39}$$

it has an electromagnetic energy $U = E_0^2 d d_t^2/4\pi$, which is transported with velocity $v = c\omega_0/\Omega_0$ during a lifetime $\Delta t \simeq d^2\Omega_0^3/c^2\omega_p^2$ over a distance $l \simeq v\Delta t \simeq d^2\Omega_0^3/c\omega_p^2$. We can see that the pulse exhibits a small pulsation with frequency ω_p^2/Ω_0, which may be neglected.[18]

[18]The energy conservation from Maxwell's equations (1) and equation of motion (2) reads:

$$\frac{\partial}{\partial t}(E^2+H^2)/8\pi + \frac{\partial}{\partial t}(\frac{1}{2}nm\dot{u}^2) + \frac{c}{4\pi}div(\mathbf{E}\times\mathbf{H}) = 0$$

There are a few situations of electromagnetic waves propagating in matter. First, we may imagine a plane wave propagating in an infinite body: if free, it cannot be propagated, according to the extinction theorem; it can only be propagating if it satisfies the polaritonic dispersion relation. This situation is equivalent with creating an electromagnetic disturbance in a body with a sufficient spatial extension; then the propagation of the disturbance can be analyzed in terms of polaritonic plane waves. This situation corresponds to the case presented in this paper, with special emphasis on a disturbance centered on a main frequency, *i.e.* generating a main frequency for a finite (sufficiently long) duration, and localized in a certain spatial region (preferrably small). Under such circumstances we get the pulsed polariton described here. In practice, one sends usually a plane wave (usually from the vacuum) on a body with a definite surface. In this case, the plane wave penetrates the body as a polaritonic wave propagating along a refraction direction. We emphasize that the conditions of geometric optics can be fulfilled in such a case. Perhaps, it is more suitable in this case to talk of a propagating beam or ray. Moreover, an optical system of focusing can be used in this case (a lens), and the rays can be focalized (through the surface) somewhere in the body. In the focus region we can create then a pulsed polariton. Once created, under favourable conditions (sufficiently localized), the pulsed polariton may become an autonomous entity, obeying not anylonger the laws of geometrical optics, and propagating as a well-defined entity, sometimes over pretty long distances. Finally, there could be another situation, where a very narrow pulse is created in vacuum, *i.e.* a highly localized spatial region of electromagnetic field, usually with a main frequency, and send through the surface of a body (either at normal or oblique incidence). In this case, the pulse may retain its individuality to an appreciable extent, propagating in the body almost as in vacuum (in particular with the speed of light), although it may create a polaritonic response of the body both inside (transmitted field) and outside (reflected field) over large distances.

It is worth looking for a refraction law for pulses. While the pulse which retains its individuality is not likely to be refracted, the polaritonic pulse formed inside a body may suffer refraction, in principle, though experimental conditions for such a situation are difficult to be realized. It is likely that the Snell law in this case is $\sin r/\sin i = c/v$, where v is the group velocity. This is to be contrasted with the previous formulae, where $\sin r/\sin i = v/c$, where v is the phase velocity (Ω/k). For conductors the two formuale are the same (and the product of the two velocities is c^2). For dielectrics (bound charges) the situation is more complex. The regular dielectric function gives a less-than-unity refractive index for high frequencies, which increases with increasing frequency. Similarly, the atomic-polaritons give a less-than-unity refractive index, which is limited to ω_c. In general, on increasing the energy we may have a cross-over from a phase-velocity driven refractive index to a group-velocity refractive index, a situation which might be encountered in the refraction of the gamma rays in Si, as reported recently.[28]

Rest frame. In the rest frame of the polaritonic pulse, *i.e.* the frame moving (along the x-direction) with velocity v with respect to the laboratory frame, the coordinate is $x' = \gamma(x - vt)$ and time is $t' = \gamma(t - vx/c^2)$, where $\gamma = (1 - v^2/c^2)^{-1/2}$ ($x = \gamma(x' + vt')$ and $t = \gamma(t' + vx'/c^2)$). Since $x - vt = x'/\gamma$, we have $\delta(x - vt)dx = \gamma\delta(x')dx$, *i.e.* $dx' = \gamma dx$, which is Lorentz contraction, as expected. It follows that the size of the pulse in the rest frame is $d' = \gamma d$, much longer than d;

(it includes the mechanical energy of the charges); it is identically satisfied everywhere except for the pulse boundaries, where the transport is governed by

$$(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x})\delta(x - vt) = 0$$

for the density of any energy (electromagnetic, mechanical). The same holds for the Poynting vector, stress forces, electromagnetic momentum, etc. We can see that for a polaritonic pulse the electromagnetic or mecahnical conservation laws become irrelevant, in the sense that they are satisfied identically.

indeed, it is difficult to construct a wavepacket in the rest frame, since time is very slow, lengths are very long and dispersion is very efficient. We note that $\gamma = (1 - v^2/c^2)^{-1/2} = \Omega_0/\omega_p \gg 1$ (and the transverse coordinates are not affected by the change of the reference frame).

Let us apply the Lorentz transformations to the fields $E_y = E$ and $H_z = H$ given by equations (39):

$$E_y^{'} = \gamma(E_y - \tfrac{v}{c}H_z) \ , \ E_z^{'} = \gamma(E_z + \tfrac{v}{c}H_y) \ , \qquad H_y^{'} = \gamma(H_y + \tfrac{v}{c}E_z) \ , \ H_z^{'} = \gamma(H_z - \tfrac{v}{c}E_y) \ ; \tag{40}$$

here it is important to use the exact magnetic field as given by equations (38) $H_z = (\omega_0/\Omega_0)E = \beta E$, where $\beta = v/c$. We get $H_z^{'} = 0$ and $E_y^{'} = E/\gamma = (\omega_p/\Omega_0)E \simeq (\omega_p/\Omega_0)E_0$, in accordance with the fact that we have a static situation in the rest frame (a vanishing magnetic field).

We note that in the rest frame there is a weak electric field $E_y^{'} = (\omega_p/\Omega_0)E$. We have also a transverse displacement u (along the y-axis), as given by equation (39), which produces a polarization $P = nqu = -(\omega_p^2/4\pi\Omega_0^2)E$ according to equation (39) and, consequently, a longitudinal field $E_{ly} = 4\pi P = -(\omega_p^2/\Omega_0^2)E$; now, on passing to the rest frame we note that the plasma frequency does not change, it is a material constant, so the density n in these formulae does not change; actually, for a body in motion the definition of the polarization is changed, such as the relativistic invariance be satisfied;[29, 30] this implies the polarization be changed according to the transformation of the fields (and a magnetization also occurs, which actually is the magnetic field in the laboratory frame); therefore, we have $P^{'} = \gamma P$ in the rest frame and a longitudinal field $E_{ly}^{'} = -\gamma(\omega_p^2/\Omega_0^2)E = -(\omega_p/\Omega_0)E$ which cancels out exactly the field $E_y^{'}$. Therefore, the pulsed polariton is at equilibrium due to the fact that the plasma polarization compensates the action of the external field.[19]

Emergent physics. An electromagnetic pulse of finite spatial extension and temporal duration can be formed in plasma by exciting the polaritonic eigenmodes. It arises by a local constructive interference of phases, which may arise over a finite distance and a finite lifetime, propagates with the group velocity and transports electromagnetic fields and matter.[20] Due to its localization such a wave superposition can be viewed as a particle, of approximate identity and finite existence, a quasi-stable, localized, autonomous entity, which we call pulse polariton (or polaritonic pulse). It may give rise to an emergent physics.[21]

The displacement u given by equation (39) generates a density imbalance

$$\delta n = -ndiv\mathbf{u} = \frac{nq}{m\Omega_0^2 d_t}E_0 \tag{41}$$

and a number

$$\delta N = \frac{nq}{m\Omega_0^2}dd_tE_0 \tag{42}$$

of mobile charges distributed over surface along the y-transverse direction; and a corresponding charge

$$\delta Q = \frac{\omega_p^2}{4\pi\Omega_0^2}dd_tE_0 \ . \tag{43}$$

Using the total number of mobile particles $N = ndd_t^2$ included in the pulse volume, equation (42) can also be writte as $\delta N/N = qE_0d_t/mv_0^2$, where $v_0 = \Omega_0 d_t$ may stand for a velocity; it follows

[19] The relativistic equation of motion in the laboratory frame preserves the equilibrium in these circumstances.

[20] Of course, such pulses can be formed in vacuum too, where the dispersion is absent and they may last forever as perfectly localized entities.

[21] The transverse longitudinal field derived above is a manifestation of emergent physics.

that the fraction of the particles displaced by the pulse is the ratio of the work qE_0d_t done by the field to the kinetic energy of the particles, which is a small quantity.

These particles (charges) move with velocity $v = c\omega_0/\Omega_0$, and each acquires an energy

$$\mathcal{E} = \frac{mc^2}{\sqrt{1 - v^2/c^2}} = mc^2\frac{\Omega_0}{\omega_p} \ , \tag{44}$$

which may attain high values. We have also a flow of particles

$$\Phi_N = v\delta n = c\frac{nq\omega_0}{m\Omega_0^3 d_t}E_0 \tag{45}$$

and a flow of energy

$$\Phi = \Phi_N\mathcal{E} = c^3\frac{nq\omega_0}{\Omega_0^2\omega_p d_t}E_0 \ , \tag{46}$$

which should be compared with the energy flow $\Phi_0 \simeq vE_0^2/4\pi$(intensity); we get

$$\frac{\Phi}{\Phi_0} = c^2\frac{nq}{\omega_p\Omega_0 d_t}\sqrt{\frac{4\pi c\omega_0}{\Omega_0\Phi_0}} \ . \tag{47}$$

We can see that it is favourable, in order to accelerate electrons, to have a rarefied plasma ($\omega_p \ll \Omega_0$), which gives rise to a well-defined pulse (with low dispersion); at the same time, an increase in accelerating energy is possible only at the expense of the charge flow.

For illustrative purposes we take here the plasma density $n = 10^{18}cm^{-3}$ (electrons, $q = 4.8 \times 10^{-10}esu$, $m \simeq 10^{-27}g$), which corresponds to a plasma frequency $\omega_p = 3\times10^{-2}eV$ ($\simeq 5\times10^{13}s^{-1}$), and a main frequency $\Omega_0 = 1eV$ ($2 \times 10^{15}s^{-1}$, wavelength $1\mu m$). We get an ultra-relativistic velocity of the pulse and a particle energy $\mathcal{E} \simeq 20MeV$.

We take also a typical size of the pulse $d = d_t = 15\mu m$ (corresponding to a pulse duration $\tau = 50fs = 5\times10^{-14}s$) and an energy $50J$, corresponding to an electric field $E_0 \simeq 10^6 statvolt/cm$ ($1statvolt/cm = 3 \times 10^4V/m$); the intensity of the pulse is $\Phi_0 = 4 \times 10^{20}w/cm^2$. This pulse transports $\delta N \simeq 3 \times 10^5$ particles (electrons) (*i.e.* $6TeV$), which means $\simeq 10^{24}$ particles per $cm^2 \cdot s$ and a large amount of energy, $\Phi \simeq 10^{25}MeV/cm^2 \cdot s$. The magnitude of the displacement in the pulse is $qE_0/m\Omega_0^2 \simeq 10$Å.[22]

Finally, we note that the polaritonic pulse may also be viewed as a localized negative electronic charge and a neutralizing positive ionic charge, both moving with a high velocity; as such, they generate (almost compensating) transverse fields; in addition, neutralizing currents which compensate for the static ionic charge backflow, producing disturbances (turbulence) in plasma and contributing to the charge neutralization.[23]

Concluding remarks. The theory of the electromagnetic field in (polarizable) matter is based on the displacement field of the mobile charges.[31] This theory supplements the Maxwell equations with an equation of motion for the displacement field, which allows a solution of the problem. Basically, this amounts to the well-known Lorentz-Drude (plasma) model of polarizable matter. This theory was applied here (and in Ref. [17]) to the dynamics of the laser beams focalized in (rarefied) plasma. First, it was recognized the essential role played by polaritons (polaritonic

[22] This is to be compared with nuclear polarization, where distances are $5 - 6$ orders of magnitude smaller; this is why macroscopic (or quantum macroscopic) laser induced nuclear effects are extremely small.

[23] Another example of emergent physics related to the pulsed polariton is the production of coherent X- or gamma rays by Compton (Thomson) backscattering.[17]

eigenmodes) in propagation of the electromagnetic field in matter. Then, it was shown that, by external agents, we can form electromagnetic fields of a finite spatial extension and a finite temporal duration. A constructive interference may appear in such wave superpositions, leading, through the mechanism of the stationary phase, to a quasi-localized, quasi-stable pulse of a finite duration, subjected to the destroying effect of the dispersion. The duration (lifetime) and spatial extension of such pulses have been determined. It was shown that such a pulse, termed pulsed polariton or polaritonic pulse) moves with the group velocity and transport electromagnetic field, mobile charges, electromagnetic and mechanical energy, and may serve to accelerate electrons in rarefied plasma (which provides favourable conditions).

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