Journal of Theoretical Physics

Founded and Edited by M. Apostol **238** (2015)

ISSN 1453-4428

Elastic waves equation with localized sources in isotropic half-space

B. F. Apostol
Department of Engineering Seismolgy, Institute of Earth's Physics,
Magurele-Bucharest MG-6,
POBox MG-35, Romania
email: afelix@theory.nipne.ro

Abstract

The representation of a localized faulting seismic source is reformulated, without resorting to the mechanical torque interpretation of the seismic moment tensor. A volume seismic source is introduced by means of the pressure exerted in a small spherical cavity. The elastic waves equation with localized (point) sources is discussed in an isotropic half space bounded by a plane surface, including the boundary conditions. The near-field approximation is used to get the solution of a quasi-static deformation, and the transient regime of far-field propagating elastic waves is discussed for the (time-pulse) seismic sources introduced here.

MSC: 74B05; 86A15; 74G70; 74G05; 74J10; 35Q74

PACS: 91.30.Bi; 91.30.Cd; 02.30.Jr; 46.70.Lk; 02.30.Em

Key words: seismic sources; elastic waves equation; isotropic half-space; near- and far-field solutions

Introduction. The main problem in Seismology is the propagation of the seismic waves. It gives information about processes occurring in earthquake focal region, the nature and structure of the Earth's interior, and the effect the seismic waves have on the Earth's surface. The problem originates in the works of Rayleigh, Lamb and Love.[1]-[3] In a simplified model, the Earth is viewed as an isotropic elastic half-space bounded by a plane surface, and the seismic sources are localized beneath the surface. For sufficiently long distances the localization of the seismic sources may be represented by δ-functions, or their derivatives (point sources). The double-couple representation of point seismic sources by means of the seismic moment tensor emerged gradually in the first half of the 20th century.[4]-[17]

The standard way of treating the seismic waves is to employ the (formal) Green function for the elastic waves equation and the Green theorem (the so-called Betti's representation), for a general, anisotropic, elastic body.[18]-[21] In this treatment the seismic sources are located on internal surfaces, either as faulting sources or volume sources. The faulting seismic sources are related to the discontinuity occurring in the displacement across the faulting surface (fault slip), while the volume sources are related to the dilatational strain.[6, 7] In both cases equivalent forces are derived for the seismic source representation, and the seismic moment is introduced. The interpretation of the "double-couple" representation of the sesmic moment as a mechanical torque (a couple of forces), while ensuring the vanishing of the net force and angular momentum, leaves open the question of the uniqueness of the double-couple distribution. We reformulate here the faulting source and the introduction of the seismic moment tensor, and introduces a volume source related to the pressure exerted in a small spherical cavity.

The elastic waves equation with sources for a half space bounded by a plane surface is discussed, including boundary conditions (usually, for a free surface). This equation is suitable for a stationary regime; we give an approximate near-field solution with the seismic sources introduced here for quasi-static displacements (especially for the surface). The far-field waves generated by these seismic sources (with δ-like time pulses) are given in the transient regime, where the relevant distances are sufficiently long in comparison with the wavelengths.

Seismic sources. The seismic sources are concentrated in small volumes. The linear dimensions of these regions are much smaller than the seismic wavelengths and distances of interest, so we may view them as point sources. In a faulting source the slip and the associated force occur, during an earthquake, along one direction, lying on the fault surface. Let $\mathbf{n}$ be the unit vector along this direction. The seismic load in the focus consists of two opposing forces, usually at (quasi-) equilbrium, so that the total force and angular momentum are vanishing. During an earthquake, the resistance of the rocks in the focus yields, so that we have a localized, active distribution of forces which is proportional to $\partial\delta(\mathbf{R}-\mathbf{R}_0)/\partial n$, where $\mathbf{R}_0$ is the position of the focus. This can also be written as $f_i l\partial_i\delta(\mathbf{R}-\mathbf{R}_0) = fl_i\partial_i\delta(\mathbf{R}-\mathbf{R}_0) = fln_i\partial_i\delta(\mathbf{R}-\mathbf{R}_0)$, where f_i are the components of the force with magnitude f acting along the direction $\mathbf{n}$ and l_i are the components of the spatial extension of the focus along the same direction $\mathbf{n}$; the function δ in these expressions should be understood as a function localized over the distance l along the direction $\mathbf{n}$, and, similarly, along two other transverse directions. The quantity fl is proportional to the seismic moment M (since f can be represented as $f = \mu A$, where μ is the rigidity modulus and A is the area of the fault); we prefer to use the seismic moment divided by density $m = M/\rho$; then, the force distribution per unit mass reads

$$\mathbf{F}(\mathbf{R},t) = m(t)n_i\partial_i\delta(\mathbf{R}-\mathbf{R}_0)\mathbf{n}\ ,\ F_i(\mathbf{R},t) = m(t)n_in_j\partial_j\delta(\mathbf{R}-\mathbf{R}_0)\ , \tag{1}$$

where $m(t)$ has a certain time dependence during the earthquake; usually, this function is localized over a duration T, during which the earthquake lasts. The force distribution given by equation (1) represents a point linear dipole; since a strain occurring along a direction $\mathbf{n}$ may generate forces directed both along $\mathbf{n}$ and along the two directions perpendicular to $\mathbf{n}$, the force distribution given by equation (1) may be generalized by replacing $m(t)n_in_j$ by the tensor $m_{ij}(t)$:

$$F_i(\mathbf{R},t) = m_{ij}(t)\partial_j\delta(\mathbf{R}-\mathbf{R}_0)\ ; \tag{2}$$

in order to have a vanishing angular momentum, the tensor of the seismic moment must be symmetric; Equations (1) and (2) provide the "double-couple" representation of a faulting source.

The force distribution localized in a volume source with a small radius a can be written as

$$\mathbf{F}(\mathbf{R},t) = p(t)\frac{\mathbf{R}-\mathbf{R}_0}{|\mathbf{R}-\mathbf{R}_0|}\theta\left(a-|\mathbf{R}-\mathbf{R}_0|\right)\ , \tag{3}$$

where $p(t) = f(t)/a^3$ is force per unit volume, divided by density (force per unit mass).

Elastic waves equation. The elastic waves equation[22]

$$\ddot{\mathbf{u}} - c_t^2\Delta\mathbf{u} - (c_l^2-c_t^2)grad\cdot div\mathbf{u} = \mathbf{F}\ , \tag{4}$$

where $\mathbf{u}$ is the displacement, $c_{l,t}$ are the velocities of the "longitudinal" and, respectively, "transverse" waves and $\mathbf{F}$ is force per unit mass, can be decomposed into two equations

$$\begin{gathered}\ddot{D} - c_l^2\Delta D = div\mathbf{F}\ ,\\ \ddot{\mathbf{u}} - c_t^2\Delta\mathbf{u} = \mathbf{F} + (c_l^2-c_t^2)gradD,\end{gathered} \tag{5}$$

by taking the *div* in equation (4), and denoting $D = div\mathbf{u}$. In the second equation (5) the dilatational waves governed by D become a source for the "transverse" waves. For a half space $z<0$ bounded by the plane surface $z=0$ it is convenient to use the in-plane poistion vector **r**, $\mathbf{R}=(\mathbf{r},z)$, and the in-plane (horizontal) displacement $\mathbf{u}$ by changing the notation $\mathbf{u}\rightarrow(\mathbf{u},v)$; v is the displacement component perpendicular to the surface. Similarly, it is convenient to introduce the time and in-plane Fourier transformations; the first equation (5) becomes

$$\frac{d^2D}{dz^2}+\kappa_l^2D=-\frac{1}{c_l^2}\left(i\mathbf{k}\mathbf{F}_{xy}+\frac{\partial F_z}{\partial z}\right)\ , \tag{6}$$

where $\omega^2=c_l^2(k^2+\kappa_l^2)$, $\mathbf{k}$ is the in-plane wavevector and $\mathbf{F}=(\mathbf{F}_{xy},F_z)$ (for simplicity we use the same notations for functions and their Fourier transforms, the difference being easily seen from the context). Making use of the Green function $e^{i\kappa_l|z|}/2i\kappa_l$ of the one-dimensional Helmholtz equation, the solution of equation (6) is readily obtained as

$$D=\frac{i}{2\kappa_lc_l^2}S_1+Ae^{i\kappa_l|z|}\ , \tag{7}$$

where

$$S_1=\int dz'e^{i\kappa_l\left|z-z'\right|}\left[i\mathbf{k}\mathbf{F}_{xy}(z')+\frac{\partial F_z(z')}{\partial z'}\right] \tag{8}$$

and A is a constant; the integration is performed from $z=-\infty$ to $z=0$. Similarly, the solutions of the second equation (5) is

$$\begin{gathered}\mathbf{u}=\frac{i}{2\kappa_tc_t^2}\mathbf{S}_2-\frac{i(1-\eta)}{4\kappa_t\kappa_lc_t^2}\mathbf{k}S_3-\frac{i(1-\eta)}{2\eta\kappa_t}\mathbf{k}A\left(\frac{2\kappa_t}{\kappa_t^2-\kappa_l^2}e^{i\kappa_l|z|}-\frac{1}{\kappa_t-\kappa_l}e^{i\kappa_t|z|}\right)\ ,\\ v=\frac{i}{2\kappa_tc_t^2}S_4-\frac{1-\eta}{4\kappa_t\kappa_lc_t^2}S_5+\frac{i(1-\eta)}{2\eta\kappa_t}\kappa_lA\left(\frac{2\kappa_t}{\kappa_t^2-\kappa_l^2}e^{i\kappa_l|z|}-\frac{1}{\kappa_t-\kappa_l}e^{i\kappa_t|z|}\right)+Be^{i\kappa_t|z|}\ ,\end{gathered} \tag{9}$$

where

$$\begin{gathered}\mathbf{S}_2=\int dz'e^{i\kappa_t\left|z-z'\right|}\mathbf{F}_{xy}(z')\ ,\ S_3=\int dz'e^{i\kappa_t\left|z-z'\right|}S_1(z')\ ,\\ S_4=\int dz'e^{i\kappa_t\left|z-z'\right|}F_z(z')\ ,\ S_5=\int dz'e^{i\kappa_t\left|z-z'\right|}S_1'(z')\ ,\end{gathered} \tag{10}$$

$\omega^2=c_t^2(k^2+\kappa_t^2)$ and $\eta=c_t^2/c_l^2$; it is sufficient to include a free solution of the form $Be^{i\kappa_t|z|}$ in v.

The boundary conditions for a pressure $\mathbf{P}$ (divided by density ρ) acting upon the surface $z=0$ are given by $n_j\sigma_{ij}=-\rho P_i$ for $z=0$, where $\mathbf{n}=(0,0,1)$ is the unit vector normal to the surface (with components n_j) and σ_{ij} is the stress tensor. Making use of $\sigma_{ij}=\frac{E}{1+\sigma}\left(u_{ij}+\frac{\sigma}{1-2\sigma}u_{kk}\delta_{ij}\right)$, where u_{ij} is the strain tensor, E is Young's modulus and σ is Poisson's ratio, these boundary conditions read

$$\begin{gathered}\mathbf{k}\mathbf{u}^1+ik^2v^0=-\frac{2\rho(1+\sigma)}{E}\mathbf{k}\mathbf{P}_{xy}\ ,\\ (1-\sigma)v^1+i\sigma\mathbf{k}\mathbf{u}^0=-\frac{\rho(1+\sigma)(1-2\sigma)}{E}P_z\ ,\end{gathered} \tag{11}$$

where $\mathbf{u}^0,v^0$ are the values of the functions for $z=0$, $\mathbf{u}^1,v^1$ are the values of the first derivatives of the functions for $z=0$ and $\mathbf{P}=(\mathbf{P_{xy}},P_z)$; using the expressions $c_l^2=E(1-\sigma)/\rho(1+\sigma)(1-2\sigma)$, $c_t^2=E/2\rho(1+\sigma)$ for velocities,[22] equations (11) become

$$\begin{gathered}\mathbf{k}\mathbf{u}^1+ik^2v^0=-\frac{1}{c_t^2}\mathbf{k}\mathbf{P}_{xy}\ ,\\ v^1+i(1-2\eta)ku^0=-\frac{\eta}{c_t^2}P_z\ .\end{gathered} \tag{12}$$

Using $\mathbf{u}$ and v given by equation (9), the boundary conditions become

$$\begin{gathered}\frac{(1-\eta)(\kappa_t-\kappa_l)}{2\eta\kappa_t(\kappa_t+\kappa_l)}k^2A+ik^2B=-\frac{i}{2\kappa_t c_t^2}\mathbf{k}\mathbf{S}_2'(0)+\\ +\frac{i(1-\eta)}{4\kappa_t\kappa_l c_t^2}k^2S_3'(0)+\frac{k^2}{2\kappa_t c_t^2}S_4(0)+\frac{i(1-\eta)}{4\kappa_t\kappa_l c_t^2}k^2S_5(0)-\frac{1}{c_t^2}\mathbf{k}\mathbf{P}_{xy}\ ,\\ \frac{1-\eta}{2\eta\kappa_t(\kappa_t+\kappa_l)}\left[(1-2\eta)k^2-\kappa_t\kappa_l\right]A-i\kappa_t B=-\frac{i}{2\kappa_t c_t^2}S_4'(0)+\\ +\frac{1-\eta}{4\kappa_t\kappa_l c_t^2}S_5'(0)+\frac{1-2\eta}{2\kappa_t c_t^2}\mathbf{k}\mathbf{S}_2(0)-\frac{(1-\eta)(1-2\eta)}{4\kappa_t\kappa_l c_t^2}k^2S_3(0)-\frac{\eta}{c_t^2}P_z\ .\end{gathered}\tag{13}$$

With the solution for the coefficients A and B of this system of equations the displacement in the half-space is completely determined. Although pretty unpracticable, the solution of the elastic waves equation is reduced by the above equations to quadratures.

Near-field approximation. In the near-field, or quasi static approximation we may take $\omega^2/c_{t,l}^2\simeq 0$ and get $\kappa_{t,l}\simeq ik$; in this approximation the z-dependence of the waves is damped. Equations (9) give the displacement

$$\begin{gathered}\mathbf{u}\simeq\frac{1}{2c^2k}\mathbf{S}_2+\frac{i(1-\eta)}{4c^2k^2}\mathbf{k}S_3+\frac{i(1-\eta)}{4\eta k^2}\mathbf{k}Ae^{-k|z|}\ ,\\ v\simeq\frac{1}{2c^2k}S_4+\frac{1-\eta}{4c^2k^2}S_5+\left[\frac{1-\eta}{4\eta k}A+B\right]e^{-k|z|}\ ,\end{gathered}\tag{14}$$

where we set approximately $c_t=c$ and $\eta\neq 1$; this latter approximation affects only the numerical factors. Within the same approximation the boundary conditions given by equations (13) (for a free surface) become

$$\begin{gathered}ik^2B\simeq-\frac{1}{2c^2k}\mathbf{k}\mathbf{S}_2'(0)-\frac{i(1-\eta)}{4c^2}S_3'(0)-\frac{ik}{2c^2}S_4(0)-\frac{i(1-\eta)}{4c^2}S_5(0)\ ,\\ -\frac{(1-\eta)^2}{2\eta}A+kB\simeq-\frac{1}{2c^2k}S_4'(0)-\frac{1-\eta}{4c^2k^2}S_5'(0)-\frac{i(1-2\eta)}{2c^2k}\mathbf{k}\mathbf{S}_2(0)+\\ +\frac{(1-\eta)(1-2\eta)}{4c^2}S_3(0)\ .\end{gathered}\tag{15}$$

Vertical faulting source. We consider now a vertical faulting source given by equation the force (per unit mass) given by equation (1):

$$F_z=m(t)\delta(\mathbf{r})\partial_z\delta(z-z_0)\ ,\tag{16}$$

where $z_0<0$; its Fourier transform is

$$F_z=m(\omega)\partial_z\delta(z-z_0)\ .\tag{17}$$

Making use of this force we compute the source terms $S_{1...5}$ given by equations (8) and (10); the calculations are straightforward, and we get

$$\begin{gathered}S_1=m(\omega)\left[2i\kappa_l\delta(z-z_0)-\kappa_l^2e^{i\kappa_l|z-z_0|}\right]\ ,\ S_2=0\ ,\\ S_3=m(\omega)\left[2i\kappa_l e^{i\kappa_t|z-z_0|}-\kappa_l^2 I\right]\ ,\\ S_4=im(\omega)\kappa_t sgn(z-z_0)e^{i\kappa_t|z-z_0|}\ ,\\ S_5=m(\omega)\left[-2\kappa_t\kappa_l sgn(z-z_0)e^{i\kappa_t|z-z_0|}+\kappa_l^2\frac{\partial I}{\partial z_0}\right]\ ,\end{gathered}\tag{18}$$

where

$$I = \int dz' e^{i\kappa_t \left|z-z'\right|} e^{i\kappa_l \left|z'-z_0\right|} =$$

$$= \begin{cases} -\frac{2i\kappa_l}{\kappa_t^2-\kappa_l^2} \left[e^{i\kappa_t|z-z_0|} - e^{i\kappa_l|z-z_0|} \right] - \frac{i}{\kappa_t+\kappa_l} e^{-i\kappa_t z - i\kappa_l z_0} \ , \ z < z_0 \ , \\ -\frac{2i\kappa_t}{\kappa_t^2-\kappa_l^2} \left[e^{i\kappa_t|z-z_0|} - e^{i\kappa_l|z-z_0|} \right] - \frac{i}{\kappa_t+\kappa_l} e^{-i\kappa_t z - i\kappa_l z_0} \ , \ z > z_0 \ . \end{cases} \tag{19}$$

Using $\kappa_t - \kappa_l \simeq \omega^2(1-\eta)/2ik\eta c^2$ and setting $\kappa_{t,l} \simeq ik$ we get

$$I \simeq -\tfrac{1}{2k} e^{-k|z+z_0|} \ ,$$

$$S_3 = -m(\omega)k \left[2e^{-k|z-z_0|} + \tfrac{1}{2} e^{-k|z+z_0|} \right] \ ,$$

$$S_4 = -m(\omega)ksgn(z-z_0)e^{-k|z-z_0|} \ , \tag{20}$$

$$S_5 = m(\omega)k^2 \left[2sgn(z-z_0)e^{-k|z-z_0|} - \tfrac{1}{2} e^{-k|z+z_0|} \right] \ ;$$

making use of these source-terms in equations (15) we get the constants A and B,

$$A = \frac{m(\omega)k}{c^2} a e^{-k|z_0|} \ , \ B = \frac{m(\omega)}{c^2} b e^{-k|z_0|} \ , \tag{21}$$

where a and b are numerical factors (affected by small errors within this approximation),

$$a = \frac{\eta(1-2\eta+5\eta^2)}{2(1-\eta)^2} \ , \ b = \frac{3\eta-1}{4} \ . \tag{22}$$

Now we can write the displacement given by equations (14) as

$$\mathbf{u} = -\tfrac{im(\omega)(1-\eta)}{2c^2} \tfrac{\mathbf{k}}{k} \left(e^{-k|z-z_0|} + \tfrac{\eta-2a}{4\eta} e^{-k|z+z_0|} \right) \ ,$$

$$v = -\tfrac{m(\omega)}{2c^2} \left[\eta sgn(z-z_0) e^{-k|z-z_0|} - \tfrac{2a(1-\eta)+8\eta b-\eta(1-\eta)}{4\eta} e^{-k|z+z_0|} \right] \ . \tag{23}$$

In order to compute the Fourier transforms of the displacement given by equations (23) we need the integrals

$$\int d\mathbf{k} \frac{e^{i\mathbf{kr}}}{k} e^{-k|z|} = \frac{2\pi}{R} \ , \ \int d\mathbf{k} e^{i\mathbf{kr}} e^{-k|z|} = \frac{2\pi\,|z|}{R^3} \ , \tag{24}$$

where $R = (r^2+z^2)^{1/2})$. We get finally

$$\mathbf{u} = \tfrac{m(t)(1-\eta)}{4\pi c^2} \left(\tfrac{1}{R_1^3} + \tfrac{\eta-2a\cdot}{4\eta} \tfrac{1}{R_2^3} \right) \mathbf{r} \ ,$$

$$v = \tfrac{m(t)}{4\pi c^2} \left[-\eta \tfrac{z-z_0}{R_1^3} + \tfrac{2a(1-\eta)+8\eta b-\eta(1-\eta)}{4\eta} \cdot \tfrac{|z+z_0|}{R_2^3} \right] \ , \tag{25}$$

where $R_{1,2} = \left[r^2 + (z \mp z_0)^2 \right]^{1/2}$. On the surface $z=0$ the displacement is given by

$$\mathbf{u}^0 = \tfrac{m(t)}{4\pi c^2} \cdot \tfrac{(1-\eta)(5\eta-2a)}{4\eta} \cdot \tfrac{\mathbf{r}}{R_0^3} \ ,$$

$$v^0 = \tfrac{m(t)}{4\pi c^2} \cdot \tfrac{2a(1-\eta)+8\eta b-\eta(1-5\eta)}{4\eta} \cdot \tfrac{|z_0|}{R_0^3} \ , \tag{26}$$

where $R_0 = (r^2+z_0^2)^{1/2}$; we can see that the maximum values of the displacement are of the order $m/c^2z_0^2$, reached for $r \simeq |z_0|/\sqrt{2}$ for the horizontal displacement and for $r=0$ for the vertical displacement.

Horizontal faulting source. The horizontal faulting force given by equation (1) is

$$\mathbf{F}_{xy}=m(t)n_i\partial_i\delta(\mathbf{r})\delta(z-z_0)\mathbf{n}\ , \tag{27}$$

with the Fourier tranform

$$\mathbf{F}_{xy}=im(\omega)\mathbf{kn}\delta(z-z_0)\mathbf{n}\ , \tag{28}$$

where $\mathbf{n}$ is the in-plane unit vector which defines the fault direction. The source terms $S_{1...5}$ given by equations (8) and (10) are

$$\begin{gathered}S_1=-m(\omega)(\mathbf{kn})^2e^{i\kappa_l|z-z_0|}\ ,\ S_2=im(\omega)(\mathbf{kn})\mathbf{n}e^{i\kappa_t|z-z_0|}\ ,\\ S_3=-m(\omega)(\mathbf{kn})^2I\ ,\ S_5=m(\omega)(\mathbf{kn})^2\frac{\partial}{\partial z_0}I\ ,\end{gathered}\tag{29}$$

where I is given by equation (19) ($S_4=0$). Within the near-field approximation the above expressions become

$$\begin{gathered}S_1=-m(\omega)(\mathbf{kn})^2e^{-k|z-z_0|}\ ,\ S_2=im(\omega)(\mathbf{kn})\mathbf{n}e^{-k|z-z_0|}\ ,\\ S_3=m(\omega)\frac{(\mathbf{kn})^2}{2k}e^{-k|z+z_0|}\ ,\ S_5=-\frac{1}{2}m(\omega)(\mathbf{kn})^2e^{-k|z+z_0|}\ .\end{gathered}\tag{30}$$

Equations (15) give the coefficients

$$\begin{gathered}A=-\frac{m(\omega)}{c^2k}(\mathbf{kn})^2\frac{\eta(1-6\eta+\eta^2)}{(1-\eta)^2}e^{-k|z_0|}\ ,\\ B=\frac{m(\omega)}{2c^2k^2}(\mathbf{kn})^2e^{-k|z_0|}\end{gathered}\tag{31}$$

and equations (14) give the displacement

$$\begin{gathered}\mathbf{u}=\frac{im(\omega)}{2c^2}\left[\frac{\mathbf{kn}}{k}\mathbf{n}e^{-k|z-z_0|}-\frac{1-10\eta+\eta^2}{4(1-\eta)}\frac{(k\mathbf{n})^2}{k^3}\mathbf{k}e^{-k|z+z_0|}\right]\ ,\\ v=\frac{m(\omega)}{8c^2}\frac{(k\mathbf{n})^2}{k^2}\frac{1+10\eta-3\eta^2}{1-\eta}e^{-k|z+z_0|}\ ;\end{gathered}\tag{32}$$

the Fourier tranforms of the displacement are

$$\begin{gathered}\mathbf{u}=\frac{im(t)}{2(2\pi)^2c^2}\left(I_1\mathbf{n}-\frac{1-10\eta+\eta^2}{4(1-\eta)}\mathbf{I}_2\right)\ ,\\ v=\frac{m(t)}{8(2\pi)^2c^2}\frac{1+10\eta-3\eta^2}{1-\eta}I_3\ ,\end{gathered}\tag{33}$$

where

$$\begin{gathered}I_1=\int d\mathbf{k}\frac{\mathbf{kn}}{k}e^{i\mathbf{kr}}e^{-k|z-z_0|}\ ,\\ \mathbf{I}_2=\int d\mathbf{k}\frac{(\mathbf{kn})^2}{k^3}\mathbf{k}e^{i\mathbf{kr}}e^{-k|z+z_0|}\ ,\\ I_3=\int d\mathbf{k}\frac{(\mathbf{kn})^2}{k^2}e^{i\mathbf{kr}}e^{-k|z+z_0|}\ .\end{gathered}\tag{34}$$

The calculation of these integrals is straightforward, by means of equations (24) and some special manipulations. We get immediately

$$I_1=-i\mathbf{n}\frac{\partial}{\partial\mathbf{r}}\frac{2\pi}{R_1}=2\pi\frac{i\mathbf{nr}}{R_1^3}\ , \tag{35}$$

where $R_1=\left[r^2+(z-z_0)^2\right]^{1/2}$. Then, we notice that

$$\frac{\partial I_3}{\partial\left|z+z_0\right|}=i\mathbf{n}\frac{\partial}{\partial\mathbf{r}}I_1(z_0\rightarrow -z_0)=-2\pi\left[\frac{1}{R_2^3}-\frac{3(\mathbf{nr})^2}{R_2^5}\right] \ , \tag{36}$$

where $R_2=\left[r^2+(z+z_0)^2\right]^{1/2}$; the integration can be effected straighforwardly in equation (36) leading to

$$I_3=-\pi\left[\tfrac{1}{R_2(R_2+|z+z_0|)}-(\mathbf{nr})^2\tfrac{2R_2+|z+z_0|}{R_2^3(R_2+|z+z_0|)^2}\right] \ ; \tag{37}$$

further, we notice that

$$\frac{\partial\mathbf{I}_2}{\partial\left|z+z_0\right|}=i\frac{\partial}{\partial\mathbf{r}}I_3 \ , \tag{38}$$

which gives access to $\mathbf{I}_2$.

The displacement of the surface $z=0$ is of particular interest. For $z=0$ and $r\ll|z_0|$ the integrals $I_{1...3}$ behave as

$$\begin{array}{c} I_1\simeq 2\pi\frac{i\mathbf{nr}}{|z_0|^3} \ , \ I_3\simeq-\frac{\pi}{2|z_0|^2}\left[1-\frac{3r^2}{4z_0^2}-\frac{(\mathbf{nr})^2}{2z_0^2}\right] \ , \\ I_2\simeq-\frac{i\pi}{4|z_0|^3}\left[\mathbf{r}+\frac{2}{3}(\mathbf{nr})\mathbf{n}\right] \ ; \end{array} \tag{39}$$

the corresponding displacement is

$$\begin{array}{c} \mathbf{u}^0\simeq-\frac{m(t)}{32\pi(1-\eta)c^2|z_0|^3}\left[(9-18\eta+\eta^2)(\mathbf{nr})\mathbf{n}+(1-10\eta+\eta^2)\mathbf{r}\right] \ , \\ v^0\simeq-\frac{m(t)}{64\pi c^2z_0^2}\frac{1+10\eta-3\eta^2}{1-\eta}\left[1-\frac{3r^2}{z_0^2}-\frac{(\mathbf{nr})^2}{2z_0^2}\right] \ ; \end{array} \tag{40}$$

we can see that the horizontal displacement has a maximum value $\simeq m/c^2z_0^2$ for r of the order $|z_0|$, while the vertical displacement decreases to zero from the maximum value $\simeq m/c^2z_0^2$ reached at the origin.

Volume source. The force generated by a localized volume source is

$$\mathbf{F}_{xy}=p(t)\frac{\mathbf{r}}{R_1}\theta(a-R_1) \ , \ F_z=p(t)\frac{z-z_0}{R_1}\theta(a-R_1) \tag{41}$$

(equation (2)), where $R_1=\left[r^2+(z-z_0)^2\right]^{1/2}$; its Fourier transform

$$\mathbf{F}_{xy}=p(\omega)\int d\mathbf{r}e^{-i\mathbf{kr}}\tfrac{\mathbf{r}}{R_1}\theta(a-R_1) \ , \ F_z=p(\omega)\int d\mathbf{r}e^{-i\mathbf{kr}}\tfrac{z-z_0}{R_1}\theta(a-R_1) \tag{42}$$

can be calculated straightforwardly: we introduce $\mathbf{k}=k(\cos\alpha,\sin\alpha)$, $\mathbf{r}=r(\cos\varphi,\sin\varphi)$ and change the integration over r to an integration over R_1; we get

$$\begin{array}{c} \mathbf{F}_{xy}\simeq-i\pi p(\omega)\mathbf{k}\left[\frac{1}{3}a^3+\frac{2}{3}\left|z-z_0\right|^3-a(z-z_0)^2\right] \ , \\ F_z\simeq 2\pi p(\omega)(z-z_0)\left[a-|z-z_0|\right] \ . \end{array} \tag{43}$$

for $|z-z_0|<a$ and $kr\ll 1$. Within the same approximation we get from equations (8) and (10)

$$\begin{array}{c} S_1\simeq 4\pi p(\omega)a^2e^{i\kappa_l|z-z_0|} \ , \ S_3\simeq 8\pi p(\omega)a^3e^{i\kappa_t|z-z_0|} \ , \\ S_2\simeq 0 \ , \ S_4\simeq 0 \ , \ S_5\simeq 0 \ . \end{array} \tag{44}$$

The near-field equations (14) and (15) give

$$A\simeq 8\pi\frac{\eta^2}{1-\eta}\frac{p(\omega)a^3}{c^2}e^{-k|z_0|}\ ,\ B\simeq 2\pi(1-\eta)\frac{p(\omega)a^3}{c^2k}e^{-k|z_0|} \tag{45}$$

and

$$\begin{gathered}\mathbf{u}\simeq 2\pi i\tfrac{p(\omega)a^3}{c^2}\left[(1-\eta)e^{-k|z-z_0|}+\eta e^{-k|z+z_0|}\right]\tfrac{\mathbf{k}}{k^2}\ ,\\ v\simeq 2\pi\tfrac{p(\omega)a^3}{c^2k}e^{-k|z+z_0|}\ .\end{gathered} \tag{46}$$

We can see that, within this approximation, only dilatation contributes to the displacement, as expected, and, especially, the vertical dilatation; while the horizontal displacement exhibits both a direct and an image source, the vertical displacement includes only the image source. The inverse Fourier transforms of the displacement given by equations (46) lead to

$$\begin{gathered}\mathbf{u}=i\tfrac{p(t)a^3}{2\pi c^2}\left[(1-\eta)\int d\mathbf{k}\tfrac{\mathbf{k}}{k^2}e^{i\mathbf{kr}}e^{-k|z-z_0|}+\eta\int d\mathbf{k}\tfrac{\mathbf{k}}{k^2}e^{i\mathbf{kr}}e^{-k|z+z_0|}\right]\ ,\\ v=\tfrac{p(t)a^3}{2\pi c^2}\int d\mathbf{k}\tfrac{e^{i\mathbf{kr}}}{k}e^{-k|z+z_0|}=\tfrac{p(t)a^3}{2\pi c^2}\cdot\tfrac{1}{R_2}\ ,\end{gathered} \tag{47}$$

where $R_2=\left[r^2+(z+z_0)^2\right]^{1/2}$. The integral

$$I_4=\frac{1}{2\pi}\int d\mathbf{k}\frac{\mathbf{k}}{k^2}e^{i\mathbf{kr}}e^{-k|z|} \tag{48}$$

which appears in equations (47) can be calculated either by using Bessel functions or by using the identity

$$\frac{\partial}{\partial\,|z|}I_4=i\frac{\partial}{\partial\mathbf{r}}\int d\mathbf{k}\frac{1}{k}e^{i\mathbf{kr}}e^{-k|z|} \tag{49}$$

and integrating over $|z|$. The result is

$$I_4=\frac{i\mathbf{r}}{R(R+|z|)} \tag{50}$$

and

$$\mathbf{u}=-\frac{p(t)a^3}{c^2}\left[(1-\eta)\frac{1}{R_1\left(R_1+|z-z_0|\right)}+\eta\frac{1}{R_2\left(R_2+|z+z_0|\right)}\right]\mathbf{r}\ , \tag{51}$$

where $R=\left(r^2+z^2\right)^{1/2}$. By setting $z=0$ in $\mathbf{u}$ and v given here we get the quasi-static surface displacement due to a volume source.

Force on the surface. We give here the displacement caused by a force $\mathbf{f}$ localized on the surface $z=0$; in equations (13) $\mathbf{P}=\mathbf{f}\delta(\mathbf{r})$ (where $\mathbf{f}$ is divided by density). For a slow time variation ($\omega\simeq 0$) we may use the near-field (quasi-static) approximation $\kappa_{t,l}\simeq ik$; equations (13) give the constants

$$A=\frac{2\eta}{c_t^2(1-\eta)^2}\left(\frac{i\mathbf{kf}_{xy}}{k}+\eta f_z\right)\ ,\ B=\frac{i}{c_t^2}\frac{\mathbf{kf}_{xy}}{k^2} \tag{52}$$

and equations (14) give the displacements

$$\begin{gathered}\mathbf{u}=-\tfrac{1}{2c_t^2(1-\eta)}\left[\tfrac{\mathbf{k}(\mathbf{kf}_{xy})}{k^3}-i\eta f_z\tfrac{\mathbf{k}}{k^2}\right]e^{-k|z|}\ ,\\ v=\tfrac{1}{2c_t^2(1-\eta)}\left(\tfrac{3-2\eta}{2}\tfrac{i\mathbf{kf}_{xy}}{k^2}+\eta f_z\tfrac{1}{k}\right)e^{-k|z|}\ .\end{gathered} \tag{53}$$

The inverse Fourier transforms lead to

$$\mathbf{u}=-\frac{1}{4\pi c_t^2(1-\eta)}\left[\frac{1}{2\pi}\int d\mathbf{k}\frac{\mathbf{k}(\mathbf{k}\mathbf{f}_{xy})}{k^3}e^{i\mathbf{k}\mathbf{r}}e^{-k|z|}-\frac{i\eta}{2\pi}f_z\int d\mathbf{k}\frac{\mathbf{k}}{k^2}e^{i\mathbf{k}\mathbf{r}}e^{-k|z|}\right] \ ,$$
$$v=\frac{3-2\eta}{8\pi c_t^2(1-\eta)}\frac{i}{2\pi}\int d\mathbf{k}\frac{\mathbf{k}\mathbf{f}_{xy}}{k^2}e^{i\mathbf{k}\mathbf{r}}e^{-k|z|}+\frac{\eta}{4\pi c_t^2(1-\eta)}f_z\frac{1}{2\pi}\int d\mathbf{k}\frac{e^{i\mathbf{k}\mathbf{r}}}{k}e^{-k|z|} \ . \tag{54}$$

The last integral in the second equation (54) is $1/R$, where $R=(r^2+z^2)^{1/2}$ (equation (24)); the last integral in the first equation (54) is I_4 given by equation (50). We denote

$$I_5=\frac{1}{2\pi}\int d\mathbf{k}\frac{\mathbf{k}\mathbf{f}_{xy}}{k^2}e^{i\mathbf{k}\mathbf{r}}e^{-k|z|} \tag{55}$$

and notice that

$$\frac{\partial I_5}{\partial|z|}=-\frac{1}{2\pi}I_1=-\frac{i\mathbf{f}_{xy}\mathbf{r}}{R^3} \ , \tag{56}$$

where I_1 is given by equation (35); we get by integration

$$I_5=\frac{2i\mathbf{f}_{xy}\mathbf{r}}{R(R+|z|)} \tag{57}$$

and

$$v=-\frac{1}{4\pi c_t^2(1-\eta)R}\left[(3-2\eta)\frac{\mathbf{f}_{xy}\mathbf{r}}{R+|z|}-\eta f_z\right] \ . \tag{58}$$

The horizontal component from equations (54) is given by

$$\mathbf{u}=-\frac{1}{4\pi c_t^2(1-\eta)}\left[\mathbf{I}_6+\eta f_z\frac{\mathbf{r}}{R(R+|z|)}\right] \ , \tag{59}$$

where

$$\mathbf{I}_6=\frac{1}{2\pi}\int d\mathbf{k}\frac{\mathbf{k}(\mathbf{k}\mathbf{f}_{xy})}{k^3}e^{i\mathbf{k}\mathbf{r}}e^{-k|z|} \ ; \tag{60}$$

since

$$\frac{\partial\mathbf{I}_6}{\partial|z|}=i\frac{\partial}{\partial\mathbf{r}}I_5=-2\frac{\partial}{\partial\mathbf{r}}\frac{\mathbf{f}_{xy}\mathbf{r}}{R(R+|z|)} \tag{61}$$

we may have access to $\mathbf{I}_6$ by integration (on the surface $\mathbf{I}_6\simeq\mathbf{f}_{xy}/r$, $(\mathbf{f}_{xy}\mathbf{r})\mathbf{r}/r^3$). We can see that the displacement is of the order f/c_t^2r.

The above nera-field results are similar with the static displacement problems for an isotropice elastic half-space with an internal or an external point force, up to numerical factors in the amplitudes (the so-called Boussinesq and Mindlin problems).[23]-[34]

Transient regime. If the wavelengths are much shorter than the distances of interest we are in the far-field regime; it may happen in this case that the wave source ceases its activity much before the waves reach the surface of the half-space; then we are in a transient regime, where the waves equation can be treated as in an infinite body, andthe boundary conditions may be neglected (they are included when the waves are reflected - and refracted - by the surface).

In an infinite body the solution of thewave equation (4) can be decomposed into "longitudinal" (l) and "transverse" (t) waves by means of the Helmholtz potentials Φ and $\mathbf{A}$ as

$$\mathbf{u}=\mathbf{u}_l+\mathbf{u}_t \ , \ \mathbf{u}_l=grad\Phi \ , \ \mathbf{u}_t=curl\mathbf{A} \ (div\mathbf{A}=0) \ ; \tag{62}$$

similarly, the force can be decomposed as

$$\mathbf{F}=\mathbf{F}_l+\mathbf{F}_t\ ,\ \mathbf{F}_l=grad\phi\ ,\ \mathbf{F}_t=curl\mathbf{H}\ (div\mathbf{H}=0)\ ; \tag{63}$$

taking the div in $\mathbf{F}=grad\phi+curl\mathbf{H}$, we get

$$\Delta\phi=div\mathbf{F}\ ; \tag{64}$$

taking the $curl$ in $\mathbf{F}=grad\phi+curl\mathbf{H}$ and making use of $curl\cdot curl=-\Delta+grad\cdot div$ we get

$$\Delta\mathbf{H}=-curl\mathbf{F}\ ; \tag{65}$$

therefore, the decomposition (63) for the force is attained by solving the two Poisson equations (64) and (65); their solutions (for vanishing boundary conditions at infinity) are

$$\phi=-\frac{1}{4\pi}\int d\mathbf{R}'\frac{1}{|\mathbf{R}-\mathbf{R}'|}div'\mathbf{F}\ ,\ \mathbf{H}=\frac{1}{4\pi}\int d\mathbf{R}'\frac{1}{|\mathbf{R}-\mathbf{R}'|}curl'\mathbf{F} \tag{66}$$

Introducing $\mathbf{u}$ given by equation (62) in the wave equation (4) we get

$$\ddot{\Phi}-c_l^2\Delta\Phi=\phi\ ,\ \ddot{\mathbf{A}}-c_t^2\Delta\mathbf{A}=\mathbf{H} \tag{67}$$

the wave equation (4) is reduced to two standard wave equations. The u_l-waves ($grad\Phi$) are called P-waves (primary, compressional waves), while the u_t-waves ($curl\mathbf{A}$) are the S-waves (shear waves).

Faulting source. For a faulting source represented by

$$\mathbf{F}(\mathbf{R},t)=m(t)n_i\partial_i\delta(\mathbf{R})\mathbf{n}\ , \tag{68}$$

where $m(t)$ is the seismic moment (divided by density) and $\mathbf{n}$ is the direction of the fault slip, we get, from equations (66),

$$\begin{aligned}\phi&=-\tfrac{m(t)}{4\pi}n_in_j\int d\mathbf{R}'\tfrac{\partial_i'\partial_j'\delta(\mathbf{R}')}{|\mathbf{R}-\mathbf{R}'|}=\\&=-\tfrac{m(t)}{4\pi}(\mathbf{n}grad)^2\int d\mathbf{r}\tfrac{\delta(\mathbf{R}-\mathbf{r})}{r}=-\tfrac{m(t)}{4\pi}(\mathbf{n}grad)^2\tfrac{1}{R}\end{aligned} \tag{69}$$

and

$$H_i=\frac{m(t)}{4\pi}\varepsilon_{ijk}n_kn_l\partial_j\partial_l\frac{1}{R}\ . \tag{70}$$

Making use of these sources we get the solutions of the wave equations (67)

$$\begin{aligned}\Phi&=-\tfrac{m(t)}{(4\pi c_l)^2}\int d\mathbf{R}'\tfrac{m(t-\left|\mathbf{R}-\mathbf{R}'\right|/c_l)}{\left|\mathbf{R}-\mathbf{R}'\right|}(\mathbf{n}grad')^2\tfrac{1}{R'}=\\&=-\tfrac{m(t)}{(4\pi c_l)^2}(\mathbf{n}grad)^2\int d\mathbf{r}\tfrac{m(t-r/c_l)}{r}\tfrac{1}{|\mathbf{R}-\mathbf{r}|}=\\&=-\tfrac{mT}{(4\pi c_l)^2}(\mathbf{n}grad)^2\int dr\cdot r\delta(t-r/c_l)\int do_{\mathbf{r}}\tfrac{1}{|\mathbf{R}-\mathbf{r}|}\end{aligned} \tag{71}$$

and

$$A_i=\frac{mT}{(4\pi c_t)^2}\varepsilon_{ijk}n_kn_l\partial_j\partial_l\int dr\cdot r\delta(t-r/c_t)\int do_{\mathbf{r}}\frac{1}{|\mathbf{R}-\mathbf{r}|}\ \ , \tag{72}$$

where we assumed a δ-time impulse $m(t) = mT\delta(t)$ with duration T (much shorter than the times we measure). The angular integral can be calculated straightforwardly; we get

$$\begin{gathered}\Phi = -\frac{mTt}{4\pi}(\mathbf{n}grad)^2\left[\frac{1}{R}\theta(R-c_lt) + \frac{1}{c_lt}\theta(c_lt-R)\right] \ , \\ A_i = \frac{mTt}{4\pi}\varepsilon_{ijk}n_kn_l\partial_j\partial_l\left[\frac{1}{R}\theta(R-c_tt) + \frac{1}{c_tt}\theta(c_tt-R)\right] \ .\end{gathered} \tag{73}$$

The total displacement is readily obtained as

$$\begin{gathered}\mathbf{u} = \frac{mTt}{4\pi}(\mathbf{n}grad)^2 grad\cdot \\ \cdot\left[\frac{1}{R}\theta(R-c_tt) + \frac{1}{c_tt}\theta(c_tt-R) - \frac{1}{R}\theta(R-c_lt) - \frac{1}{c_lt}\theta(c_lt-R)\right] - \\ -\frac{mTt}{4\pi}\mathbf{n}(\mathbf{n}grad)\Delta\left[\frac{1}{R}\theta(R-c_tt) + \frac{1}{c_tt}\theta(c_tt-R)\right] \ .\end{gathered} \tag{74}$$

For the near-field contribution we limit ourselves here to the part continuous in time (proportional to functions θ); it is given by

$$\mathbf{u}_n = \frac{mTt}{4\pi}(\mathbf{n}grad)^2(grad\frac{1}{R})\left[\theta(R-c_tt) - \theta(R-c_lt)\right] \tag{75}$$

(it is the double-couple result). The far-field displacement is

$$\mathbf{u}_f = \frac{mT}{4\pi}\frac{(\mathbf{nR})^2}{c_lR^4}\mathbf{R}\delta^{'}(R-c_lt) + \frac{mT}{4\pi}\frac{\mathbf{nR}}{c_tR^2}\left[\mathbf{n} - \frac{(\mathbf{nR})\mathbf{R}}{R^2}\right]\delta^{'}(R-c_tt) \ . \tag{76}$$

We note that the far-field displacement given by equation (76) is the far-field displacement given by the double-couple model (note that $\delta^{'}(R-ct) = (1/c^2)\delta^{'}(t-R/c)$). In general, the double-couple model is obtained by replacing mn_in_j in equation (76) by the parameters m_{0ij}. The order of magnitude of the far-field displacement given by the above formulae is $u_f \simeq m/c^3RT$; for a seismic moment $M = 10^{26}dyn\cdot cm$(magnitude $M_w = 7$, density $\rho = 5g/cm^3$), velocity $c = 5km/s$, distance $R = 100km$ and a duration $T = 10$s we get $u_f \simeq 1cm$. The energy flux density for a scalar wave u is $\mathbf{S} = -\rho c^2\dot{u}gradu$; for the far-field displacement given here we have $S \simeq \rho m^2/c^5R^2T^4$; with the same numerical values as above we get $S \simeq 5\times10^{-3}w/cm^2$.

Volume source. The force distribution for a volume source is $\mathbf{F} = p(t)(\mathbf{R}/R)\theta\,(a-R)$ (equation (3)); since $curl\mathbf{F} = 0$ the potential $\mathbf{H}$ is zero, and so is $\mathbf{u}_t$; we have only l-waves, given by $\ddot{\mathbf{u}}_l - c_l^2\Delta\mathbf{u}_l = grad\phi$, where $\Delta\phi = div\mathbf{F}$; since $\Delta = div\cdot grad$ we may take $grad\phi = \mathbf{F}$, so that we have

$$\mathbf{u}_l = \frac{pT}{4\pi c_l^2}\int d\mathbf{R}^{'}\frac{\delta(\left(t-\left|\mathbf{R}-\mathbf{R}^{'}\right|/c_l\right)}{|\mathbf{R}-\mathbf{R}^{'}|}\frac{\mathbf{R}^{'}}{R^{'}}\theta(a-R^{'}) \tag{77}$$

for a time impulse with duration T. In equation (77) we change the variable $R^{'}$into $\mathbf{r} = \mathbf{R}-\mathbf{R}^{'}$,

$$\mathbf{u}_l = \frac{pT}{4\pi c_l^2}\int d\mathbf{r}\frac{\delta((t-r/c_l)}{r}\frac{\mathbf{R}-\mathbf{r}}{|\mathbf{R}-\mathbf{r}|}\theta(a-|\mathbf{R}-\mathbf{r}|) \ , \tag{78}$$

and notice that $\mathbf{u}_l = u_l\mathbf{R}/R$, as expected; the integral becomes

$$\begin{gathered}u_l = \frac{pT}{2c_l^2}\int dr\cdot r\delta((t-r/c_l)\int d\theta\sin\theta\frac{R-r\cos\theta}{(R^2+r^2-2Rr\cos\theta)^{1/2}}\cdot \\ \cdot\theta\left[a-(R^2+r^2-2Rr\cos\theta)^{1/2}\right] \ .\end{gathered} \tag{79}$$

Introducing $u=\cos\theta$ we get the integration limits from

$$\frac{R^2+r^2-a^2}{2Rr}<u<\frac{R^2+r^2}{2Rr} \ ; \tag{80}$$

making use of the graphical representation of the functions given in equation (80) we get the integration limits $(R^2+r^2-a^2)/2Rr<u<1$ for u, providing $R-a<r<R+a$. The integration with respect to u is performed by using the change of variable $R^2+r^2-2Rru=x^2$; we get

$$\begin{gathered} u_l=\frac{pT}{4c_l^2R^2}\int_{R-a}^{R+a}dr\delta(t-r/c_l)\cdot= \\ \cdot\left\{(R^2-r^2+\tfrac{1}{3}a^2)a-\left[R^2-r^2+\tfrac{1}{3}(R-r)^2\right]\left|R-r\right|\right\}= \\ =\frac{pT}{4c_lR^2}\left[\tfrac{1}{3}a^3+(R^2-c_l^2t^2)a-(R^2-c_l^2t^2)\left|R-c_lt\right|-\tfrac{1}{3}\left|R-c_lt\right|^3\right] \end{gathered} \tag{81}$$

for $R-a<c_lt<R+a$. This displacement has the form of a shock (similar with a function $\delta'(t-R/c_l)$), which, in the (far-field) limit R, $c_lt\gg a$ has two extrema $\simeq\pm\frac{1}{2}(pT/4c_l)c_lta^2/R^2$ at $R=c_lt\pm\frac{a}{2}$.

Concluding remarks. The double-couple seismic source is reformulated here without resorting to the mechanical torque representation; a volume source is also put forward, based on the pressure exerted on the internal surface of a small spherical cavity. The elastic waves equations and the boundary conditions are conveniently written down for anisotropic elastic half-space by using in-plane (horizontal) Fourier transformations; it is shown that the dilatational waves act a sources for "transverse" waves. The problem of the elastic waves in a half-space is thereby reduced to quadratures, at least in principle. Near-field (quasi-static) approximations are solved explicitly for the sources introduced here, including the displacement caused by a localized force acting upon the surface. The transient regime of far-field waves is also solved for both types of sources.

Acknowledgments. The author is indebted to the members of the Institute for Earth's Physics at Magurele-Bucharest for useful discussions. This work has been supported partially by Romanian Government Research Grant #PN09-03 02/27.02.2015.

References

[1] Lord Rayleigh, "On waves propagated along the plane surface of an elastic solid", Proc. London Math. Soc. **17** 4-11 (1885) (J. W. Strutt, Baron Rayleigh, *Scientific Papers*, vol.2, 441-447, Cambridge University Press, London (1900)).

[2] H. Lamb, "On the propagation of tremors over the surface of an elastic solid", Phil. Trans. Roy. Soc. (London) **A203** 1-42 (1904).

[3] A. E. H. Love, *Some Problems of Geodynamics*, Cambridge University Press, London (1926).

[4] H. Nakano, "Notes on the nature of the forces which give rise to the earthquake motions", Seism. Bull. Central Metrological Observatory of Japan **1** 92-120 (1923).

[5] H. Honda, "Earthquake mechanism and seismic waves", J. Phys. Earth **10** 1-98 (1962).

[6] R. Burridge and L. Knopoff, "Body force equivalents for seismic dislocations", Bull. Seism. Soc. Am. **54** 1875-1888 (1964).

[7] J. D. Eshelby, "The determination of the elastic field of an ellipsoidal inclusion, and related problems", Proc. Roy. Soc. London **A241** 376-396 (1957).

[8] L. R. Sykes, "Mechanism of earthquakes and nature of faulting on the mid-oceanic ridges", J. Geophys. Res. **72** 2131-2153 (1967).

[9] T. Maruyama, "On force equivalents of dynamic elastic dislocations with reference to the earthquake mechanism", Bull. Earthq. Res. Inst., Tokyo Univ., **41** 467-486 (1963).

[10] A. Ben Menahem, "Radiation of seismic surface waves from finite moving sources", Bull. Seism. Soc. Am. **51** 401-435 (1961).

[11] A. Ben Menahem, "Radiation of seismic body waves from finite moving sources", J. Geophys. Res. **67** 345-350 (1962).

[12] N. A. Haskell, "Total energy spectral density of elastic wave radiation from propagating faults", Bull. Seism. Soc. Am. **54** 1811-1841 (1964).

[13] N. A. Haskell, "Total energy spectral density of elastic wave radiation from propagating faults". Part II., Bull. Seism. Soc. Am. **56** 125-140 (1966).

[14] J. N. Brune, "Seismic moment, seismicity, and rate of slip along major fault zones", J. Geophys. Res. **73** 777-784 (1968).

[15] G. Backus and M. Mulcahy, "Moment tensors and other phenomenological descriptions of seismic sources. I. Continuous displacements", Geophys. J. Roy. Astron. Soc. **46** 341-361 (1976).

[16] G. Backus and M. Mulcahy, "Moment tensors and other phenomenological descriptions of seismic sources. II. Discontinuous displacements", Geophys. J. Roy. Astron. Soc. **47** 301-329 (1976).

[17] B. V. Kostrov and S. Das, *Principles of Earthquake Source Mechanics*, Cambridge University Press, NY (1988).

[18] K. Aki and P. G. Richards, *Quantitative Seismology*, University Science Books, Sausalito, CA (2009).

[19] R. Madariaga, "Seismic Source Theory", in *Treatise of Geophyiscs*, vol. 4, *Earthquake Seismology*, ed. H.Kanamori, Elseveir (2015).

[20] L. Knopoff, "Diffraction of elastic waves", J. Acoust. Soc. Am. **28** 217-229 (1956).

[21] A. T. de Hoop, "Representation theorems for the displacement in an elastic solid and their applications to elastodynamic diffraction theory", D. Sc. Thesis, Technische Hogeschool, Delft (1958).

[22] L. Landau and E. Lifshitz, *Course of Theoretical Physics*, vol. 7, *Theory of Elasticity*, Elsevier, Oxford (1986).

[23] J. V. Boussinesq, *Applications des potentiels a l'etude de l'equilibre et du mouvement des solides elastiques*, Gauthier-Villars, Paris (1885).

[24] A. Sommerfeld, *Vorlesungen uber Theoretische Physik*, Bd. II, *Mechanik der deformierbaren Medien*, Akad. Verlag., Leipzig (1970).

[25] A. I. Lurie, *Spatial Problems in the Theorie of Elasticity* (in Russian), Gostehizdat, Moscow (1955).

[26] J. Dougall, "A general method of solving the equations of elasticity", Proc. Edinburgh Math. Soc. **16** 82-98 (1897).

[27] E. Melan, "Der Spannungszustand der durch eine Einzelkraft in Innern beanspruchten Halbscheibe", Z. Angew. Math. Mech. **12** 343-346 (1932) (correction Z. Angew. Math. Mech. **20** 368 (1940)).

[28] R. Mindlin, "Force at a point in the interior of a semi-infinite solid", Physics **7** 195-202 (1936).

[29] R.Mindlin, "Note on the Galerkin and Papkovitch stress functions", Bull. Am. Math. Soc. **42** 373-376 (1936).

[30] R. Mindlin, "Force at a point in the interior of a semi-infinite solid", Tech. Note #8, Columbia Univ. (1953).

[31] G. D. Grodskii, "Integration of general equations of equilibrium of an isotropic elastic body by means of Newtonian potentials and harmonic functions" (in Russian), Izv. Akad. Nauk SSSR, Mat. Estestv. Nauk, **4** 587-614 (1935).

[32] H. Neuber, "Ein neuer Ansatz zur Losung raumlicher Probleme der Elastizitatstheorie. Der Hohlkegel unter Einzellast als Beispiel", Z. Angew. Math. Mech. **14** 203-212 (1934).

[33] P. F. Papkovitch, "A review of some general solutions of basic differential equations of rest for an isotropic elastic body" (in Russian), PPM (Appl. Math. Mech) **1** 117-132 (1937).

[34] L. Rongved, "Force at a point in the interior of a semi-infinite solid with fixed boundary", J. Appl. Mech. **22** 545-546 (1955).