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On the solar neutrino problem

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Abstract

The coupled Dirac equations for solar e- and μ-neutrinos are solved in the presence of a small uniform interaction of the e-neutrinos with matter (the Sun). It is shown that the interaction leads to an increase of the amplitude of the neutrinos oscillations and a decrease of the localization probability of the e-neutrinos. However, the strength of this interaction is insufficient to explain the magnitude of the discrepancy between the solar neutrino flux and the neutrino flux measured on the Earth. A speculation related to a possible superfluid neutrino state is advanced, which may throw some light on the solar neutrino problem.

It is well known that the solar e-neutrinos measured on the Earth are less than the value expected from models of neutrino production in Sun.[1] In order to solve this solar neutrino problem an interaction of the e-neutrinos with matter was suggested.[2, 3] It is claimed that such an interaction produces a difference in mass, which would favour the μ-neutrinos.[4]-[7] However, mass is a Casimir invariant of the Lorentz group which is not affected by interaction. The interaction affects momentum and energy. The coupled Dirac equations of the e- and μ-neutrinos are solved here for a small uniform interaction. It is shown that interaction leads to an increase in the oscillation amplitude of the e-electrons and a decrease of their localization probability. This effect may be related to the solar neutrino problem, though it is much weaker than what would be needed to explain the discrepancy.

We consider two neutrino eigenstates of the free hamiltonian ν_α, $\alpha = 1, 2$, corresponding to admixtures of e- and μ-neutrinos; with usual notations their wavefunctions (spinors) are $\nu_\alpha(\mathbf{r}, t) = e^{-iE_\alpha t + i\mathbf{pr}}\nu_\alpha$, where $E_\alpha = \sqrt{m_\alpha^2 + p^2}$ is energy, $\mathbf{p}$ is momentum and m_α is the mass. The wavefunctions of the e- and μ-neutrinos are

$$\begin{gathered}\psi_e = \cos\theta \cdot \nu_1 + \sin\theta \cdot \nu_2 \quad , \quad \psi_\mu = -\sin\theta \cdot \nu_1 + \cos\theta \cdot \nu_2 \ , \\ \nu_1 = \cos\theta \cdot \psi_e - \sin\theta \cdot \psi_\mu \quad , \quad \nu_2 = \sin\theta \cdot \psi_e + \cos\theta \cdot \psi_\mu \ ,\end{gathered} \tag{1}$$

where θ is the admixture angle. We write also

$$\psi = U\nu \ , \ \nu = U^{-1}\psi \ , \tag{2}$$

where $\psi = (\psi_e, \psi_\mu)^T$, $\nu = (\nu_1, \nu_2)^T$ and the matrix U is

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \ . \tag{3}$$

The wavefunction $\psi_e(\mathbf{r},t)$ can be written as

$$\begin{gathered}\psi_e(\mathbf{r},t)=\cos\theta e^{-iE_1t+i\mathbf{pr}}\nu_1+\sin\theta e^{-iE_2t+i\mathbf{pr}}\nu_2= \\ =\left[\cos^2\theta e^{-iE_1t+i\mathbf{pr}}+\sin^2\theta e^{-iE_2t+i\mathbf{pr}}\right]\psi_e- \\ -\cos\theta\sin\theta\left(e^{-iE_1t+i\mathbf{pr}}-e^{-iE_2t+i\mathbf{pr}}\right)\psi_\mu\ ,\end{gathered}\tag{4}$$

hence the probability of localization

$$\begin{gathered}|\psi_e(\mathbf{r},t)|^2=\left(1-\sin^2 2\theta\sin^2\tfrac{\Delta Et}{2}\right)|\psi_e|^2\ , \\ |\psi_\mu(\mathbf{r},t)|^2=\sin^2 2\theta\sin^2\tfrac{\Delta Et}{2}\,|\psi_\mu|^2\ ,\end{gathered}\tag{5}$$

where $\Delta E=E_2-E_1$; we can see the oscillations $e\longleftrightarrow\mu$. Similar oscillations occur for $E_1=E_2$ and distinct $\mathbf{p}$'s.

Such neutrinos are produced in Sun with energies of the order a few MeV's; as it is well known, their mass is very small (or even zero). In Sun, the e-neutrinos interact with electrons; for $1MeV$ the neutrino wavelength is $\simeq 2\times10^{-11}cm$, which is comparable with the electron Compton wavelength. It follows that the interaction is a forward scattering. The strength of the interaction of a neutrino with an electron is of the order G/Ω, where $G\simeq10^{-49}erg\cdot cm^3$ is the weak-interaction coupling constant and Ω is the volume; it follows that the interaction of an e-neutrino with the medium can be written as

$$V=\frac{G}{N}\sum_i\delta(\mathbf{r}-\mathbf{r}_i)\ ,\tag{6}$$

where N is the number of scattering centers, labelled by i; in a first approximation it can be taken as $V=Gn\simeq10^{-24}erg$ ($\simeq10^{-12}eV$), where n is the density of electrons (in Sun $n\simeq10^{25}cm^{-3}$).

Equations (1) and the Dirac equation

$$i\frac{\partial\nu_\alpha}{\partial t}=(\boldsymbol{\alpha}\mathbf{p}+\beta m_\alpha)\nu_\alpha\tag{7}$$

lead to the Dirac equations

$$\begin{gathered}i\tfrac{\partial\psi_e}{\partial t}=\left[\boldsymbol{\alpha}\mathbf{p}+(m-\Delta m\cos2\theta)\beta\right]\psi_e+\Delta m\sin2\theta\cdot\beta\psi_\mu\ , \\ i\tfrac{\partial\psi_\mu}{\partial t}=\left[\boldsymbol{\alpha}\mathbf{p}+(m+\Delta m\cos2\theta)\beta\right]\psi_\mu+\Delta m\sin2\theta\cdot\beta\psi_e\end{gathered}\tag{8}$$

for the e,μ-neutrinos ($\boldsymbol{\alpha}$ and β are the Dirac matrices), where $2m=m_1+m_2$ and $2\Delta m=m_2-m_1$; their solutions are given by equations (1). These equations can also be written as

$$i\frac{\partial\psi}{\partial t}=U\overline{E}\nu\ ,\ \overline{E}=\left(\begin{array}{cc}E_1&0\\0&E_2\end{array}\right)\ .\tag{9}$$

In the presence of the interaction of the e-neutrinos with matter, these equations become

$$i\frac{\partial\widetilde{\psi}}{\partial t}=U\overline{E}\widetilde{\nu}+\overline{V}\widetilde{\psi}\ ,\tag{10}$$

where

$$\overline{V}=V\left(\begin{array}{cc}1&0\\0&0\end{array}\right)\ ,\tag{11}$$

or

$$i\frac{\partial\widetilde{\nu}}{\partial t}=\overline{E}\widetilde{\nu}+U^{-1}\overline{V}U\widetilde{\nu}\ . \tag{12}$$

We seek the solution as

$$\widetilde{\nu}=e^{-i\overline{E}t}\mu\ , \tag{13}$$

where μ satisfies the system of equations

$$\begin{aligned} i\tfrac{\partial\mu_1}{\partial t}&=V\cos^2\theta\cdot\mu_1+V\cos\theta\sin\theta e^{-i\Delta Et}\mu_2\ ,\\ i\tfrac{\partial\mu_2}{\partial t}&=V\cos\theta\sin\theta e^{i\Delta Et}\mu_1+V\sin^2\theta\cdot\mu_2\ . \end{aligned} \tag{14}$$

We solve this system of equations in the limit $V\ll\Delta E$; the solution is

$$\begin{aligned} \widetilde{\nu}_1&\simeq e^{-i(E_1-V\cos^2\theta)t}\nu_1-\tfrac{V}{2\Delta E}\sin 2\theta\left(e^{-iE_1t}-e^{-iE_2t}\right)\nu_2\ ,\\ \widetilde{\nu}_2&\simeq e^{-i(E_2-V\sin^2\theta)t}\nu_2-\tfrac{V}{2\Delta E}\sin 2\theta\left(e^{-iE_1t}-e^{-iE_2t}\right)\nu_1\ . \end{aligned} \tag{15}$$

The probability of localization of the e-neutrinos, interaction included, is

$$\left|\widetilde{\psi}_e(\mathbf{r},t)\right|^2=\left\{1-\sin^2 2\theta\left[\sin^2\frac{(\Delta E+V\cos 2\theta)t}{2}+\frac{2V}{\Delta E}\sin^2\frac{\Delta Et}{2}\right]\right\}\left|\widetilde{\psi}_e\right|^2\ . \tag{16}$$

The time average of this probability shows that the interaction leads to an increase of the oscillations amplitude (decrease of the probability) by a factor $1+2V/\Delta E$ ($\Delta E>0$). It is worth noting that for $\Delta E=0$ (equal, or vanishing mass) the oscillations are given by $1-\sin^2 2\theta\sin^2(Vt\cos 2\theta/2)$. Similar calculations can be done for an admixture of three neutrino flavours, to include the τ-neutrino, with similar results.

In conclusion, a small uniform interaction between e-neutrinos and matter is considered here. The coupled Dirac equations of the e, μ-neutrinos are solved for this interaction and the neutrino oscillations are computed. It is shown that the oscillation amplitude of the e-neutrinos decreases in the presence of the interaction and their localization probability increases. Although the effect may be related to the solar neutrino problem, it is much weaker than what would be needed to explain the magnitude of the solar neutrino problem.

It is likely that the solar neutrinos have a very small temperature, as a result of their extremely weak interaction with matter. Consequently, they may be viewed as being in their ground state, which is a Fermi sea. Since the electron and neutrino Compton wavelengths are close to each other we may expect an attractive neutrino-neutrino interaction from their forward scattering. This interaction may be mediated by the longitudinal compression solar density waves. Consequently, a superfluid ("superconducting") neutrino-pairing state may be expected. The combination of the weak interaction strength and the state density may lead to a critical temperature which lies above the neutrino temperature, and a corresponding energy gap. Therefore, a sizeable depletion of neutrino states might be expected to paired states. A paired state, extended in the whole space, can be viewed as a composite particle with an internal cohesion energy; such a state would not be detectable, which may give a hint to the solar neutrino problem.

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