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Ions "diffracted" on electromagnetic radiation

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Abstract

A new diffraction phenomenon is described, related to non-relativistic ions scatterred by (and embedded in) a field of high-intensity electromagnetic radiation; for convenience, we include also the scattering on a nuclear potential. In typical ion scattering experiments the motion of the ion projectile is quasi-classical, and the electromagnetic radiation, even at high intensities, affects little this motion; the quasi-classical ions must be tightly collimated. Along its longitudinal motion (parallel witth the initial direction of motion) the ion is able to emits multiple "photons" with momentum $n\hbar\omega/v$, where n is any integer, ω is the radiation frequency and v is the velocity of the incoming ion ($\hbar$ being the Planck's constant). The scattering amplitude is computed here in the Born approximation. It is shown that, apart from a fine structure due to the transverse motion, the scattering amplitude exhibits difraction maxima at angles corresponding to emission of multiple "photons".

Strongly focused optical laser beams are able to generate high intensity electromagnetic radiation in the focal region;[1, 2] electrons accelerated in such fields up to energies in the GeV range can be used as projectiles in nuclear physics experiments.[3]-[7] Via non-linear Compton scattering promising ways are opened up to investigating fundamental phenomena like electron-positron pairs generation, vacuum polarization, vacuum breakdown or non-linear quantum electrodynamics effects.[8]-[12] Special efforts are devoted to connect typical nuclear physics with high-intensity laser radiation.[13] We present here a new diffraction phenomenon which may appear in the ion scattering on high-intensity laser radiation. It resembles the well known Dirac-Kapitza electron diffraction in an electromagnetic standing wave.[14]

We consider the scattering of a non-relativistic ion with mass M on an electromagnetic radiation field; the scattering process is embedded in the radiation field; for convenience, we include also a target placed at the origin, which gives rise to a potential $V(\mathbf{r})$ (Fig. 1); the scattering potential $V(\mathbf{r})$ is a typical nuclear potential (Coulomb potential included). The electromagnetic radiation is described by a vector potential $\mathbf{a} = \mathbf{a}_0 \cos(\omega t - \mathbf{kr})$, $\mathbf{ka}_0 = 0$ (linear polarization), confined to a finite region denoted by R in Fig. 1; the region R corresponds to the focal region of a laser beam; ω is the radiation frequency and $\mathbf{k}$ is the wavevector, $\omega = ck$, where c is the speed of light in vacuum; t and $\mathbf{r}$ denote the time and, respectively, the position vector. The size d of the focal region R is of the order of a few tens of radiation wavelengths $\lambda = 2\pi/k$.

We consider the energy of the projectile in the range $1MeV$ to $\simeq 100MeV$, with a typical value, used for illustrating numerical estimations, $E_0 = 10MeV$. For this energy the velocity of the projectile is $v = \sqrt{2E_0/M} \simeq 3 \times 10^9/\sqrt{A}cm/s$, where A is the mass number of the projectile; we can see that we may consider $v/c \ll 1$ in this range of energies, *i.e.* non-relativistic ions. The

potential $V(\mathbf{r})$ is sufficiently small and localized, such that the scattering on it may be treated within the Born approximation; we shall show that the Born approximation is warranted also for the scattering on the radiation field, even for high-intensity radiation. This is due to the fact that the motion of the ion projectile is quasi-classical. For instance, for a laser intensity $I = 10^{20} w/cm^2$ in the focus, we get an electric field $E_0 = \sqrt{8\pi I/c} \simeq 10^9 esu$ ($esu \simeq 3 \times 10^4 V/m$), and a magnetic field with the same strength. For an optical radiation with $\omega = 10^{15} s^{-1}$ the vector potential is of the order $a_0 \simeq (c/\omega)E_0 \simeq 3 \times 10^4 statvolt$, which implies an energy $ea_0 \simeq 10^{-5} erg \simeq 6MeV$ ($e = 4.8 \times 10^{-10} esu$ being the electron charge). The wavelength of the projectile is of the order $\Lambda = 2\pi\hbar/\sqrt{2ME_0} \simeq 10^{-12}/\sqrt{A} cm$ (for $E_0 = 10MeV$), which is much shorter than the radiation wavelength $\lambda = 2\pi c/\omega \simeq 1.8 \times 10^{-4} cm$ (for $\omega = 10^{15} s^{-1}$); also, it is much shorter than the size d of the focal region R. It follows that the projectile motion is quasi-classical, and we may restrict to analyze it inside the focal region R. The quasi-classical ions can be tightly localized, and they are, we assume, tightly collimated.

The Schrodinger equation of the projectile (embedded in the radiation field) reads

$$i\hbar\frac{\partial\psi}{\partial t} = \left[\frac{1}{2M}(\mathbf{P} - \frac{Ze}{c}\mathbf{a})^2 + V(\mathbf{r})\right]\psi \quad , \tag{1}$$

where $\mathbf{P}$ is the momentum and Z is the charge number of the projectile. First, we note that the effect of the radiation on the projectile is governed by the parameter Zea_0/Mc^2 (it may be viewed as a measure of the change of the ratio v/c); with our numerical values we get $Zea_0/Mc^2 \simeq 10^{-2}/A$; we can see that this parameter is much smaller than unity, even for high laser intensities, due to the large mass of the ion. Therefore, we may conclude that the electromagnetic radiation does not affect much the ion motion. Second, we note that the spatial phase $\mathbf{kr}$ in the potential $\mathbf{a} = \mathbf{a}_0 \cos(\omega t - \mathbf{kr})$ is of the order kl, where l is the length of the path of the ion, while the temporal phase ωt is of the order $(c/v)kl$; since $c/v \gg 1$, it follows that we may retain only the temporal phase ωt in the vector potential and may neglect the spatial phase $\mathbf{kr}$ (this amounts to recognizing that the effect of the magnetic field is much smaller than the effect of the electric field, due to the relativistic factor v/c). In addition, by virtue of the quasi-classical motion of the projectile, we may write $\omega t = \omega l/v$; since v is not changed appreciably by the radiation, we may use the initial velocity $v = \sqrt{2E_0/M}$; further on, by the same token (and in accordance with the assumptions of the perturbation theory), we may neglect the transverse distances x, y in the path length, and write $l = z$ (we take the origin of the time at the spatial origin). Finally, we get for the Schrodinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = \left[\frac{1}{2M}P^2 - \frac{Ze}{Mc}\mathbf{P}\mathbf{a}_0 \cos\omega z/v + \frac{Z^2e^2}{2Mc^2}a_0^2\cos^2\omega z/v + V(\mathbf{r})\right]\psi \quad , \tag{2}$$

where

$$U(\mathbf{r}) = -\frac{Ze}{Mc}\mathbf{P}\mathbf{a}_0 \cos\omega z/v + \frac{Z^2e^2}{2Mc^2}a_0^2\cos^2\omega z/v + V(\mathbf{r}) \tag{3}$$

is viewed as a perturbation; the terms including the radiation field extends to z of the order of the size d of the focal region about the origin; since it does not depend on the transverse coordinates, it holds even for very collimated ion beams. The approximation involved by neglecting the transverse coordinates in equation (3) is valid for small scattering angles.[1]

[1]Initially, the charge is localized over a volume of the order of the atomic volume (wavepacket); in the accelerating electric field the charge acquires rapidly the energy $\mathcal{E}$ (say, $\mathcal{E} = 10MeV$) in the direction of the field; it preserves its localization over transverse distances, while being delocalized over the propagation distance; its wavelength along the propagation distance Λ corresponds to energy $\mathcal{E}$. From the plane wave $e^{-\frac{i}{\hbar}(P^2/2M)t+\frac{i}{\hbar}Px}$ we can see that the

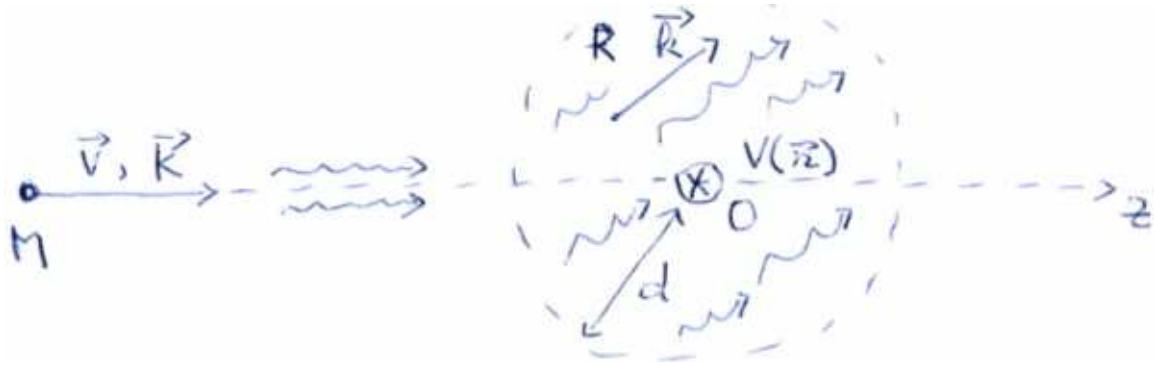

Figure 1: Ion beam colliding an electromagnetic radiation field (region R) and a nuclear target placed at the origin O.

Recently, laser-assisted proton collisions on light nuclei has been discussed in Ref. [15], where the spatial phase $\mathbf{kr}$ is neglected in comparison with the temporal phase ωt for independent t and $\mathbf{r}$ (the so-called dipole approximation). This approximation is valid as long as the range of variation of $\mathbf{r}$ is small, while the range of variation of t is large; it applies to a quantum particle which resides for a long time in the laser focal region. This assumption does not apply to the problem of ion scattering, where the quasi-classical ion spends little time in the focal region. In addition, the limitation on the range of variation of the position $\mathbf{r}$ imposes appreciable constraints upon the scalar products used in Ref. [15]. In particular, the scattering amplitude given in Ref. [15] vanishes in the absence of the nuclear potential.

The perturbation $U(\mathbf{r})$ is treated in the Born approximaton. We get the scattering amplitude

$$f = f_{rad} + f_V \quad , \tag{4}$$

where

$$f_{rad} = \frac{Ze}{2\pi c\hbar^2}\left[Mva_{0z}\int d\mathbf{r}\cos\omega z/v\cdot e^{i\mathbf{qr}} + \frac{Ze}{2c}a_0^2\int d\mathbf{r}\cos^2\omega z/v\cdot e^{i\mathbf{qr}}\right] \tag{5}$$

is the scattering amplitude due to the radiation and

$$f_V = \frac{M}{2\pi\hbar^2}\int d\mathbf{r}V(\mathbf{r})e^{i\mathbf{qr}} \tag{6}$$

is the well-known Born amplitude of the scattering potential $V(\mathbf{r})$; $\mathbf{q} = \mathbf{K} - \mathbf{K}'$ is the momentum exchanged by the projectile during collision. We can see that apart from the regular scattering

diffusion length of the particle is of the order $\delta = \sqrt{\hbar t/M}$; this length becomes of the order d (the dimension of the scattering region) in time $\tau \simeq Md^2/\hbar$; in order to preserve the localization of the scattering process we should have $Md^2/\hbar \gg LM\Lambda/\hbar$, *i.e.* the time τ should be much longer than the time needed for the particle to arrive at the scattering region placed at distance L from the particle source; we get $L \ll d^2/\Lambda$; with our numerical data $L \ll 10^6 cm$, a condition satisfied in experiments.

In our conditions the wavelength Λ is much shorter than the transveres localization a (atomic dimension). Let us consider a test process which gives the particle a momentum Δp and an energy $\Delta\varepsilon$, much smaller than the momentum and, respectively, energy of the particle; this test simulates the scattering; the particle is displaced over the distance $l = (\Delta p/M)\Delta t = (\Delta p/M)(\hbar/\Delta\varepsilon) = \hbar/Mv = \Lambda$, where v is the particle velocity in the accelaration field; this shows that in the scattering process the particle is localized over distance Λ, *i.e.* it has a definite position with accuracy Λ. Therefore, we may define a velocity $v = x/t$ with the accuracy $\Delta v/v \simeq \Lambda/x$ (a similar accuracy follows from the time uncertainty).

In general, the position of a particle at rest with mass M can be tested by another particle with mass m, momentum Δp and energy $\Delta\varepsilon = (\Delta p)^2/2m$, which leads to the displacement (accuracy of the position) $d = (\Delta p/M)\Delta t = (m/M)\lambda$, where λ is the wavelength of the test particle; or by radiation, which leads to $d = \hbar/Mc = (\hbar\omega/Mc^2)\lambda$, where ω and λ are the radiation frequency and, respectively, wavelength.

Born amplitude f_V due to the potential $V(\mathbf{r})$ there exists, in the presence of the radiation field, an additional scattering amplitude f_{rad} due to the radiation.

The evaluation of the integrals in equation (5) requires special attention. We can write

$$f_{rad} = \frac{Ze}{2\pi c\hbar^2}\left[Mva_{0z}\int dz\cos\omega z/v\cdot e^{iq_z z} + \frac{Ze}{2c}a_0^2\int dz\cos^2\omega z/v\cdot e^{iq_z z}\right]\int d\mathbf{r}_\perp e^{i\mathbf{q}_\perp\mathbf{r}_\perp} \quad , \tag{7}$$

where q_z is the z-component of $\mathbf{q}$ and $\mathbf{q}_\perp$, $\mathbf{r}_\perp$ are the transverse components of $\mathbf{q}$ and, respectively $\mathbf{r}$. From $\mathbf{q} = \mathbf{K} - \mathbf{K}'$ and $K = K'$ we have $q_z = 2K\sin^2\theta/2$ and $q_\perp = K\sin\theta$ ($q = 2K\sin\theta/2$), where θ is the scattering angle. Due to the quasi-classical character of the ion motion, the transverse integration over $\mathbf{r}_\perp$ for each ion extends over a very small region of cross-section d'^2, where $d'$is of the order of the ion wavelength Λ, placed at the impact parameter $\mathbf{r}_{\perp i}$ for the $i-th$ ion; the transverse factor in equation (7) is

$$F_t = \int d\mathbf{r}_\perp e^{i\mathbf{q}_\perp\mathbf{r}_\perp} \simeq d'^2 e^{i\mathbf{q}_\perp\mathbf{r}_{\perp i}} \simeq \Lambda^2 e^{i\mathbf{q}_\perp\mathbf{r}_{\perp i}} \quad ; \tag{8}$$

this is valid for tightly collimated ions. Then, we note that the order of magnitude of the coefficients in the front of the z-integrals in equation (7) are

$$\frac{ZeMva_{0z}}{2\pi c\hbar^2}F_t \simeq \frac{Zea_0}{\hbar\omega}\frac{1}{\lambda\Lambda}F_t \simeq Z\frac{ea_0}{\hbar\omega}\frac{\Lambda}{\lambda}e^{i\mathbf{q}_\perp\mathbf{r}_{\perp i}} \tag{9}$$

and

$$\frac{Z^2e^2a_0^2}{4\pi c^2\hbar^2}F_t \simeq \left(Z\frac{ea_0}{\hbar\omega}\frac{\Lambda}{\lambda}\right)^2 e^{i\mathbf{q}_\perp\mathbf{r}_{\perp i}} \quad ; \tag{10}$$

it follows that the parameter

$$\xi = Z\frac{ea_0}{\hbar\omega}\frac{\Lambda}{\lambda} \tag{11}$$

controls the validity of the Born approximation; making use of our numerical values, we get $\xi \simeq 10^{-2}Z/\sqrt{A}$; we can see that $\xi \ll 1$, even for higher intensities of the radiation field. In addition, we can see that the contribution of the integral including $\cos^2\omega z/v$ in equation (7) is much smaller than the contribution of the first integral, which includes $\cos\omega z/v$ (providing $a_{0z} \neq 0$).

For a beam of ions with transverse extension $d_\perp$, separated by a mean distance b the contribution of the transverse factor includes the summation

$$\sum_i e^{i\mathbf{q}_\perp\mathbf{r}_{\perp i}} = \frac{1}{b^2}\int_0^{d_\perp} d\mathbf{r}_\perp\cdot e^{i\mathbf{q}_\perp\mathbf{r}_\perp} = \frac{2\pi}{b^2q_\perp^2}\int_0^{q_\perp d_\perp} dx\cdot xJ_0(x) \quad , \tag{12}$$

where $J_0(x)$ is the Bessel function of zero-th order. For $\theta \neq 0$ the integration limit $q_\perp d_\perp$becomes rapidly very large and the "interference" factor in equation (12) oscillates rapidly; it is of order $N_s\sqrt{\xi}$, where N_s is the number of ions in the cross-section of the beam.

The integration over z in equation (7) extends from $-d_\parallel$ to $d_\parallel$, where $d_\parallel$ is the extension of the focal region along the z-direction; if we take $d_\parallel \to \infty$, we get

$$\begin{aligned} f_{rad} \simeq \tfrac{Ze}{2c\hbar^2}\{Mva_{0z}[\delta(q_z+\omega/v)+\delta(q_z-\omega/v)]+ \\ +\tfrac{Ze}{4c}a_0^2[2\delta(q_z)-\delta(q_z+2\omega/v)+\delta(q_z-2\omega/v)]\}\int d\mathbf{r}_\perp e^{i\mathbf{q}_\perp\mathbf{r}_\perp} \quad , \end{aligned} \tag{13}$$

which exhibits localized spots at $\theta = \pm\sqrt{\hbar\omega/E_0}$, $\pm\sqrt{2\hbar\omega/E_0}$; these maxima correspond to emission of one, or two "photons", with energy $\hbar\omega$ and momentum $\hbar\omega/v$. In the scattering amplitudes δ^2 should be replaced by $d_\parallel\delta$. We note that $d_\parallel$ ocurring in these amplitudes (via the δ-functions) does not contribute to the estimation of the validity of the Born approximation, since the scattered wave $e^{iK^{'}r}/r$ extends in this case over length $d_\parallel$ and contributes a factor $1/d_\parallel$. For a finite $d_\parallel$ the functions $\delta(Q)$ with $Q - q_z$, $Q = q_z \pm \omega/v$, $Q = q_z \pm 2\omega/v$ in equation (13) should be replaced by $2\sin Qd_\parallel/Q$.

Similar results are obtained for circularly polarized radiation. In higher orders of the perturbation theory (for the radiation part of the scattering amplitude) maxima are obtained at multiple-"photon" emission (absorption) defined by the condition $\sin^2\theta/2 = \pm n\hbar\omega/vP$, with integer n (with smaller scattering amplitudes). Also, we can see that the radiation part of the scattering exists independently of the scattering potential $V(\mathbf{r})$; it is a "diffraction" effect suffered by ions in the radiation field, where the role of the radiation is played by ions, while the role of the diffraction-grating distance is played by the radiation wavelength; indeed, the condition of diffraction is then $\theta \simeq n\Lambda/\lambda = n\hbar\omega/cP$. The effect resembles the electron diffraction in a standing electromagnetic wave. The "diffraction" effect presented here for ions in a traveling electromagnetic wave is, in general, absent for electrons, which, due to their small mass, are rapidly accelerated, especially in high-intensity traveling radiation fields.

Comment. The calculations described above are improper for a few reasons. First, if we use the standard non-relativistic hamiltonian given by equation (1), we assume that the electromagnetic momentum is included, which means that the asymptotic states include this electromagnetic momentum; in this case there will be no scattering on laser light; indeed, the projectile travels a long distance (and a long time) in radiation to be able to absorb the radiation into its energy levels, which become (slightly) time dependent. The radiation potential included in equation (1) is not available anymore for scattering, it is included in the energy exponent of the wavefunction, which is given by the Kramers-Henneberger transformation. The only available scattering remains the one by an external potential, as provided by the Kroll-Watson formula.

For the scattering of a charge on the laser light confined in the beam focus (*i.e.* in a limited region of space) we must use the dipole hamiltonian

$$i\hbar\frac{\partial\psi}{\partial t} = [\frac{1}{2M}P^2 - e\mathbf{r}\mathbf{E} + V(\mathbf{r})]\psi \quad , \tag{14}$$

with asymptotic scattering states of the free charge (free-particle momenta); in this case the radiation potential (dipole potential) remains availabe for scattering. In the scattering region, even if the projectile moves quasi-classically trough the light pulse, it has sufficient time to absorb (or emits) photons, *i.e.* to take the radiation into its energy (time-dependent) energy levels, since the light pulse has the dimension of a few tens of light wavelengths; therefore, the charge sees a few tens of radiation period; this circumstance is achieved by the Goepert-Mayer transformation, which takes the radiation potential (dipole potential) into the energy exponent of the wavfunction; in addition, this exponent has a large amplitude, which prevents the approximation of the wavefunction exponential by its series expansion. In this case we have the dipole potential available for scattering and, at the same time, the scattered wavefuncton is modified by the dipole potential. In the former case we must perform first the Kramers-Henneberger transform and then formulate the scattering problem, while in the latter case we must formulate first the scattering problem and then perform the Goeppert-Mayer transform (and the Kramers-Henneberger transform, whose small effects may ne neglected).

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