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Ions diffracted by electromagnetic radiation. Charges scattered by radiation field

M. Apostol
Department of Theoretical Physics, Institute of Atomic Physics,
Magurele-Bucharest MG-6, POBox MG-35, Romania
email: apoma@theory.nipne.ro

Abstract

A new diffraction phenomenon is described, related to non-relativistic ions scatterred by a field of low- or moderate-intensity electromagnetic radiation; for convenience, we include also the scattering on a nuclear potential. In typical scattering experiments the motion of the ion projectile is quasi-classical, and the low- or moderate-intensity electromagnetic radiation affects little this motion; however, along its longitudinal motion (parallel with the initial direction of motion), while preserving its energy (elastic scattering), the ion is able to change the direction of its momentum by multiple "photons" momenta $n\hbar\omega/v$, where n is any integer, ω is the radiation frequency and v is the velocity of the incoming ion ($\hbar$ being the Planck's constant). The scattering amplitude of ions in such fields of electromagnetic radiation is computed here in the Born approximation. It is shown that, apart from a fine structure due to the transverse motion, the scattering amplitude exhibits difraction maxima at angles corresponding to "emission of multiple photons". In high-intensity radiation the ion scattering may become quantum-mechanical and the (inelastic) scattering amplitude is time dependent. In this case the effect of the radiation can be accounted for by the well-known Goeppert-Mayer transform, which allows the computation of the cross-section. The calculations are performed for the illustrative example of ions, but they can be applied to any charge scattered by electromagnetic radiation.

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It is well known that a quantum-mechanical charge in low-intensity electromagnetic radiation may suffer transitions by absorbing or emitting photons. However, very often, in typical scattering experiments the projectile momentum (energy) is sufficiently high to make the projectile motion a (non-relativistic) quasi-classical one. Then, if the energy of the electromagnetic field is sufficiently low, the quasi-classical character of the motion is preserved in scattering and the charge may be diffracted (elastically scattered) by the radiation field. For higher radiation intensity the projectile motion in the radiation field becomes again quantum-mechanical; then, inelastic scattering may appear, by absorption and emission of multiple "photons" (a mechanism distinct from the quantum-mechanical photon transitions). For very high-intensity radiation the charge may be rapidly accelerated by radiation up to relativistic velocities. Such aspects of charge scattering in electromagnetic radiation are presented in this paper. The calculations are done for ions, but they may be applied to any charged particle. The present interest in this subject is derived from the recent development in high-intensity laser beams.

Strongly focused optical laser beams are able to generate high-intensity electromagnetic radiation in the focal region;[1, 2] electrons accelerated in such fields, up to energies in the GeV range, can be used as projectiles in nuclear physics experiments.[3]-[7] Via non-linear Compton scattering promising ways are opened up to investigating fundamental phenomena like electron-positron pairs generation, vacuum polarization, vacuum breakdown or non-linear quantum electrodynamics effects.[8]-[12] In this context, special efforts are devoted to connect typical nuclear physics with laser radiation.[13] We present here a description of quasi-classical charges scattered by radiation field. It is shown that a new diffraction phenomenon may appear in the scattering by low- and moderate-intensity laser radiation. It may be likened to the well known Kapitza-Dirac electron diffraction in an electromagnetic standing wave.[14] Also, the inelastic scattering cross-section is computed for high-intensity radiation.

The scattering in the presence of the electromagnetic radiation has been approached previously for projectiles dressed with radiation;[15]-[22] the interest in these investigations was to estimate the radiation effect on the scattering by a potential. In these approaches the incoming and outgoing wavefunctions of the projectile include the efects of the radiation (the scattering experiment is embedded in the radiation field). In this paper the asymptotic wavefunctions of the projectile are free wavefunctions, the scatterer being the radiation field and, possibly, a potential like the nuclear potential.

First, we consider the scattering of a non-relativistic ion with mass M in an electromagnetic radiation field; for convenience, we include also a target placed at the origin, which gives rise to a potential $V(\mathbf{r})$ (Fig. 1); the scattering potential $V(\mathbf{r})$ may be viewed as a typical nuclear potential. The electromagnetic radiation is described by a vector potential $\mathbf{a} = \mathbf{a}_0 \cos(\omega t - \mathbf{kr})$, $\mathbf{ka}_0 = 0$ (linear polarization), confined to a finite region denoted by F in Fig. 1; the region F corresponds to the focal region of a laser beam; ω is the radiation frequency and $\mathbf{k}$ is the wavevector, $\omega = ck$, where c is the speed of light in vacuum; t and $\mathbf{r}$ denote the time and, respectively, the position vector. Usually, the size d of the focal region F may be of the order of a few tens of radiation wavelengths $\lambda = 2\pi/k$.

We consider the energy of the projectile in the range $1MeV$ to $\simeq 100MeV$, with a typical value $\mathcal{E} = 10MeV$, used for illustrating numerical estimations. For this energy the velocity of the projectile is $v = \sqrt{2\mathcal{E}/M} \simeq 3 \times 10^9/\sqrt{A} cm/s$, where A is the mass number of the projectile; we assume $v/c \ll 1$, *i.e.* non-relativistic ions (electron mass $\simeq 10^{-27}g$). The potential $V(\mathbf{r})$ is sufficiently small and localized, such that the scattering on it may be treated within the Born approximation; we shall show that the Born approximation is warranted also for the scattering by low- and moderate-intensity radiation field. For instance, for a laser intensity $I = 10^{18}w/cm^2$ in the focus, we get an electric field of the order $E_0 = \sqrt{8\pi I/c} \simeq 10^8 esu$ ($\simeq 3 \times 10^3 V/m$) (and a magnetic field with the same strength). For an optical radiation with frequency $\omega = 10^{15}s^{-1}$ the vector potential is of the order $a_0 \simeq (c/\omega)E_0 \simeq 3 \times 10^3 statvolt$, which implies an energy $ea_0 \simeq 10^{-6}erg \simeq 1MeV$ ($e = 4.8 \times 10^{-10}esu$ being the electron charge). The wavelength of the projectile is of the order $\Lambda = 2\pi\hbar/\sqrt{2M\mathcal{E}} \simeq 10^{-12}/\sqrt{A}cm$ (for $\mathcal{E} = 10MeV$), which is much shorter than the radiation wavelength $\lambda = 2\pi c/\omega \simeq 1.8 \times 10^{-4}cm$ (for $\omega = 10^{15}s^{-1}$) and the size d of the focal region F. It follows that the projectile motion may be viewed as quasi-classical.

The Schrodinger equation of the projectile in the region F reads

$$i\hbar\frac{\partial\psi}{\partial t} = \left[\frac{1}{2M}\mathbf{P}^2 - Ze\mathbf{rE} + V(\mathbf{r})\right]\psi \quad , \tag{1}$$

where $\mathbf{P}$ denotes momentum, Z is the charge number of the projectile and $\mathbf{E} = -(1/c)\partial\mathbf{a}/\partial t = \mathbf{E}_0 \sin(\omega t - \mathbf{kr})$, $\mathbf{E}_0 = \omega\mathbf{a}_0/c$, is the electric field. First, we note that the effect of the radiation on the projectile motion is governed by the parameter $\xi = ZeE_0 d/\mathcal{E}$; this is also the parameter which controls the validity of the Born approximation. With our numerical data we get ξ of the order unity (for $\mathcal{E} = 10MeV$ and d a few tens of λ). We assume that the radiation has a sufficiently low intensity and the projectile energy is sufficiently high such that $\xi \ll 1$; in these conditions the quasi-classical character of the projectile motion in the radiation field is preserved and the Born approximation is meaningful. Second, we note that the spatial phase $\mathbf{kr}$ in the potential $\mathbf{a} = \mathbf{a}_0 \cos(\omega t - \mathbf{kr})$ is of the order kd at most, while the temporal phase ωt is of the order $(c/v)kd$; since $c/v \gg 1$, it follows that we may retain only the temporal phase ωt in the vector potential and may neglect the spatial phase $\mathbf{kr}$, a typical non-relativistic approximation (this amounts to recognizing that the effect of the magnetic field is much smaller than the effect of the electric field, due to the relativistic factor v/c). Therefore, we may use $\mathbf{a} = \mathbf{a}_0 \cos \omega t$ for the vector potential and $\mathbf{E} = \mathbf{E}_0 \sin \omega t$ for the electric field ($\mathbf{E}_0 = \omega \mathbf{a}_0/c$). In addition, by virtue of the quasi-classical motion of the projectile, we may write $\omega t = \omega l/v$, where l is the length of the projectile path in the radiation region; since v is not changed appreciably by the radiation, we may use the initial velocity $v = \sqrt{2\mathcal{E}/M}$; further on, by the same token (and in accordance with the assumptions of the perturbation theory), we may neglect the transverse distances x, y in the path length, and write $l = z$ (we take the origin of the time at the spatial origin). Finally, we get for the Schrodinger equation

$$i\hbar \frac{\partial \psi}{\partial t} \simeq \left[\frac{1}{2M} \mathbf{P}^2 - ZezE_{0z} \sin \omega z/v + V(\mathbf{r}) \right] \psi \quad , \tag{2}$$

where

$$U(\mathbf{r}) = -ZezE_{0z} \sin \omega z/v + V(\mathbf{r}) \tag{3}$$

is viewed as a perturbation; the coordinate z extends to a range of the order of the size d of the focal region about the origin. The approximation caused by neglecting the transverse coordinates in equation (3) is valid for small scattering angles (forward scattering).

Recently, laser-assisted proton collisions on light nuclei has been discussed in Ref. [22], where the spatial phase $\mathbf{kr}$ is neglected in comparison with the temporal phase ωt for independent t and $\mathbf{r}$ (the so-called dipole approximation). This approximation is valid as long as the energy is changed appreciably by the radiation and the quasi-classical approximation does not hold anymore; it applies to a quantum particle in the laser focal region, where it is dressed with radiation. This assumption does not apply to the problem of ion scattering discussed here, where the ion motion is quasi-classical. In particular, the scattering amplitude given in Ref. [22] vanishes in the absence of the nuclear potential.

In the Born approximation for the scattering potential $U(\mathbf{r})$ in equation (2) we get the scattering amplitude

$$f = f_{rad} + f_V \quad , \tag{4}$$

where

$$f_{rad} = \frac{MZeE_{0z}}{2\pi\hbar^2} \int d\mathbf{r} \cdot z \sin \omega z/v \cdot e^{i\mathbf{qr}} \tag{5}$$

is the scattering amplitude due to the radiation and

$$f_V = \frac{M}{2\pi\hbar^2} \int d\mathbf{r} V(\mathbf{r}) e^{i\mathbf{qr}} \tag{6}$$

is the well-known Born amplitude of the scattering potential $V(\mathbf{r})$; $\mathbf{q} = \mathbf{K} - \mathbf{K}'$ is the wavevector exchanged by the projectile during collision. We can see that apart from the regular scattering Born amplitude f_V due to the potential $V(\mathbf{r})$, there exists, in the presence of the radiation field, an additional scattering amplitude f_{rad} due to the radiation.

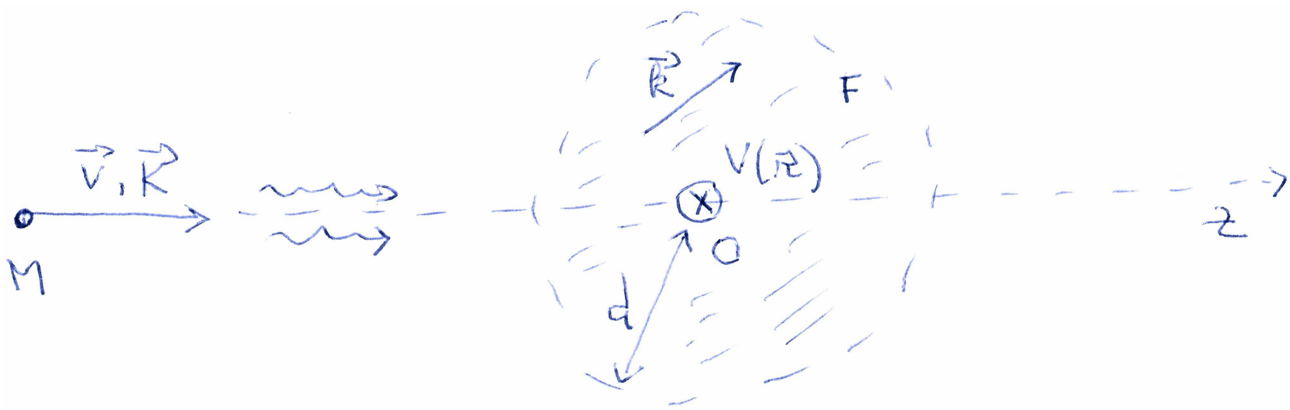

Figure 1: Ion beam colliding an electromagnetic radiation field (region F) and a nuclear target placed at the origin O.

The evaluation of the integral in equation (5) requires special attention. We can write

$$f_{rad} = \frac{MZeE_{0z}}{2\pi\hbar^2} \int dz \cdot z \sin \omega z/v \cdot e^{iq_z z} \int d\mathbf{r}_\perp e^{i\mathbf{q}_\perp \mathbf{r}_\perp} \quad , \tag{7}$$

where q_z is the z-component of $\mathbf{q}$ and $\mathbf{q}_\perp$, $\mathbf{r}_\perp$ are the transverse components of $\mathbf{q}$ and, respectively $\mathbf{r}$. From $\mathbf{q} = \mathbf{K} - \mathbf{K}'$ and $K = K'$ we have $q_z = 2K \sin^2 \theta/2$ and $q_\perp = K \sin\theta$ ($q = 2K \sin\theta/2$), where θ is the scattering angle. The integration over z gives a longitudinal factor

$$F_l = -\frac{\partial}{\partial q_z} \left[\frac{\sin(q_z + \omega/v)d/2}{q_z + \omega/v} - \frac{\sin(q_z - \omega/v)d/2}{q_z - \omega/v} \right] \quad , \tag{8}$$

which, for large d, may be approximated by $F_l \simeq -\pi \left[\delta'(q_z + \omega/v) - \delta'(q_z - \omega/v) \right]$. Due to the quasi-classical character of the ion motion, the transverse integration over $\mathbf{r}_\perp$ for each ion extends over a very small region of cross-section d'^2, where d' is of the order of the ion wavelength Λ, placed at the impact parameter $\mathbf{r}_{\perp i}$ for the $i-th$ ion; the transverse factor in equation (7) is

$$F_t = \int d\mathbf{r}_\perp e^{i\mathbf{q}_\perp \mathbf{r}_\perp} \simeq d'^2 e^{i\mathbf{q}_\perp \mathbf{r}_{\perp i}} \simeq \Lambda^2 e^{i\mathbf{q}_\perp \mathbf{r}_{\perp i}} \quad ; \tag{9}$$

we get

$$f_{rad} \simeq \frac{MZeE_{0z}}{2\hbar^2} \Lambda^2 \left[\delta'(q_z - \omega/v) - \delta'(q_z + \omega/v) \right] e^{i\mathbf{q}_\perp \mathbf{r}_{\perp i}} \quad . \tag{10}$$

The summation over i can be transformed in an integral which includes the Bessel function $J_0(q_\perp d)$; making use of the integral[23]

$$\int_0^1 dx \cdot x J_0(q_\perp dx) = \frac{1}{q_\perp d} J_1(q_\perp d) \tag{11}$$

and using the asymptotic expression of the Bessel function J_1, we get

$$f_{rad} \simeq \sqrt{\pi/2}\xi \cos\alpha \frac{\Lambda^{3/2}n^{2/3}}{d^{1/2}\theta^{3/2}} \left[\delta^{'}(q_z - \omega/v) - \delta^{'}(q_z + \omega/v)\right] \cos(\theta d/\Lambda - 3\pi/4) \quad , \tag{12}$$

where α is the angle between the electric field and the direction of ion motion and n is the ion density.

We can see that the scattering amplitude f_{rad} in the radiation field (Born approximation) exhibits a "double" maximum at $q_z = \omega/v$, *i.e.* at scattering angles $\theta = \pm\sqrt{2\omega/vK} = \pm\sqrt{\hbar\omega/\mathcal{E}}$, which correspond to emission of one "photon" with momentum $\hbar\omega/v$ (only the $\delta^{'}(q_z - \omega/v)$ must be preserved in equation (12)); in higher-orders of the perturbation theory similar diffraction maxima (with richer internal structure) appear at angles given by $\sin^2\theta/2 = n\hbar\omega/4\mathcal{E}$, n any integer, corresponding to multiple-"photon" emission with momenta $n\hbar\omega/v$, where n is any integer; these maxima affect the regular scattering amplitude generated by the potential $V(\mathbf{r})$. It is easy to see that the order of magnitude of the scattering amplitude f_{rad} is given by $\xi\cos\alpha(\Lambda d/\theta)^{3/2}n^{2/3}$; in the scattering cross-section $\delta^{'2}$ may be replaced by $d\left|\delta^{'}\right|$ and the rapidly varying factor $\cos^2(\theta d/\Lambda - 3\pi/4)$ must be averaged. We emphasize that the radiation part of the scattering exists independently of the scattering potential $V(\mathbf{r})$; it is a diffraction effect suffered by ions in the radiation field, where the role of the radiation is played by ions, while the role of the diffraction-grating distance is played by the radiation wavelength; in contrast with the usual Bragg diffraction condition $\theta \simeq n\Lambda/\lambda = n\hbar\omega/cP$, the condition of ion diffraction given here reads $\theta \simeq \pm\sqrt{n\hbar\omega/\mathcal{E}}$. The effect may be likened to the electron diffraction in a standing electromagnetic wave.[1][14] We note that the "photons" involved in this diffraction effect can be viewed as photons dressed with interaction, in view of the ion velocity v which appears in their momenta $n\hbar\omega/v$, instead of the light speed c. Similar results can be obtained for a general polarization of the radiation field.

If the parameter ξ is much larger than unity the quasi-classical approximation and the Born approximation are not valid anymore; the ion suffers a quantum-mechanical motion in the radiation field and time t is disentangled from the position $\mathbf{r}$; this happens for high-intensity radiation or low-energy projectiles. In order to account for the effect of the radiation field in this case we may use the Goeppert-Mayer unitary tranform[24]

$$\psi = e^{-\frac{iZe}{\hbar\omega}\mathbf{E}_0\mathbf{r}(\cos\omega t - 1)}\phi \quad , \tag{13}$$

where ψ is the wavefunction in equation (1) and ϕ satisfies the Schrodinger equation

$$i\hbar\frac{\partial\phi}{\partial t} = \frac{1}{2M}\left[\mathbf{P} - \frac{Ze}{c}(\mathbf{a} - \mathbf{a}_0)\right]^2 \phi + V(\mathbf{r})\phi \quad , \tag{14}$$

with $\psi(t=0) = \phi(t=0)$ (we note that $\mathbf{E}(t=0) = 0$); we recognize in equation (14) the standard form of the non-relativistic hamiltonian of the charge Ze in the radiation field with the vector potential $\mathbf{a}$ (the constant term $\mathbf{a}_0$ appears as a consequence of the initial conditions).[25, 26] At this point we note that the hamiltonain given by equation (14) is not suitable for the scattering problem discussed here with quasi-classical projectiles since it includes the radiation contribution in the asymptotic momentum $\mathbf{P}$; this would correspond to a scattering process embedded in the radiation region, in contrast with the scattering process discussed here, where we assume that the radiation field is confined to a finite region F. Moreover, it is worth noting that the radiation effects in

[1]The diffraction of electrons in standing electromagnetic wave (Kapitsa-Dirac effect) is a Bragg diffraction due to stimulated Compton scattering; in Bragg diffraction the change in the momentum is transverse with respect to the propagtion direction; in the diffraction presented here the momentum change is longitudinal.

the hamiltonian given by equation (14) are controlled by the ratio $Zea_0/cP = (Zea_0/Mc^2)(c/v)$, which includes the well-known "relativistic" parameter $\eta = Zea_0/Mc^2$; usually, this parameter is very small for ions, even for high-intensity radiation; *e.g.*, with our numerical data we get $\eta \simeq 10^{-4}/A$. It follows that for reasonably high velocities the ratio Zea_0/cP is small, and we may neglect the radiation interaction in equation (13); it follows that the effect of the radiation is, practically, included in the unitary transform given by equation (13).

As a matter of fact, if we still maintain the radiation interaction in equation (14) (for low velocities v), we can use a transformation of the Kramers-Henneberger type[27]-[29]

$$\phi = e^{-\frac{iZ^2e^2}{8Mc^2\hbar\omega}a_0^2(\sin 2\omega t - 8\sin\omega t + 6\omega t)}e^{\frac{iZe}{Mc\hbar\omega}(\sin\omega t - \omega t)\cdot\mathbf{a}_0\mathbf{P}}\chi \quad , \tag{15}$$

where χ satisfies the Schrodinger equation

$$i\hbar\frac{\partial\chi}{\partial t} = \left[\frac{1}{2M}P^2 + \widetilde{V}(\mathbf{r})\right]\chi \tag{16}$$

with the radiation-dressed potential

$$\widetilde{V}(\mathbf{r}) = e^{-\frac{Ze}{M\omega^2}(\sin\omega t - \omega t)\cdot\mathbf{E}_0 grad}V(\mathbf{r}) \ ; \tag{17}$$

we can see that in a very short time (given by $(ZeE_0/M\omega^2 a)(\omega t)^3/6$, where a is the range of the potential $V(\mathbf{r})$) the potential $V(\mathbf{r})$ is practically removed from the problem, and the wavefunction χ is a free-particle wavefunction; since the time-dependent exponents in the wavefunction ϕ (equation (15)) are immaterial for our problem, we arrive again at the conclusion that the effect of the electric field is included mainly in the transform given by equation (13). This is an interesting effect, which shows that in high-intensity radiation the ion scattering on short-range nuclear potentials is obliterated by the scattering by the radiation field, which has a large cross-section, as we see below (for Coulomb potentials the situation is different).

For reasonably high ion velocity we may consider the incoming ion wavefunction $e^{-\frac{i}{\hbar}\mathcal{E}t + i\mathbf{K}\mathbf{r}}$ and the outgoing wavefunction $e^{-\frac{i}{\hbar}\mathcal{E}' + i\mathbf{K}'\mathbf{r}}$ as χ-type wavefunctions and write the radiation scattering amplitude as

$$T = \frac{1}{v}\int d\mathbf{r} e^{-\frac{iZe}{\hbar\omega}\mathbf{E}_0\mathbf{r}(\cos\omega t - 1)}e^{i\mathbf{q}\mathbf{r}}e^{-\frac{i}{\hbar}(\mathcal{E} - \mathcal{E}')t} \quad , \tag{18}$$

where $\mathcal{E} = \hbar^2K^2/2M$ and $\mathcal{E}' = \hbar^2K'^2/2M$ are the energies of the incoming and, respectively, outgoing particles and $v \simeq d^3$ is the volume of the region F containing the radiation. This quantity is a (time-dependent) tunneling coefficient; it is easy to see that the transition probability, *i.e.* the number of scattered particles with momentum $\hbar K'$ and energy $\mathcal{E}'$ is given by

$$|T|^2 = \frac{1}{v}\int d\mathbf{r} e^{-\frac{iZe}{\hbar\omega}\mathbf{E}_0\mathbf{r}(\cos\omega t - 1)}e^{i\mathbf{q}\mathbf{r}} \tag{19}$$

(for a sufficiently large volume v). The integration in equation (19) gives a momentum transfer $\hbar\mathbf{q}$ in the direction of the electric field $\mathbf{E}_0$; we denote it by $\hbar q_0$; the integration along the coordinate in the field direction gives

$$|T|^2 = \frac{2\pi}{d}\delta\left[q_0 - \frac{ZeE_0}{\hbar\omega}(\cos\omega t - 1)\right] \quad , \tag{20}$$

or

$$|T|^2 = \frac{2\pi}{d}\delta\left[q_0 - \frac{\xi v}{2\omega d}K(\cos\omega t - 1)\right] \ . \tag{21}$$

Usually, the cross-section is obtained as the ratio of the transition probability per unit time to the current density v/d^3; thereafter, the differential cross-section is obtained by multiplying with the number of available scattered states, which in our case is $d\mathbf{K}'d^3/(2\pi)^3$, *i.e.*, $K'^2 d^3 dK' \sin\theta d\theta/2\pi^2$ (it is worth noting that the volume available for the transition process to $\mathbf{K}'$-states is d^3). The time derivative $\partial |T|^2/\partial t$ gives the probability per unit time for scattering both to and from the state $\mathbf{K}'$, since it may be both positive and negative. Consequently, it is more convenient to compute the cross-section as the ratio of the number of scattered particles $|T|^2$ to the incoming particle flux $1/d^2$; the differential cross-section is given by

$$d^2\sigma = \frac{d^4}{\pi} K'^2 \delta\left[q_0 - \frac{\xi v}{2\omega d} K(\cos\omega t - 1)\right] \sin\theta \cdot dK' d\theta \ . \tag{22}$$

We can see in equation (22) that the scattering implies in this case absorption and emission of special "photons" which change the particle momentum along the direction of the electric field by the time-dependent amount $(\hbar\xi v K/2\omega d)(\cos\omega t - 1)$ (and the energy by a corresponding amount). In order to integrate with respect to K' it is convenient to take the z-axis along the direction of the electric field $\mathbf{E}_0$; then $q_0 = K - K'\cos\theta$ and

$$d\sigma = \frac{d^4 K^2}{\pi}\left[1 - \frac{\xi v}{2\omega d}(\cos\omega t - 1)\right]^2 \frac{\sin\theta}{\cos^3\theta} d\theta \ , \ \ 0 < \theta < \pi/2 \ ; \tag{23}$$

for $\pi/2 < \theta$ the cross-section is zero. We may take the time average in equation (23) and get

$$d\sigma \simeq \frac{3d^4 K^2}{8\pi}\left(\frac{\xi v}{\omega d}\right)^2 \frac{\sin\theta}{\cos^3\theta} d\theta = \frac{3d^2}{2\pi}\left(\frac{ZeE_0 d}{\hbar\omega}\right)^2 \frac{\sin\theta}{\cos^3\theta} d\theta \ ; \tag{24}$$

we can see that the cross-section corresponding to the scattering by the high-intensity radiation field is large, with a singularity at right angle ($\theta = \pi/2$) with respect to the direction of the electric field. We note the coefficient $ZeE_0 d/\hbar\omega$ entering the cross-section, which counts the number of "photons" $\hbar\omega$ included in the amount of energy $ZeE_0 d$ of the ion in the radiation field.[2]

[2] It is worth computing the usual Born scattering cross-section according to the method used here (*i.e.*, normalizing to the particle flux, not to the current density). The Born scattered wavefunction is given by

$$\psi_1 = -\frac{M}{2\pi\hbar^2}\int d\mathbf{r}' V(\mathbf{r}') e^{i\mathbf{k}\mathbf{r}'} \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\ \mathbf{r}-\mathbf{r}'\ |} \simeq f\frac{e^{ikr}}{r} \ ;$$

the tunneling coefficient is

$$T_B = f\int d\mathbf{r} e^{-i\mathbf{k}'\mathbf{r}} \frac{e^{ikr}}{r} \ ,$$

where $\mathbf{k}'$ is directed along $\mathbf{r}$; the evaluation of this integral (over the scattering region) gives $T_B \simeq f/d$ (normalized to the scattering volume); the cross-section obtained as the ratio to the incoming particle flux is $d^2 |T_B|^2 = |f|^2$, *i.e.* the Born cross-section. The density of states in the elastic scattering is the solid angle $d\Omega$, while the density of states in the inelastic scattering is $K'^2 d^3 dK' d\Omega/(2\pi)^3$. In the elastic scattering the energy is conserved, while in the inelastic scatetring the energy is not conserved; the amplitudes of the two types of scattering cannot be added, such that cross-sections of the type $|Tf|$ (arising from $|T+f|^2$) are meaningless. If we use radiation-dressed wavefunctions in the Born formula we may get an amplitude $\sim J_n f$, where J_n is a Bessel function, by artificially selecting an n-photon exchange process; this procedure leads to the Kroll-Watson formula $d\sigma \sim J_n^2 d\sigma_{el}$; but it corresponds both to absortion of n photons and emission of n photons; The energy of the emitted photons as well as the energy of the absorbed photons inside the radiation region goes to charges which suffer a continuous energy exchange with the radiation in that region.

In scattering problems in the presence of the radiation the potential $V(\mathbf{r})$ contributes to diffraction, which appears at low- or moderate-intensity radiation, while in high-intensity radiation the scattering on the potential $V(\mathbf{r})$ is blurred by the large cross-section of the inelastic scattering by radiation.

It is worth discussing the case of electron projectiles. First, in order to preserve the non-relativistic character of the motion the radiation intensity must be limited to $I < 10^{15}w/cm^2$. Second, the critical value $\xi = 1$ for $I = 10^{15}w/cm^2$ restricts the electron energy to $\mathcal{E} < 5MeV$; it follows that for all allowed energies the parameter ξ is larger than unity and, consequently, the electron suffers an inelastic scattering. For usual sources of light the parameter ξ is much smaller than unity, and the electrons suffer diffraction.

In conclusion, one may say that non-relativistic quasi-classical charged projectiles may suffer a diffraction effect during their scattering by a low- or moderate-intensity radiation field, while in higher-intensity radiation the scattering becomes inelastic. The behaviour of a non-relativistic charged particle in the field of electromagnetic radiation raises an interesting problem, related to the dipole hamiltonian given by equation (1) and the standard form of the non-relativistic hamiltonian given by equation (14). Classically, the two hamiltonians lead to equivalent equations of motion, while, quantum-mechanically, they are related through the Goeppert-Mayer unitary transform and, thus, are considered equivalent. In spite of this formal equivalence, there exist differences between the two hamiltonians, depending on the problem they are applied to. In scattering problems the radiation is confined to a finite region (denoted by F in this paper) and the asymptotic wavefunctions are eigenfunctions of the momentum $\mathbf{P} = M\mathbf{v}$; consequently, the appropriate starting point in these problems is the dipole hamiltonian given by equation (1). This has relevant effects in scattering problems, as shown in this paper, distinct from the results obtained by starting directly with the standard form of the non-relativistic hamiltonian (equation (14)). These effects, which are related to the phase factor given by equation (13), are lost when starting with the hamiltonian given by equation (14). Apart from other peculiar points (like the selection of transition amplitudes associated with n-photon exchange), the standard non-relativistic hamiltonian is the starting point in Refs. [15]-[22]. On the contrary, for problems related to quantum-mechanical bound-states in radiation field it is convenient to start with the standard form of the non-relativistic hamiltonian (equation (14)), which includes the radiation effects in the momentum $\mathbf{P} = M\mathbf{v} + (Ze/c)\mathbf{a}$ (the phase factor given in equation (13) has little effect in this case).

Comment. The quasi-classical approximation is not appropriate, since the projectile has still a quantum-mechanical behaviour along its trajectory; the energy is not resolved in equation (23).

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