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Seismic moment deduced from quasi-static surface displacement in seismogen zones

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Abstract

It is shown how to determine the seismic-moment tensor of a shear faulting from measurements of the (quasi)-static displacement in seismogen zones at Earth's surface. This is known as the inverse problem in Seismology. In a previous paper its solution has been given for measurements of the displacement vector at Earth's surface produced by seismic waves (an earthquake). Usually, the problem is treated in non-covariant forms, which leads to results dependent on the reference frame. Here we describe a manifestly covariant procedure.

Introduction. It is widely admitted that the continuous accumulation of the tectonic strain may be gradually dis-charged, to some extent, causing quasi-static crustal deformations of the Earth's surface in seismogen zones.[1]-[7] We show here that measurements of these deformations (which are very small) may give, besides qualitative information about the seismic activity, the depth of the focus, and the focal volume, the opportunity to determine the tensor of the seismic moment for a shear faulting. This is known as the inverse problem in Seismology. Usually, it is tackled by non-covariant forms of the experimental data, which produce results dependent on the reference frame. Since the seismic moment is a tensor, it is emphasized here that only covariant forms of the data are relevant for its determination. This covariant procedure has been described in a previous paper for data provided by measurements of the displacement vector at Earth's surface in the seismic waves produced by an earthquake.[8] Here we solve the similar problem for (quasi)-static displacement in seismogen zones for a seismic moment correspondng to a shear faulting. We use the displacement derived previously for a homogeneoaus isotropic half-space with tensorial point forces generated by a seismic moment in a focus localized inside the half-space.[9]

Static displacement. In Ref. [9] the static deformations produced by a tensorial point force in a homogeneous isotropic elastic half-space have been computed. The equation of elastic equilibrium with the force density $\overline{\mathbf{f}}$ is

$$\Delta\mathbf{u}+\frac{1}{1-2\sigma}\mathrm{graddiv}\mathbf{u}=-\frac{2(1+\sigma)}{E}\mathbf{f}\ \ , \tag{1}$$

where $\mathbf{u}$ is the displacement vector (with components u_i, $i=1,2,3$), E is the Young modulus and σ is the Poisson ratio. The components of the force are given by

$$f_i=M_{ij}\partial_j\delta(\mathbf{r}-\mathbf{r}_0)\ \ , \tag{2}$$

where $\mathbf{r}_0$ is the position of the focus and M_{ij} is the tensor of the seismic moment. It is convenient to write $\overline{\mathbf{f}}=-\left[2(1+\sigma)/E\right]\mathbf{f}$ and $\overline{M}_{ij}=-\left[2(1+\sigma)/E\right]M_{ij}$ (reduced force and seismic moment).

Equation (1) is solved for a half-space $z<0$, with free surface $z=0$, the position of the focus being $\mathbf{r}_0=(0,\,0,\,z_0)$, $z_0<0$; we use the radial coordinate $\rho=(x^2+y^2)^{1/2}$ for the in-plane coordinates x, y and $x_1=x$, $x_2=y$, $x_3=z$. The coordinates of the displacement vector of the surface $z=0$ are given by[9]

$$\begin{gathered}2\pi\cdot u_\alpha=-\overline{M}_{\alpha\beta}I_\beta^{(1)}+\overline{M}_{\alpha 3}I^{(0)}-\tfrac{1}{2}\overline{M}_{\beta\gamma}\partial_\beta\partial_\gamma[2\sigma I_\alpha^{(3)}-z_0I_\alpha^{(2)}]\\-z_0\overline{M}_{3\beta}\partial_\beta I_\alpha^{(1)}+\tfrac{1}{2}\overline{M}_{33}[2\sigma I_\alpha^{(1)}+z_0I_\alpha^{(0)}]\ ,\\2\pi\cdot u_3=-\tfrac{1}{2}\overline{M}_{\alpha\beta}\partial_\beta[(1-2\sigma)I_\alpha^{(2)}+z_0I_\alpha^{(1)}]\\+z_0\overline{M}_{3\alpha}I_\alpha^{(0)}+\tfrac{1}{2}\overline{M}_{33}[(1-2\sigma)I^{(0)}-z_0\tfrac{\partial}{\partial z_0}I^{(0)}]\ ,\end{gathered}\tag{3}$$

where

$$\begin{gathered}I^{(0)}=-\tfrac{z_0}{r^3}\ ,\ \ I^{(1)}=\tfrac{1}{r}\ ,\ \ I_\alpha^{(2)}=-\tfrac{x_\alpha}{r(r+|z_0|)}\ ,\\I_\alpha^{(3)}=-\tfrac{x_\alpha}{r+|z_0|}\ ,\end{gathered}\tag{4}$$

$I_\alpha^{(n)}=\partial_\alpha I^{(n)}$ $(n=0,\,1,\,2,\,3)$ and $r=(\rho^2+z_0^2)^{1/2}$; we use $\alpha,\,\beta,\,\gamma=1,2$. The components u_α are vanishing for $\rho\longrightarrow 0$ and go like $1/\rho^2$ for $\rho\longrightarrow\infty$; they have a maximum value for ρ of the order $\mid z_0\mid$. The component u_3 goes like $1/z_0^2$ for $\rho\longrightarrow 0$ and $1/\rho^2$ for $\rho\longrightarrow\infty$. It is convenient to give these displacement components for ρ close to zero, *i.e.* in the seismogen zone (close to a presumable epicentre). We get

$$\begin{gathered}u_\alpha=\tfrac{1}{16\pi}\left[4(1-2\sigma)\overline{M}_{33}-(3+2\sigma)\overline{M}_0\right]\tfrac{x_\alpha}{|z_0|^3}+\tfrac{1}{8\pi}(1-2\sigma)\tfrac{\overline{M}_{\alpha\beta}x_\beta}{|z_0|^3}+...\ ,\\u_3=\tfrac{1}{8\pi z_0^2}\left[2(3-2\sigma)\overline{M}_{33}-(1+2\sigma)\overline{M}_0\right]+\tfrac{\overline{M}_{3\alpha}x_\alpha}{2\pi|z_0|^3}+...\ ,\end{gathered}\tag{5}$$

where $\overline{M}_0=\overline{M}_{ii}$ is the trace of the tensor $\overline{M}_{ij}$. Equations (5) depend on the reference frame (their form is not covariant); consequently, we cannot use them for determining the components of the seismic-moment tensor.

A simplified numerical estimation of these unknowns can be obtained as follows. We assume $M_0=0$ (as for a shear faulting), replace all the components of the seismic-moment tensor in equations (5) by a mean value $\overline{M}$ and average over the orientation of the vector $\boldsymbol{\rho}$; we denote the resulting u_3 by u_v (vertical component) and introduce u_h(horizontal component) by $u_h=\left(u_1^2+u_2^2\right)^{1/2}$; we get approximately

$$u_h\simeq\frac{(1-2\sigma)\left|\overline{M}\right|}{4\pi}\frac{\rho}{\mid z_0\mid^3}\ ,\ \ u_v\simeq\frac{(3-2\sigma)\overline{M}}{4\pi z_0^2}\ ;\tag{6}$$

hence, we get immediately the depth of the focus

$$\mid z_0\mid\simeq\frac{1-2\sigma}{3-2\sigma}\left|u_v\right|/(\partial u_h/\partial\rho)\tag{7}$$

and the mean value $\overline{M}=4\pi z_0^2u_v/(3-2\sigma)$ of the (reduced) seismic moment. Making use of $\overline{M}_{ij}=-\left[2(1+\sigma)/E\right]M_{ij}$ we have

$$M_{av}\simeq-\frac{2\pi E}{(1+\sigma)(3-2\sigma)}z_0^2u_v\tag{8}$$

for the mean value M_{av} of the seismic moment M_{ij}. For $M_{av}=10^{22}dyn\cdot cm$ (which would correspond to an earhquake with magnitude $M_w=4$ by the Gutenberg-Richter law $\lg M_{av}=$

$1.5M_w + 16.1$), Young modulus $E = 10^{11} dyn/cm^2$, $\sigma = 0.25$ and depth $\mid z_0 \mid = 100km$ we get a vertical displacement $u_v \simeq 1\mu m$; we can see that the static surfece displacement is very small.

A rough estimate for the elastic energy stored by the static deformation is given by $\mathcal{E} \simeq 4\pi z_0^2 E \mid u_v \mid \simeq 2(1+\sigma)(3-2\sigma) \mid M_{av} \mid$; it is also given by $\mathcal{E} \simeq \mu V$, where μ is the Lame coefficient and V is the focal volume ($\mu = E/2(1+\sigma)$; the other Lame coefficient is $\lambda = E\sigma/(1-2\sigma)(1+\sigma)$); making use of the approximations introduced above, we get $V \simeq 8\pi(1+\sigma)z_0^2 \mid u_v \mid$. For $\mid z_0 \mid = 100km$ and $u_v = 1\mu m$ ($\sigma = 0.25$) we get a volume $V \simeq 10^5 \pi m$, *i.e.* a linear dimension $l \simeq 500m$. Similarly, from equations (5) we get an estimate $u_{ij} \sim V/\mid z_0 \mid^3$ for the surface strain; using the numerical data above, it is ofÅ order.

The displacement components given by equation (5) can be written in a covariant form ($M_0 = 0$) as

$$u_i = \left\{ \left[2(3-2\sigma)\overline{M}_4^{(n)} - (9-10\sigma)\overline{M}_4^{(nv)} \right] n_i - 4\overline{M}_4^{(n)} v_i + (1-2\sigma)\overline{M}_{ij} v_j \right\} \frac{1}{8\pi z_0^2} \quad , \tag{9}$$

where

$$\begin{gathered} \mathbf{n} = (x_\alpha,\, z - z_0)/\mid z_0 \mid \; , \;\; \mathbf{v} = (x_\alpha,\, z)/\mid z_0 \mid \; , \\ \overline{M}_4^{(n)} = \overline{M}_{ij} n_i n_j \;\; , \;\; \overline{M}_4^{(nv)} = \overline{M}_{ij} n_i v_j \;\; ; \end{gathered} \tag{10}$$

in equations (9) and (10) we retain only contributions linear in x_α and in the limit $z \to 0$. Within these restrictions the form given by equation (9) is unique. In these equations

$$\overline{M}_i = \overline{M}_{ij} v_j \simeq \frac{\overline{M}_{i\alpha} x_\alpha}{\mid z_0 \mid} \tag{11}$$

are the components of a vector and

$$\overline{M}_4^{(n)} \simeq 2\overline{M}_3 + \overline{M}_{33} \;\; , \;\; \overline{M}_4^{(nv)} \simeq \overline{M}_3 \tag{12}$$

are scalars. Taking the scalar product $\mathbf{nu} \simeq u_3$ in equation (9), we get

$$\overline{M}_4^{(n)} = \frac{4\pi z_0^2 u_3 + 4(1-\sigma)M_3}{3-2\sigma} \quad ; \tag{13}$$

inserting this $\overline{M}_4^{(n)}$ and $\overline{M}_4^{(nv)} \simeq \overline{M}_3$ in equation (9) we get

$$u_\alpha = \frac{1-2\sigma}{3-2\sigma} \frac{x_\alpha}{\mid z_0 \mid} u_3 + \frac{1-2\sigma}{8\pi z_0^2} \overline{M}_\alpha \tag{14}$$

(and the identity $u_3 = u_3$). This equation gives

$$\overline{M}_\alpha = 8\pi z_0^2 \left(\frac{1}{1-2\sigma} u_\alpha - \frac{1}{3-2\sigma} \frac{x_\alpha}{\mid z_0 \mid} u_3 \right) \tag{15}$$

(and $M_\alpha = -[E/2(1+\sigma)]\, \overline{M}_\alpha$) as functions of the measured quantities u_α, u_3 and x_α; $\overline{M}_4^{(nv)}$ and $\overline{M}_4^{(n)}$ are given by equations (12) and (13) as functions of u_3 and the parameter $\overline{M}_3$. This is the maximal information provided by measuring the static displacement in a seismogen zone; the parameter z_0 remains undetermined; we can use its numerical estimation given above (equation (7)).

Seismic moment. We assume that the components M_α of the vector $\mathbf{M}$ are determined from data, according to equation (15); the component M_3 will be determined shortly. The scalars

$M_4^{(nv)} \simeq M_3$ and $M_4^{(n)}$ are given by equations (12) and (13), respectively; they depend on the parameter M_3. Parameters z_0 (focus depth) and the focal volume V remain undetermined. Order-of-magnitude estimations given above (equation (7) and below) may be used for them.

In order to determine the seismic moment we use its expression derived in Ref. [8] for a shear faulting. According to Ref. [8] this tensor is given by

$$M_{ij} = M^0(s_i a_j + s_j a_i) \ , \ i, j = 1, 2, 3 \ , \tag{16}$$

where $M^0 = 2\mu V$ and s_i, a_i are the components of two orthogonal unit vectors $\mathbf{s}$ and $\mathbf{a}$: $\mathbf{s}$ is normal to the fault plane and $\mathbf{a}$ is directed along the fault displacement (fault sliding).[10] We can see that equation (10) implies $M_0 = M_{ii} = 0$. We assume that the measured data of the static displacement satisfy this condition. In addition, we assume that M^0 is a known parameter.

We introduce the scalar products $A = \mathbf{av}$ and $B = \mathbf{sv}$ and write

$$A\mathbf{s} + B\mathbf{a} = \mathbf{m} \ , \ B\mathbf{s} + A\mathbf{a} = \mathbf{v} \tag{17}$$

from equation (16), where $\mathbf{m} = \mathbf{M}/M^0$; we solve this system of equations for $\mathbf{s}$ and $\mathbf{a}$ with the conditions $s^2 = a^2 = 1$, $\mathbf{sa} = 0$. We note that equation (16) is invariant under the symmetry operations $\mathbf{s} \longleftrightarrow \mathbf{a}$ and $\mathbf{s}$, $\mathbf{a} \longleftrightarrow -\mathbf{s}$, $-\mathbf{a}$ (and $\mathbf{s} \longleftrightarrow -\mathbf{a}$); consequently, it is sufficient to retain one solution of the system of equations (17) (it has multiple solutions), all the other being given by these symmetry operations. We get

$$\mathbf{s} = \tfrac{A}{A^2-B^2}\mathbf{m} - \tfrac{B}{A^2-B^2}\mathbf{v} \ , \ \mathbf{a} = -\tfrac{B}{A^2-B^2}\mathbf{m} + \tfrac{A}{A^2-B^2}\mathbf{v} \tag{18}$$

and

$$A^2 + B^2 = m^2 = v^2 \ , \ 2AB = v^2 m_4 \ , \tag{19}$$

where $m_4 = \mathbf{mv}/v^2 = M_{ij}v_i v_j/v^2 M^0$. From $m^2 = v^2$ we get the component M_3 as given by

$$M_3^2 = M^{02}v^2 - M_\alpha^2 \ ; \tag{20}$$

we may take

$$A = v\sqrt{\frac{1+\sqrt{1-m_4^2}}{2}} \ , \ B = sgn(m_4) \cdot v\sqrt{\frac{1-\sqrt{1-m_4^2}}{2}} \tag{21}$$

as a solution of the system of equations (19); this solves the problem of determining the seismic moment from the measurements of the surface static displacement. From equation (16) the seismic-moment tensor is given by

$$M_{ij} = \frac{M^0}{v^2(1-m_4^2)}\left[m_i v_j + m_j v_i - m_4\left(m_i m_j + v_i v_j\right)\right] \ ; \tag{22}$$

the vector $\mathbf{v}$ is known from equation (10) ($z \rightarrow 0$, $v = \rho/ \mid z_0 \mid$) and the vector $\mathbf{m}$ is known from equations (15) and (20) (with z_0 and M^0 known parameters); the scalar m_4 is given by $m_4 = M_\alpha v_\alpha / v^2 M^0$. The component M_3 does not enter the expression of m_4; it is included in M_{ij}. The quadratic form $M_{ij}x_i x_j = const$ is a hyperbola with an arbitrary orientation in space; its asymptotes indicate the fault plane (vector $\mathbf{s}$) and the fault slip (vector $\mathbf{a}$).

The isotropic case $M_{ij} = -M^{is}\delta_{ij}$, where $M^{is} = 2(2\mu + \lambda)V$, implies a surface displacement

$$\mathbf{u} = \frac{M^{is}(1+\sigma)}{4\pi z_0^2 E}\left[(3-10\sigma)\mathbf{n} - (3-\sigma)\mathbf{v}\right] \ , \tag{23}$$

the vector $\mathbf{M}$ being given by $\mathbf{M} = -M^{is}\mathbf{v}$. The energy can be estimated as $\mathcal{E} = M^{is}/2 = 4\pi z_0^2 E \mid u_v \mid$,which leads to a focal volume $V = [4\pi(1+\sigma)(1-2\sigma)/(1-\sigma)]z_0^2 \mid u_v \mid$.

Concluding remarks. It is shown in this paper that the tensor of the seismic moment of a shear faulting can be deduced from measurements of the (quasi)-static displacement at Earth's surface in seismogen zones by using covariant forms of the measured data. The derivation is made possible by using the static deformations derived for a homogeneous isotropic half-space with tensorial point forces generated by a seismic moment in a focus localized inside the half-space. The procedure described here is a solution of the inverse problem in Seismology; its practical application may be hampered by the errors implied by the very small (quasi)-static surface displacements. A similar problem has been solved in Ref. [8] for the surface displacement in the seismic waves produced by an earthquake.

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References

[1] F. C. Frank, "Deduction of earth strains from survey data", Bull. Seism. Soc. Am. **56** 35-42 (1966).

[2] J. C. Savage and R. O. Burford, "Geodetic determination of relative plate motion in central California", J. Geophys. Res. **78** 832-845 (1973).

[3] J. C. Savage, "Strain accumulation in western United States", Ann. Rev. Earth Planet. Sci. **11** 11-43 (1983).

[4] K. L. Feigl, D. C. Agnew, Y. Bock, D. Dong, A. Donnellan, B. H. Hager, T. A. Herring, D. D. Jackson, T. H. Jordan, R. W. King, S. Larsen, K. M. Larson, M. M. Murray, Z. Shen and F. W. Webb, "Space geodetic measurement of crustal deformation in central and southern California, 1984-1992", J. Geophys. Res. **98** 21677-21712 (1993).

[5] S. N. Ward, "A multidisciplinary approach to seismic hazard in southern California", Bull. Seism. Soc. Am. **84** 1293-1309 (1994).

[6] Working Group on California Earthquake Probabilities, "Sesimic hazards in southern California: probable earthquakes, 1994-2024", Bull. Seism. Soc. Am. **85** 379-439 (1995).

[7] J. C. Savage and R. W. Simpson, "Surface strain accumulation and and the seismic moment tensor", Bull. Seism. Soc. Am. **87** 1345-1353 (1997).

[8] B. F. Apostol, "The inverse problem in Seismology. Seismic moment and energy of earthquakes. The seismic hyperbola", J. Theor. Phys. **278** (2017).

[9] B. F. Apostol, "Elastic displacement in a half-space under the action of a tensor force. General solution for the half-space with point forces", J. Elast. **126** 231-244 (2017).

[10] B. V. Kostrov, "Seismic moment and energy of earthquakes, and seismic flow of rock", Bull. (Izv.) Acad. Sci. USSR, Earth Physics, **1** 23-40 (1974) (English translation pp. 13-21).