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On the field-assisted proton emission from the nuclei of the heavy atoms

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Abstract

The possibility of field-assisted proton emission from stable atomic nuclei of heavy atoms in static or oscillating electric fields is discussed. The quasi-classical dynamics of the linearized Thomas-Fermi model is adopted for describing the dynamics of the electron cloud of the heavy atoms in electric fields. It is shown that the electron cloud in the heavy atoms screens to a large extent the electric field acting upon the nucleus, such that, in this respect, we may view all the available electric fields as being low fields. The time-imaginary tunneling method is used for field-asissted proton emission in low fields; it is shown that the emission rate is extremely low.

We consider a heavy atom in an oscillating electric field generated in a laser beam; a typical component of the field is given by the vector potential $\mathbf{A} = \mathbf{A}_0 \cos(\omega t - \mathbf{kr})$, where $\omega = ck$ is the frequency and $\mathbf{k}$ is the wavevector (c denotes the speed of light in vacuum); in the non-relativistic approximation we may neglect the spatial phase and use $\mathbf{A} \simeq \mathbf{A}_0 \cos\omega t$; the electric field is given by $\mathbf{E} = -(1/c)\partial\mathbf{A}/\partial t$, where $\mathbf{E} = \mathbf{E}_0 \sin\omega t$, $\mathbf{E}_0 = \omega\mathbf{A}_0/c$. We consider linear polarization, but the calculations can easily be extended to a general polarization. The non-relativistic approximation for a charge q requires $qA_0/mc^2 \ll 1$ ($qE_0/mc\omega \ll 1$). We limit ourselves to optical lasers, with typical frequency of the order $\omega = 10^{15}s^{-1}$; for electrons ($q = 4.8 \times 10^{-10}esu$) the non-relativistic conditon imposes a maximal field $E_0 \ll 10^8 esu$ (laser intensity $10^{18}w/cm^2$); for protons ($m \simeq 2 \times 10^3$ electron mass), the field is limited by $10^{11}esu$ (intensity $10^{24}w/cm^2$).

The electron cloud of the heavy atom is perturbed by the external electric field and generates an internal field; the motion of the nucleons is perturbed by the total electric field. In low fields the protons oscillate, emit higher-order harmonics of electromagnetic radiation (as the electron cloud does), internal conversion may appear and isomeric states may be affected;[1]-[3] also, spontaneous proton or alpha-particle decay may be affected.[4] We describe here the field-assisted proton emission (charge, in general) from stable nuclei. In strong fields the charges lying high in energy may suffer a fast ejection from bound states.[5] We show here that all the available electric fields may be viewed as being low fields for the atomic nuclei.

The most difficult point in treating the problem stated above is a convenient means for describing the dynamics of the electron cloud in heavy atoms placed in oscillating electric fields. In this respect, we employ the linearized Thomas-Fermi model for heavy atoms.[6] In this model, which ensures the stability of the bound state, the radial distribution of the single-electron states exhibits a maximum at the distance of the order $R = a_H/Z^{1/3}$, where a_H is the Bohr radius and Z ($Z \gg 1$) is the atomic number ($a_H = \hbar^2/mq^2 \simeq 0.53$Å, electron mass $m \simeq 10^{-27}g$, electron charge $q \simeq -4.8 \times 10^{-10} esu$, Planck's constant $\hbar \simeq 10^{-27} erg \cdot s$); the atomic cloud extends up to distances of the order a_H. The ground-state energy of the atom depends on the distance R; in the harmonic approximation the electron cloud exhibits a normal mode (a breathing-type mode) with frequency $\omega_0 \simeq Z \mid q \mid /\sqrt{ma_H^3} \simeq 4.5Z \times 10^{16} s^{-1}$; it correspnds to an energy $\hbar\omega_0 \simeq 28Z(eV)$.[6] A displacement u of the electron cloud obeys the equation of motion $\ddot{u}+\omega_0^2 u = 0$ (we neglect the loss, which is mainly electromagnetic and is very small). We recognize in ω_0 the plasma frequency $\simeq 4\pi nq^2/m$ of a mean electron density $n \simeq Z/R^3 = Z^2/a_H^3$. It corresponds to the atomic giant-dipole oscillations discussed in Ref. [6]. In the presence of an external electric field E oriented along the z-direction the electrons are displaced by u (with fixed nucleus), which produces an energy change $\simeq z^2u^2/R^2$. By averaging over z, we get a factor $1/\sqrt{3}$ in the eigenfrequency ω_0, as expected. It follows that the displacement u obeys the equation of motion $\ddot{u}+\Omega^2 u = qE/m$, where $\Omega = \omega_0/\sqrt{3}$, and the internal field is $E_i = -4\pi nqu$ (polarization $P = nqu$ and the dipole moment $p = Zqu$). For $E = E_0 \sin\omega t$ the solution of this equation is $u = u_0 \sin\omega t$, $u_0 = -(qE_0/m)/(\omega^2 - \Omega^2)$, and the internal field is $E_i = \Omega^2 E/(\omega^2 - \Omega^2)$; the total elecric field acting upon the atomic nucleus is

$$F = \frac{\omega^2}{\omega^2 - \Omega^2} E_0 \sin\omega t \ ; \tag{1}$$

since $\omega \ll \Omega$, we may use the approximation $F \simeq -(\omega^2/\Omega^2)E \simeq -10^{-3}/Z^2$ (where $\omega = 10^{15} s^{-1}$); we can see that the total field acting upon the nucleus is appreciably reduced by the electron screening.

The effect of the electric field on bound-state charges is most conveniently discussed by using a mean-field model of single-particle states for the bound state. In this model the single-particle energy levels are more dense for high-energy particles and well-separated for states lying deep in energy; there exists a large spatial degeneracy and the states may be grouped in energy shells. For static fields F a high-energy charge covers the dimension of the bound-state a in time $\Delta t = a/[(\mid q \mid F/m)\Delta t]$, *i.e.* $\Delta t = \sqrt{ma/\mid q \mid F}$; if $\Delta t \gg \hbar/\triangle\mathcal{E}$, *i.e.* $\mid q \mid Fa \ll \Delta\mathcal{E}^2/(\hbar^2/ma^2)$, where $\triangle\mathcal{E}$ is the separation between the energy levels, the field may be viewed as being introduced adiabatically; it preserves the stationary states and its effects are given by the perturbation theory. Ionization may appear in this case, by tunneling through the potential barrier created by the field in neutral bound states (*e.g.*, atoms), or proton or alpha-particle decay may be slightly affected by the external field.[4] If the field is much higher than the critical field given by this condition, the effect of the field may become comparable with the bound-state effects, and the charge may suffer a fast ejection; the rate of the fast ejection, estimated in Ref. [5], is $1/\tau \simeq (\mid q \mid F/2ma)^{1/2}$. For deep states the above condition becomes more relaxed ($\mid q \mid Fa \ll \hbar^2/ma^2$). Practically for all bound-state charges these conditions are satisfied by any value of the laboratory fields; in particular, the static fields are completely screened for atomic nuclei by the electron cloud.

In oscillating fields with amplitude F_0 a high-energy charge suffers a displacement of the order $\mid q \mid F_0/m\omega^2$; if this displacement is much smaller than the dimension a of the bound state, *i.e.* if $\mid q \mid F_0 a \ll (\hbar\omega)^2/(\hbar^2/ma^2)$, then the charge oscillates, emits higer-order harmonics of electromagnetic radiation and may get ionized through the potential barrier; in neutral bound states (like atoms), this potential barrier is created by the field and the ionization rate can be computed by the method of imaginary-time tunneling;[7]-[9] the low field may also affect the proton or alpha-particle decay rates by small corrections.[4] For protons in atomic nuclei the critical field given by this condition is $F_{0c} = 10^2 esu$. For deep-lying states this condition is relaxed to $\mid q \mid F_0 a \ll \hbar\omega \cdot \Delta\mathcal{E}/(\hbar^2/ma^2)$.[1] For fields much higher than the critical field the charge may suffer again a fast ejection. For protons the non-relativistic condition limits the fields to

[1] Indeed, the time $\Delta t = a/(\mid q \mid F_0/m\omega)$ should be much longer in this case than $\hbar/\Delta\mathcal{E}$, which leads to the

$E_0 = 10^{11} esu$, which corresponds to a very high intensity of the laser beam ($I = 10^{24} w/cm^2$); the screening factor ω^2/ω_0^2 in equation (1) is of the order $\simeq 10^{-7}$ for any reasonable heavy atom; it follows that the field acting upon the atomic nucleus is reduced to $F_0 \simeq 10^4 esu$. Therefore, in the terms of the above discussion, we may consider the atomic nuclei as being placed, approximately, in low fields.

In low fields the imaginary-time tunneling gives the ionization rate

$$w/t_a \simeq (1/t_a) e^{-\frac{\mathcal{E}_b}{\hbar\omega} \ln \frac{2\omega\sqrt{2m\mathcal{E}_b}}{|q|F_0}} \quad , \tag{2}$$

where w is the tunneling probability, t_a is the atempt time (of the order $t_a \simeq 10^{-21} s$[10]) and $\mathcal{E}_b$ is the binding energy of the charge; theresult is valid for $\mid q \mid F_0 \ll \omega\sqrt{m\mathcal{E}_b}$, a condition weaker than the low-field condition (since $\hbar\omega \ll \sqrt{\mathcal{E}_b/(\hbar^2/ma^2)}$). The proton emission rate given by equation (2) is extremely low.

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condition given in the text; the interaction is introduced adiabaticaly in this case and the energy levels have a slight time dependence.