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Derivation of the seismic moment from seismic waves

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Abstract

The tensor of the seismic moment is derived from the displacement caused by the seismic waves on the Earth's surface, by using its vectorial representation for a shear-faulting focal region, energy conservation and a covariance condition. It is assumed that the seismic waves are produced in a homogeneous isotropic body by tensorial forces localized both in space and time. It is shown that only three components of the seismic moment are independent, according to the number of independent parameters in the seismic-wave displacement. Additional information is obtained, regarding the earthquake energy, duration, focal volume, focal strain, the rate of focal strain and focal slip. The results are extended to an isotropic seismic moment, which may correspond to explosions.

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Introduction. This paper presents a derivation of the seismic-moment tensor from the displacement produced by the seismic waves at Earth's surface. This derivation may be viewed as a solution to the inverse problem in Seismology.

It is generally accepted[1]-[5] that typical tectonic earthquakes originate in a localized focal region with dimensions much smaller than the distance to the observation point. Also, it is accepted that the duration of the earthquakes is much shorter than any time of interest. The seismic waves are generated by a tensorial force density

$$F_i = M_{ij}T\delta(t)\partial_j\delta(\mathbf{R}-\mathbf{R}_0) \ , \tag{1}$$

where M_{ij} is the symmetrical tensor of the seismic moment (i, $j = 1, 2, 3$ denote the cartesian coordinates), T is the (short) duration of the earthquake, t denotes the time, $\mathbf{R}$ is the position of the observation point and $\mathbf{R}_0$ is the position of the focus; δ denotes the Dirac delta function. Summation over repeating suffixes is assumed throughout this paper. The force density given by equation (1) ensures a vanishing total force and a vanishing total angular momentum. The force density which implies the δ-functions corresponds to an elementary earthquake. A superposition of such elementary forces describes structured focal regions, extended both in space and time. The function $\delta(t)$ in equation (1) is localized over a short duration T centered on $t = 0$; its magnitude is of the order $1/T$. Similarly, the function $\delta(\mathbf{R}-\mathbf{R}_0)$ is localized over a small volume $V \simeq l^3$ with dimension l, centered on $\mathbf{R}_0$, its magnitude being of the order $1/V$; V is a measure of the focal volume.

The elastic displacement $\mathbf{u}$ produced by the force $\mathbf{F}$ in a homogeneous isotropic body is solution of the equation

$$\ddot{u}_i - c_t^2 \Delta u_i - (c_l^2 - c_t^2)\partial_i div\mathbf{u} \ = \ \frac{1}{\rho} M_{ij} T\delta(t)\partial_j \delta(\mathbf{R}-\mathbf{R}_0) \ \ , \tag{2}$$

where ρ is the density of the body and $c_{l,t}$ are the velocities of the longitudinal and transverse waves; in the far-field region these waves are given by[1]-[5]

$$\mathbf{u}_l = -\frac{T\delta^{'}(t-R/c_l)}{4\pi\rho c_l^3 R} M_4 \mathbf{n} \ , \ \mathbf{u}_t = \frac{T\delta^{'}(t-R/c_t)}{4\pi\rho c_t^3 R} \left(M_4\mathbf{n} - \mathbf{M}\right) \ \ , \tag{3}$$

where $\mathbf{R}$ denotes the position measured from the focus (placed at $\mathbf{R}_0$), $M_i = M_{ij}n_j$ and $M_4 = M_{ij}n_i n_j$, $\mathbf{n} = \mathbf{R}/R$ being the unit vector along the direction from the focus to the observation point. The waves given by equations (3) are shell spherical waves, localized over radial distances of the order $\Delta R \simeq c_{l,t}T$ (thickness of the shell). The longitudinal and transverse waves are known as P- and S-wave, respectively. Since $\delta^{'}(t-R/c_{l,t})$ is of the order $1/T^2$, we may write the displacement produced by these waves as

$$\mathbf{v}_l = -\frac{1}{4\pi\rho T c_l^3 R} M_4 \mathbf{n} \ , \ \mathbf{v}_t = \frac{1}{4\pi\rho T c_t^3 R} \left(M_4\mathbf{n} - \mathbf{M}\right) \ . \tag{4}$$

We consider these displacements as input data for our problem. We can see that they imply three independent parameters: the magnitude of the longitudinal displacement v_l (one parameter) and the transverse vector $\mathbf{v}_t$ (two parameters, $\mathbf{v}_l\mathbf{v}_t = 0$); we consider that the unit vector $\mathbf{n}$ is known. From equations (4) we get

$$\mathbf{M} = -4\pi\rho T R \left(c_l^3 \mathbf{v}_l + c_t^3 \mathbf{v}_t\right) \ \ ; \tag{5}$$

we note, from the second equation (4), that $M^2 > M_4^2$ ($v_t^2 > 0$). Also, we assume that the parameters ρ, $c_{l,t}$ and R are known. The problem is to determine the components M_{ij} of the seismic moment by using equations (5), where the displacement is measured on Earth's surface. This may be viewed as the inverse problem in Seismology. We can see, on one hand, that we need additional information for solving this problem and, on the other hand, only three out of six components M_{ij} are independent. We show below that additional information is provided by the vectorial representation of the seismic moment for a shear-faulting focal region (Kostrov representation), the energy conservation and the requirement of covariance of the equations (with respect to rotations and translations); the covariance condition warrants results independent of the reference frame. The results are extended to the special case of an isotropic seismic moment.

At first sight, we may say that for given displacements $\mathbf{v}_{l,t}$ and given T we may solve equations (5) and get the three independent components of the seismic moment M_{ij}. Unfortunately, leaving aside that the other three components of the seismic moment are left as free parameters by such a procedure, the measurement of the duration T from $\Delta r/c_{l,t}$, where Δr is the projection of ΔR on Earth's surface, is dependent on the local frame, and, consequently, would not provide a suitable input data for covariant equations.

The seismic moment and seismic energy are basic concepts in the theory of earthquakes.[1]-[4] The seismic moment has emerged gradually in the first half of the $20th$ century, the first estimation of a seismic moment being done by Aki in 1966.[6] The relations between the seismic moment, seismic energy, the mean displacement in the focal region, the rate of the seismic slip and the earthquake magnitude are recognized today as very convenient tools for characterizing the earthquakes.[7]-[9]

The inverse (inversion) problem in Seismology[10] is solved usually by determining the seismic-moment components from information provided by far-field seismic waves at different locations and times,[11]-[15] or free oscillations of the earth, long-period surface waves, supplemented, in general, with additional relevant information (constraints; see Ref. [16] and references therein). Besides noise, the information provided by such data may reflect particularities of the structure of the focal region and the focal mechanism which are not included, usually, in equations, like the structure factor of the focal region, both spatial and temporal, or deviations from homogeneity and isotropy. In particular, waves measured at different locations (or times) may lead to overdetermined systems of equations for the unknowns M_{ij}, and the solutions must be "compatibilized". A proper procedure of compatibilization may lead, in fact, to redundant equations, if the covariance of the equations is not ensured. Indeed, the experimental data may often be used in a non-covariant form, which makes the results dependent on the reference frame. We may add that the normal modes of the pure free oscillations do not imply a source of waves, while surface waves, having sources on the surface, have a very indirect connection to the body waves generated in the focal region. Surface displacement in the main shock of an earthquake is often used, which has a very indirect relevance for the earthquake source and mechanism.

Vectorial representation. The additional information needed for solving equations (5) is the vectorial representation of the seismic moment for a fault (Kostrov, dyadic, representation).[1, 2, 8, 9] It reads

$$M_{ij} = 2\mu V(s_i a_j + a_i s_j) \quad , \tag{6}$$

where $\mathbf{s}$ is the unit vector perpendicular to the fault surface, $\mathbf{a}$ is the unit displacement vector along the fault surface ($\mathbf{as} = 1$), μ is the Lame coefficient ($\mu = \rho c_t^2$) and V is the volume of the focal region. It is worth noting an uncertainty (indeterminacy) of the dyadic construction of the seismic-moment tensor given by equation (6). Indeed, first, we can see that the seismic moment is invariant under the inter-change $\mathbf{s} \longleftrightarrow \mathbf{a}$. This means that from the knowledge of the seismic moment M_{ij} we cannot distinguish between the two orthonormal vectors $\mathbf{s}$ and $\mathbf{a}$ (fault direction and fault slip). This symmetry of equation (6) indicates, in fact, that we have formally two mutually perpendicular faults; this may explain the presence of the volume V in equation (8) (instead of the fault area). Another symmetry of the seismic moment given by equation (6) is $\mathbf{s} \longleftrightarrow -\mathbf{a}$ (and $\mathbf{s} \longleftrightarrow -\mathbf{s}$, $\mathbf{a} \longleftrightarrow -\mathbf{a}$), which means that we cannot distinguish between the signs of the vectors $\mathbf{s}$ and $\mathbf{a}$.

A formal derivation of equation (6) is provided by assuming that the torque represented by the seismic-moment components can be written as $M_{ij} = f_i h_j$, where f_i is the i-th component of the force and h_j is the j-th component of the arm of the force. For a fault, the force component can be written as $f_i \simeq 2\mu S u_i^0/l$, where $u_i^0 \simeq l a_i$ is the i-th component of the slip along the fault and $S \simeq l^2$ is the area of the fault (the pre-factor 2 arises from the integration over the two oriented surfaces of the fault). The arm of the force is $h_j \simeq l s_j$, such that we get $M_{ij} \simeq 2\mu V a_i s_j$, which, by symmetrization, leads to equation (6). Also, such qualitative considerations lead to the representation

$$u_{ij}^0 = \frac{1}{2}(s_i a_j + a_i s_j) = \frac{1}{4\mu V} M_{ij} \tag{7}$$

for the focal strain.

We can see that equation (6) reduces the number of independent components of the seismic moment to four; indeed, apart from the parameter V, we have three other independent parameters in equation (6) from the two mutually orthonormal vectors $\mathbf{s}$ and $\mathbf{a}$. The reduction of the number of independent components is reflected by the two conditions $M_0 = M_{ii} = 0$ (traceless tensor M_{ij}) and $M_{ij} s_j s_i = 0$ (or $M_{ij} a_i a_j = 0$).

Energy conservation. As it is well known,[17] from equation (2) we get the law of energy conservation

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\rho\dot{u}_i^2+\frac{1}{2}\rho c_t^2(\partial_j u_i)^2+\frac{1}{2}\rho(c_l^2-c_t^2)(\partial_i u_i)^2\right]- \\ -\rho c_t^2\partial_j(\dot{u}_i\partial_j u_i)-\rho(c_l^2-c_t^2)\partial_j(\dot{u}_j\partial_i u_i)=\dot{u}_iM_{ij}T\delta(t)\partial_j\delta(\mathbf{R}) \ . \tag{8}$$

According to this equation, the external force performs a mechanical work in the focus ($\dot{u}_iM_{ij}T\delta(t)\partial_j\delta(\mathbf{R})$ per unit volume and unit time). The corresponding energy is transferred to the waves (the term in the square brackets in equation (8)), which carry it through the space (the term including the *div* in equation (8)). It is worth noting that outside the focal region the force is vanishing. Also, the waves do not exist inside the focal region. Therefore, limiting ourselves to the displacement vector of the waves, we have not access to the mechanical work done by the external force in the focal region. This circumstance arises from the localized character of the focus.

From equation (8) the mechanical work in the focal region is given by

$$W=\int dt\int d\mathbf{R}\dot{u}_i^0(t)M_{ij}T\delta(t)\partial_j\delta(\mathbf{R}) \ \ , \tag{9}$$

where $\mathbf{u}^0$ is the focal displacement (slip) and $\dot{\mathbf{u}}^0$ is the rate of focal slip; equation (9) can also be written as

$$W=\frac{1}{2}\int d\mathbf{R}u_i^0M_{ij}\partial_j\delta(\mathbf{R}) \ \ , \tag{10}$$

where the estimation of the integral leads to $W\simeq\frac{1}{2}a_iM_{ij}s_j$; using equation (6) we get $W\simeq\mu V=\rho c_t^2V$. We can see that the mechanical work done in the focal region is of the order of the elastic energy stored in this region, as expected.

In the far-field region we can use the decomposition into longitudinal and transverse waves ($curl\mathbf{u}_l=0$, $div\mathbf{u}_t=0$) in equation (8); this decomposition leads to

$$\frac{\partial e_{l,t}}{\partial t}+c_{l,t}div\mathbf{s}_{l,t}=0 \ \ , \tag{11}$$

where

$$e_{l,t}=\frac{1}{2}\rho\left(\dot{\mathbf{u}}_{l,t}\right)^2+\frac{1}{2}\rho c_{l,t}^2\left(\partial_i u_{l,tj}\right)^2 \tag{12}$$

'is the energy density and

$$s_{l,ti}=-\rho c_{l,t}\dot{u}_{l,tj}^f\partial_i u_{l,tj}^f \tag{13}$$

are the components of the energy flux densities per unit time (the flow vectors). From equation (11) we can see that the energy is transported with velocities $c_{l,t}$ (as it is well known). The volume energy $E=\int d\mathbf{R}(e_l+e_t)$ is equal to the total energy flux

$$\Phi=-\int dtd\mathbf{R}\left(c_l div\mathbf{s}_l+c_t div\mathbf{s}_t\right)=-\int dt\oint d\mathbf{S}\left(c_l\mathbf{s}_l+c_t\mathbf{s}_t\right) \ \ . \tag{14}$$

Making use of equations (3), we get

$$E=\Phi=\frac{4\pi\rho}{T}R^2\left(c_lv_l^2+c_tv_t^2\right) \ \ ; \tag{15}$$

this relation gives the energy released by the earthquake in terms of the displacement measured in the far-field region and the (short) duration of the earthquake.

By equating $W=\rho c_t^2V$ obtained above with energy E (equation (15)), we get

$$V=\frac{4\pi}{c_t^2T}R^2\left(c_lv_l^2+c_tv_t^2\right) \ \ , \tag{16}$$

an equation which relates the focal volume to the earthquake duration.

Seismic moment. We turn now to equation (6). Making use of the reduced moment $m_{ij} = M_{ij}/2\mu V$ and $m_i = M_i/2\mu V = M_{ij}n_j/2\mu V$, equation (6) leads to

$$s_i(\mathbf{na}) + a_i(\mathbf{ns}) = m_i \ . \tag{17}$$

By using equations (5) and (16) the components m_i of the reduced moment are given by

$$m_i = -\frac{T^2}{2R} \cdot \frac{c_l^3 v_{li} + c_t^3 v_{ti}}{c_l v_l^2 + c_t v_t^2} \ . \tag{18}$$

We solve here the equations (17) for the unit vectors **a** and **s**, subject to the conditions

$$s_i^2 = a_i^2 = 1 \ , \ \ s_i a_i = 0 \ . \tag{19}$$

Since $M_0 = 0$ and $M^2 > M_4^2$, we have $m_0 = m_{ii} = 0$ and $m^2 > m_4^2$ (where $m_4 = m_{ij}n_i n_j$ and $m^2 = m_i^2$). From equation (18) we have $m_i < 0$. We write equations (17) as

$$\alpha \mathbf{s} + \beta \mathbf{a} = \mathbf{m} \ \ , \tag{20}$$

where we introduce two new notations $\alpha = (\mathbf{na})$ and $\beta = (\mathbf{ns})$. We assume that the vectors **s**, **a** and **n** lie in the same plane, *i.e.*

$$\beta \mathbf{s} + \alpha \mathbf{a} = \mathbf{n} \ . \tag{21}$$

This condition determines the system of equations and ensures the covariance of the solution; it is the covariance condition. From equations (20) and (21) we get

$$2\alpha\beta = m_4 \ , \ \ \alpha^2 + \beta^2 = m^2 = 1 \ . \tag{22}$$

The equality $m^2 = 1$ (covariance condition) has important consequences. It implies $M^2 = (2\mu V)^2$, such that we can write the seismic moment from equation (6) as

$$M_{ij} = M \left(s_i a_j + a_i s_j \right) \ ; \tag{23}$$

it follows the magnitude of the seismic moment $\left(M_{ij}{}^2\right)^{1/2} = \sqrt{2}M$.[18] In addition, from $E = W = \mu V$ (equation (10)) we have $E = M/2 = \left(M_{ij}{}^2\right)^{1/2}/2\sqrt{2}$. The magnitude $\left(M_{ij}{}^2\right)^{1/2} = \sqrt{2}M = 2\sqrt{2}E$ may be used in the Gutenberg-Richter relation $\lg\left(M_{ij}{}^2\right)^{1/2} = 1.5M_w + 16.1$, which defines the magnitude M_w of the earthquake; in terms of the earthquake energy this relation becomes $\lg E = 1.5(M_w - \lg 2) + 16.1$ (where $\lg 2 \simeq 0.3$). We note that an error of an order of magnitude in the seismic moment (M, E, $\left(M_{ij}{}^2\right)^{1/2}$) induces an error $\simeq 0.3$ in the magnitude M_w.

Further, from equation (18), the equality $m^2 = 1$ implies

$$T = (2R)^{1/2} \frac{\left(c_l v_l^2 + c_t v_t^2\right)^{1/2}}{\left(c_l^6 v_l^2 + c_t^6 v_t^2\right)^{1/4}} \tag{24}$$

which gives the duration T of the earthquake in terms of the displacements $v_{l,t}$ measured at distance R. Inserting T in equation (16), we get the focal volume

$$V = \frac{\pi (2R)^{3/2}}{c_t^2} \left(c_l v_l^2 + c_t v_t^2\right)^{1/2} \left(c_l^6 v_l^2 + c_t^6 v_t^2\right)^{1/4} \ \ , \tag{25}$$

the magnitude parameter M and the energy E of the earthquake

$$M=2E=2\mu V=2\pi\rho(2R)^{3/2}\left(c_l v_l^2+c_t v_t^2\right)^{1/2}\left(c_l^6 v_l^2+c_t^6 v_t^2\right)^{1/4} \tag{26}$$

in terms of the displacements $v_{l,t}$ measured at distance R. In addition, eliminating R between equations (24) and (25) we can express the focal volume as

$$V=\frac{\pi T^3}{c_t^2}\cdot\frac{c_l^6 v_l^2+c_t^6 v_t^2}{c_l v_l^2+c_t v_t^2}\ . \tag{27}$$

The solutions of the system of equations (22) are given by

$$\alpha=\sqrt{\frac{1+\sqrt{1-m_4^2}}{2}}\ ,\ \ \beta=sgn(m_4)\sqrt{\frac{1-\sqrt{1-m_4^2}}{2}} \tag{28}$$

and $\alpha\longleftrightarrow\pm\beta$, α, $\beta\longleftrightarrow-\alpha$, $-\beta$. Making use of equations (18) and (24), we get the parameters m_i and m_4

$$m_i=-\frac{c_l^3 v_{li}+c_t^3 v_{ti}}{\left(c_l^6 v_l^2+c_t^6 v_t^2\right)^{1/2}}\ ,\ \ m_4=-\frac{c_l^3(\mathbf{v}_l\mathbf{n})}{\left(c_l^6 v_l^2+c_t^6 v_t^2\right)^{1/2}} \tag{29}$$

in terms of the wave displacements. Finally, we get the vectors

$$\begin{aligned}\mathbf{s}&=\tfrac{\alpha}{\alpha^2-\beta^2}\mathbf{m}-\tfrac{\beta}{\alpha^2-\beta^2}\mathbf{n}\ ,\\ \mathbf{a}&=-\tfrac{\beta}{\alpha^2-\beta^2}\mathbf{m}+\tfrac{\alpha}{\alpha^2-\beta^2}\mathbf{n}\ ;\end{aligned} \tag{30}$$

from equations (20) and (21). These solutions are symmetric under the operations $\mathbf{s}\longleftrightarrow\mathbf{a}$ ($\alpha\longleftrightarrow-\beta$) and $\mathbf{s}\longleftrightarrow-\mathbf{a}$ ($\alpha\longleftrightarrow\beta$, or α, $\beta\longleftrightarrow-\alpha$, $-\beta$). The seismic moment given by equation (23) is determined up to these symmetry operations. Inserting these vectors in equation (23) we get the solution for the components of the seismic moment

$$M_{ij}=\frac{M}{1-m_4^2}\left[m_i n_j+m_j n_i-m_4\left(m_i m_j+n_i n_j\right)\right]\ . \tag{31}$$

Equation (30) is manifestly covariant. The covariance condition $m_i^2=1$ reduces to three the four independent components of the seismic tensor.

Having known M_{ij} and T we may give an estimate of the focal strain $u_{ij}^0=M_{ij}/2M$ and the focal strain rate u_{ij}^0/T; the focal slip is of the order $l\simeq V^{1/3}$, where V is given by equation (25).

The eigenvalues of the seismic moment (given by equation (23)) are $\pm M$ (we leave aside the eigenvalue zero); the corresponding eigenvectors $\mathbf{w}$ are given by $\mathbf{aw}=\pm\mathbf{sw}$, which imply $\mathbf{mw}=\pm\mathbf{nw}$; the vectors $\mathbf{w}$ are directed along the bisectrices of the angles made by $\mathbf{s}$ and $\mathbf{a}$, or $\mathbf{m}$ and $\mathbf{n}$ ($\mathbf{w}\sim\mathbf{s}\pm\mathbf{a}$). The associated quadratic form $M_{ij}x_i x_j=const$ is a rectangular hyperbola in the reference frame defined by the vectors $\mathbf{s}$ and $\mathbf{a}$; by using the coordinates $u=\mathbf{sx}$ and $v=\mathbf{ax}$ in equation (23), the equation of this hyperbola is $uv=const/2M$. Actually, in the local frame (coordinates x_i), the quadratic form $M_{ij}x_i x_j=const$ is a degenerate hyperboloid, consisting of a family of parallel hyperbolas displaced along the third axis (perpendicular to the u- and v-axes). This "seismic hyperbola" is an image of the geometry and the focal mechanism of the fault. Making use of equations (23) and (30), this quadratic form can also be written as

$$2\xi\eta-m_4\left(\xi^2+\eta^2\right)=const\ \ , \tag{32}$$

where the coordinates $\xi = m_i x_i$ and $\eta = n_i x_i$ are directed along the vectors $\mathbf{m}$ and $\mathbf{n}$, respectively. The asymptotics of this hyperbola are $\xi = m_4\eta/\left(1+\sqrt{1-m_4^2}\right)$ and $\eta = m_4\xi/\left(1+\sqrt{1-m_4^2}\right)$ (corresponding to the asymptotics $u = (\alpha\xi - \beta\eta)/(\alpha^2-\beta^2) = 0$ and $v = (-\beta\xi+\alpha\eta)/(\alpha^2-\beta^2) = 0$).

Isotropic seismic moment. An isotropic seismic moment $M_{ij} = -M\delta_{ij}$ is an interesting particular case, since it can be associated with seismic events caused by explosions.[?] In this case the transverse displacement (in equations (4)) is vanishing and $\mathbf{M} = -M\mathbf{n}$, $M_4 = -M$. From equations (4) and (15) we get

$$\mathbf{M} = -4\pi\rho T R c_l^3 \mathbf{v}_l \ , \ \ E = \frac{4\pi\rho R^2}{T} c_l v_l^2 \ . \tag{33}$$

We can see that $\mathbf{v}_l\mathbf{n} > 0$ corresponds to $M > 0$ (explosion), while the case $\mathbf{v}_l\mathbf{n} < 0$ corresponds to an implosion. The focal zone is a sphere and the vectors $\mathbf{s}$ and $\mathbf{a}$ are equal ($\mathbf{s} = \mathbf{a}$) and depend on the point on the focal surface. The force acting on the surface of this sphere generates a pressure, and the seismic moment changes sign in the Kostrov respresentation

$$M_{ij} = -2\rho c_l^2 V \delta_{ij} \ . \tag{34}$$

Similarly, the energy is $E = W = \frac{1}{2}M$ $(M > 0)$, such that, making use of equations (33), we get $c_l T = \sqrt{2Rv_l}$,

$$M = 2\pi\rho c_l^2 (2Rv_l)^{3/2} = 2\rho c_l^2 V \ , \tag{35}$$

and the focal volume $V = \pi(2Rv_l)^{3/2}$. These equations determine the seismic moment and the volume of the focal region from the displacement v_l measured at distance R. A superposition of shear faulting and isotropic focal mechanisms cannot be resolved, because the longitudinal displacement $\mathbf{v}_l$ includes indiscriminately contributions from both mechanisms.

Concluding remarks. This paper presents a derivation of the tensor of the seismic moment from the displacement caused by the seismic waves at Earth's surface (inverse problem in Seismology). The derivation is made for earthquakes produced by tensorial forces localized both in space and time in a homogeneous isotropic body. It makes use of the vectorial representation of the seismic moment for a fault, the energy conservation and manifestly covariant equations. Addditional information regarding the earthquakes energy and duration, as well as the focal volume, the focal strain, the focal slip and the rate of the focal strain and the focal slip is given. The results are extended to isotropic seismic moments, which correspond to explosions.

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