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Qualitative features of the seismic waves on Earth's surface

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Abstract

Qualitative features of the seismic waves on Earth's surface are presented, based on the mathematical results given in Ref. [1]-[3]. It is emphasized that a typical seismogram includes the small P and S waves and the main shock which has a wall-like profile. The displacement, velocity and acceleration may exhibit much enhanced values in the main shock in comparison with the epicentral P and S waves.

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Key words: seismic sources; primary seismic waves; secondary waves; seismic main shock

Introduction. This paper deals with a physical description of the qualitative behaviour of the seismic waves on the Earth's surface. It is based on the mathematical results given in Refs. [1]-[3]. We asssume a tectonic earthquake with a localized focus placed at depth H, and an elastic half-space $z < 0$ with a plane surface $z = 0$ as a model for the region of the earth we are interested in. Also, we assume a small finite duraton T of the earthquake (seimic activity localized in time). Such an earthquake is called elementary earthquake in Refs. [1]-[3].

Seismic focal force. The equation of the elastic waves in a homogeneous, isotropic body was solved by Stokes[4] with a force density $\mathbf{F}\delta(\mathbf{R})$, where $\mathbf{F}$ is the force and the focus is placed at $\mathbf{R} = 0$ (δ is the Dirac delta-function). This force density has the drawback that it is not zero, which would mean an unphysical displacement of the Earth as a whole. Then, the solution $\mathbf{u}(\mathbf{R})$ of the displacement is computed for $\mathbf{R} \to \mathbf{R} + \mathbf{h}$ (*i.e.* $\mathbf{u}(\mathbf{R} + \mathbf{h})$ is computed), where $\mathbf{h}$ is an infinitesimal displacement and the subtraction

$$\mathbf{u}(\mathbf{R} + \mathbf{h}) - \mathbf{u}(\mathbf{R}) \simeq h_i \partial_i \mathbf{u}(\mathbf{R}) \tag{1}$$

is estimated, as for a fault. Since $\mathbf{u}(\mathbf{R})$ is proportional to $\mathbf{F}$, equation (1) includes the product $h_i F_j$, which represents a couple of forces; such a procedure has the drawback that the couple is not zero, which would mean an unphysical rotation of the Earth as a whole. To cure this drawback a local distribution of couples is assumed, with a zero total angular momentum. This is the approach in use today.[5, 6] Meantime, it was noticed that $h_i F_j$ in equation (1) may be replaced by a general tensor M_{ij}; this is the tensor of the seismic moment. If this tensor is symmetric, then the total angular momentum is zero. The force density

$$f_i = m_{ij}(t) \partial_j \delta(\mathbf{R}) \quad , \tag{2}$$

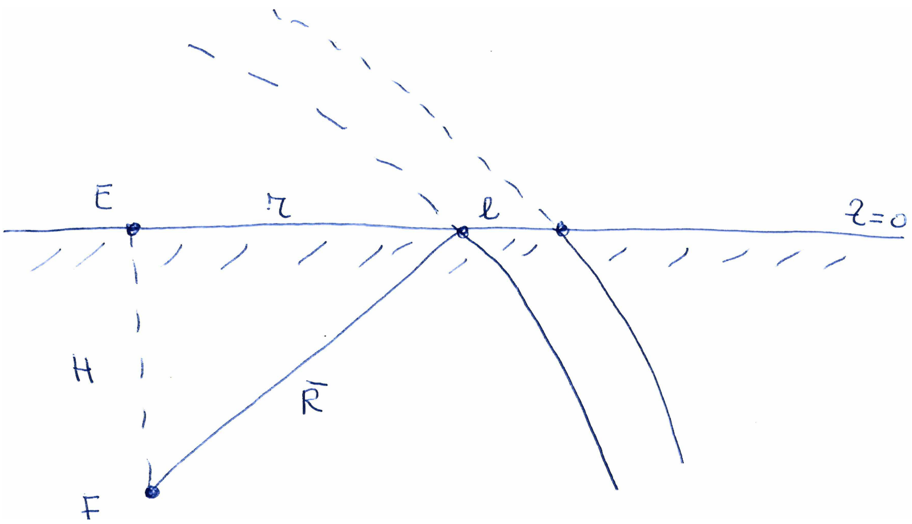

Figure 1: Intersection of a primary wave with Earth's surface ($z = 0$). F denotes the seismic focus placed at depth H, E is the epicentre and l is the width of the shell wave.

where m_{ij} is the symmetric tensor M_{ij} of the seismic moment divided by the density of the body was established in Refs. [1]-[3]. The solution of the equation of the elastic waves in a homogeneous, isotropic body (and a half-space) with the force source given by equation (2) and $m_{ij}(t) = m_{ij}T\delta(t)$ was given in Refs. [1]-[3]. A regularization procedure[7] was needed in order to get the correct solution.

P and S waves. In a homogeneous, isotropic body, in the far-field region, the force density given by equation (2) with $m_{ij}(t) = m_{ij}T\delta(t)$ generates two spherical-shell waves, propagating with velocities c_l and c_t (*e.g.*, $c_l = 5km/s$, $c_t = 3km/s$); the width of these shell waves is $l = c_{l,t}T$. The wave which propagates with the velocity c_l is a longitudinal wave, whose displacement is directed along the radius $\mathbf{R}$, while the wave propagating with the velocity c_t is a transverse wave, which produces a displacement perpendicular to the radius vector $\mathbf{R}$. The longitudial wave is called the P wave in Seismology (primary wave), while the transverse wave is called the S wave (secondary, shear wave). We call both primary waves. They are responsible for the feeble preliminary tremor of an earthquake.[8, 9]

In order to simplify the matters for a qualitative discussion we leave aside the difference between the longitudinal and the transverse waves (velocity c), neglect the vectorial character of the displacement u and use an average m over directions for the sesmic tensor and the displacement components; up to immaterial numerical factors, we write the displacement of a primary wave as

$$u = \frac{Tm}{cR}\delta^{'}(R - ct) \; ; \tag{3}$$

its magnitude is

$$u = \frac{m}{c^2 lR} \tag{4}$$

($\delta \simeq 1/l$).

Interaction with the surface. The intersection of a shell wave with the plane surface of the Earth is shown in Fig. 1. First, we note that the part of the wave lying above the surface (dashed line in Fig. 1) is lost; it is recovered by the wave reflected inside the body. The surface acts as a source of waves, which we call secondary waves. The secondary waves have been computed in Refs. [1]-[3]. They have a special character. Their wavefront is quasi-cylindrical and it accummulates

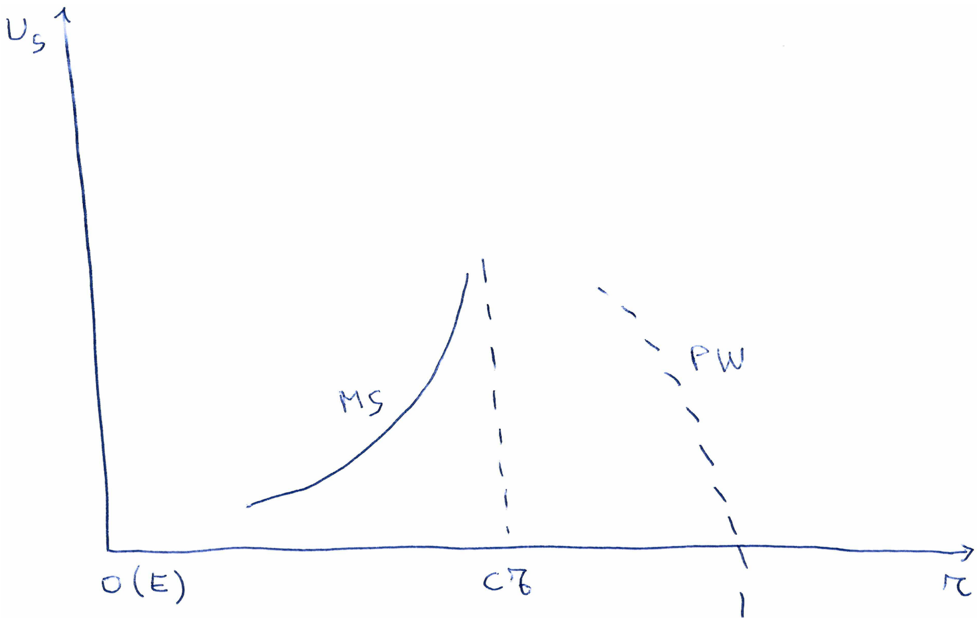

Figure 2: The wall-like profile of the displacement u_s of a secondary wave on Earth's surface (main shock, MS) *vs* the epicentral distance r; τ denotes the propagation time from the epicentre and P denotes the primary wave.

the energy released by the primary waves over an extended surface region. The wavefront of the secondary waves exhibits a characteristic wall-like profile, which is the main shock of the earthquakes. Such a main shock has been noticed in earthquakes and is discussed in Refs. [10]-[14]. The displacement, velocity and acceleration are much higher in the frontwave of the secondary wave (main shock) than in regions close to the epicentre.

We note here a few qualitative features of the secondary waves. First, we note that the distance from the focus to a point on the Earth's surface placed at distance r from the epicentre is

$$\overline{R} = \sqrt{r^2 + H^2} \tag{5}$$

(where H denotes the depth of the focus). It follows that the extension Δr of the spot of the primary wave over the Earth's surface is $\Delta r \simeq l\overline{R}/r$ (in first approximation); we can see that the spot is very large at epicentre and tends to l for large distances r. Next, we note (from equation (5)) that the spot of the primary wave on the Earth's surface propagates with velocity

$$v = c\frac{\overline{R}}{r} > c \quad , \tag{6}$$

which is higher than the velocity c of the elastic waves (primary wave inside the body). It follows that the primary waves come and pass over a small, finite annulus of width l on the Earth's surface and leave behind elastic waves which propagate with the velocity c, smaller than the velocity v of the spot left by the primary wave on the surface. This spot generates secondary waves (propagating with velocity c), which accumulate themselves in a wall-like frontwave on Earth's surface. A typical seismogram is sketched in Fig. 3.

An approximate expression for the magnitude of the secondary waves on Earth's surface (derived from the results given in Refs. [1]-[3]) is

$$u_s \simeq \frac{mH}{c\overline{R}} \cdot \frac{\tau}{(c^2\tau^2 - r^2)^{3/2}} \quad , \tag{7}$$

Figure 3: A sketch of a typical seismogram exhibiting the waves P and S and the main shock MS.

where τ is the time measured from the moment when the primary wave touches the epicentre. This expression is valid for distances r not very small (points placed not too close to the epicentre). We can see that the displacement u_s has a wall-like profile, with a long tail and a singularity at $r = c\tau$; this point is behind the spot left by the primary waves on Earth's surface. The function $u_s(r)$ given by equation (7) is sketched in Fig. 2. For $r > c\tau$ the secondary wave u_s is zero; for any finite r the secondary wave decreases ib time. The singularity occurring at $r = c\tau$ is, in fact, smoothed out by $r = c\tau \simeq l$, such that displacement in the frontwave is

$$u_{sf} = \frac{mH}{c^2 l^3} \cdot \frac{r}{\overline{\overline{R}}} \quad . \tag{8}$$

We can see that the displacement of the main shock increases gradualy with the epicentral distances r, its maximum value being

$$u_{sM} = \frac{mH}{c^2 l^3} = u_E \left(\frac{H}{l}\right)^2 \quad , \tag{9}$$

where u_E is the epicentral displacement of the primary wave given by equation (4). Therefore, we can see that the displacement is much higher at large epicentral distances, as a consequence of the elastic energy accumulated by the main shock. Similar formulae can be obtained for velocities ($\dot{u}_{sM} = \dot{u}_E (H/l)^3$) and accelerations ($\ddot{u}_{sM} = \ddot{u}_E (H/l)^4$).

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