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Seismic source parameters from local seismic recordings. Earthquake of 28.10.2018, Vrancea, Romania

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Abstract

A direct, practical and operative procedure is described for deriving the parameters of the seismic source from the ground displacement of the P and S waves recorded at a local site on Earth's surface for elementary tectonic earthquakes. The procedure gives the seismic-moment tensor, the earthquake energy, the earthquake magnitude, the orientation of the fault and the direction of the tectonic slip, the duration of the focal seismic activity (of an elementary earthquake) and the dimension of the focal region (fault). The theory underlying this procedure is given in Ref. [40]. It includes manifestly covariant equations.

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Introduction. It is widely agreed that typical tectonic earthquakes are produced by a slip of the tectonic plates in a localized fault, placed deeply inside the Earth, beneath Earth's surface. This conception occurred from the "theory" of the motion of the tectonic plates and gave support to this "theory".[1] There exist more complex earthquakes, like surface earthquakes, with an extended, or moving, seismic source, or earthquakes mixed up with volcanic activity. We limit ourselves here to tectonic earthquakes with a localized fault-like seismic source and a finite small duration. We call them elementary earthquakes. Also, as a special case we consider here localized explosions.

A localized fault with a slip of tectonic plates represents the geometry of the seismic focus. Besides the geometry of the seismic focus, the mechanism of occurrence of an earthquake in the seismic focus is equally worth interesting. During the first half of the $20th$ century it emerged gradually that the tensor of the seismic moment governs the force density in the seismic focus (see, for instance, Refs. [2]-[5]). In Refs. [6]-[9] the force density

$$f_i = M_{ij} T \delta(t) \partial_j \delta(\mathbf{R}) \tag{1}$$

has been established for a seismic focus palced at $\mathbf{R} = 0$, with a seismic activity which lasts a short duration T at the moment $t = 0$, where M_{ij} is the seismic-moment tensor; M_{ij} is a symmetrical

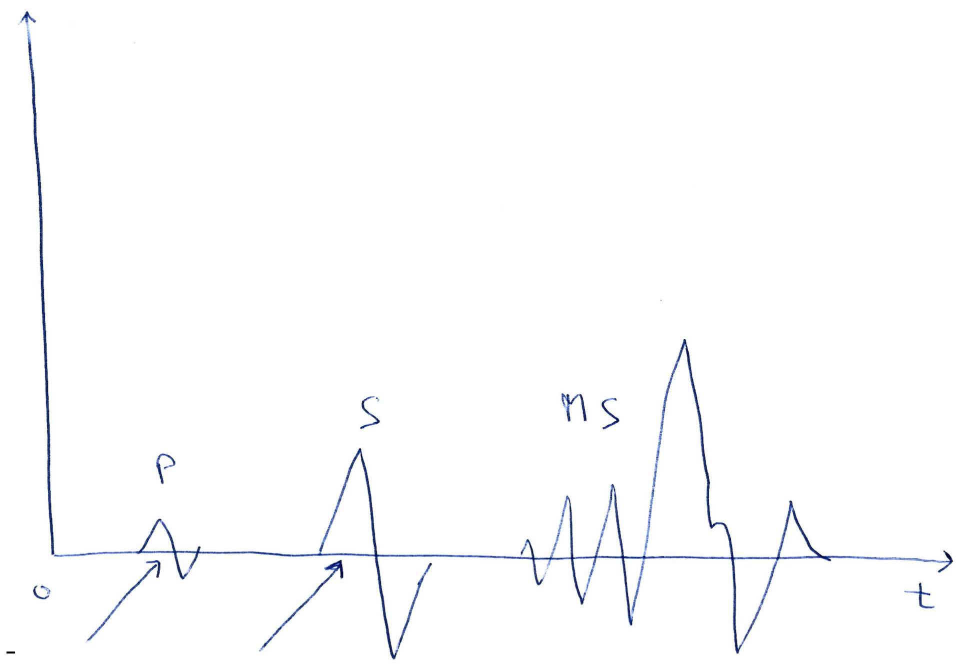

Figure 1: A typical sketch of a seismogram, displaying the P- and the S-wave and the main shock MS. The arrows indicate the "same side" of the P- and S-waves.

tensor. The equation of the elastic waves in a homogeneous, isotropic body has been solved in Refs. [6]-[8] (the static deformations produced by this force density in a homogeneous, isotropic half-space have been computed in Ref. [9]). A homogeneous, isotropic elastic half-space with a plane surface is used as a model for the Earth in the seismic regions of interest. It was shown that the force density given by equation (1) generates in the far-field region two spherical-shell waves, identified as the P (primary, longitudinal) and S (secondary, transverse) seismic waves;[10, 11] we call them primary waves. In addition, the primary waves produce on Earth's surface wave sources, with a cummulative elastic energy, which generate secondary waves; the wavefront of the secondary waves has a wall-like profile on Earth's surface, which is the main shock of the earthquakes.[12]-[16] The displacement, velocity and acceleration of the ground are much enhanced in the mean shock, far away from the epicentre, in comparison with the epicentral primary waves. A typical seismogram is sketched in Fig. 1.

The seismograms includes the ground displacement (velocity, acceleration), recorded locally. The earthquakes produce also long-time vibrations of Earth's surface,[17, 18] which are recorded.[19] The main problem of Seismology is to derive the parameters of the seismic source from such seismic recordings. The parameters of the seismic source include both the geometry of the fault and the focal mechanism. The former is defined by the fault position and orientation, the direction of the seismic slip, an estimate of the dimension of the fault, etc. The latter includes the seismic-moment tensor, the released seismic energy, the magnitude of the earthquake (through the Hanks-Kanamori,[20] Gutenberg-Richter relation[21]-[25]), an estimate of the duration of the activity of the focus, etc.

The parameters of the seismic source are currently computed for every earthquake on the Earth, almost in real time, by various international and national agencies. The Institute of Earth's Physics at Magurele produces such information for Vrancea earthquakes. The computation of the parameters is automatic, by various numerical codes. All these computations share, with variations, a common procedure. The main quantity envisaged by these computation is the seismic-moment tensor.

The determination of the seismic-moment tensor from seismic-wave data recorded at Earth's surface is called the inverse problem of Seismology. For stronger earthquakes (with magnitude higher than 5) seismic-moment tensors are routinely determined from teleseismic data.[26]-[28] For smaller earthquakes regional data are needed, usually long-period waveforms.[29, 30] Simultaneous inversions of body and surface waves are also used,[31] or intermediate-period surface waves.[32] Usually, the solutions are affected by errors and a quality assessment is needed. In such approaches synthetic seismograms with fitting parameters (like, for instance, location coordinates) are compared with data recorded from several stations. Information provided by far-field seismic waves at different locations and times is used,[33]-[38] or free oscillations of the Earth, as well as long-period surface waves, supplemented with additional information (the so-called constraints; see Ref. [39] and references therein). Besides noise, the information used in these procedures may reflect particularities of the structure of the focal region and the focal mechanism which are not included in equations, like the structure factor of the focal region, both spatial and temporal, or deviations from homogeneity and isotropy. Adjusting parameters are then introduced. It is worth noting that waves measured at different locations (or times) may lead to overdetermined systems of equations for the unknowns M_{ij}, and the solutions must then be "compatibilized". A proper procedure of compatibilization may lead, in fact, to redundant equations, if the covariance of the equations is not ensured (in which case it would not be necessary). The experimental data may often be used in a non-covariant form, which makes the results dependent on the reference frame. The covariance is understood in the present paper as the invariance of the form of the equations to translations and rotations (independence of the reference frame). In addition, the normal modes of the pure free oscillations do not imply a source of waves, while surface waves, having sources on the surface, have a very indirect connection to the body waves generated in the focal region. Surface displacement in the main shock of an earthquake is often used, which has a very indirect relevance for the earthquake source and mechanism.

Basically, these approaches use a set of equations for the displacement produced by the seismic waves (or velocities, accelerations), which relate these quantities to the components M_{ij} of the seismic-moment tensor via so-called Green functions. These equations should be covariant, *i.e.* invariant to translations and rotations of the local frame on Earth's surface, a circumstance which, very likely, is overlooked. The seismograms recorded from several stations are decomposed in temporal Fourier components and a class of components is retained which is compared with synthetic seismograms. The synthetic seismograms are fitted to the recorded seismograms and parameters averaged over the data set provided by several stations are given. If the equations are not covariant such an average procedure is inappropriate, while, for a covariant set of equations there is no need to use data recorded from several statiosn.

We present in this paper a direct, practical and operative procedure of deriving the parameters of the seismic source from the ground displacement of the P and S waves recorded at a local site on Earth's surface for elementary tectonic earthquakes. The procedure gives the seismic-moment tensor, the earthquake energy, the earthquake magnitude, the orientation of the fault and the direction of the tectonic slip, the duration of the focal seismic activity (of an elementary earthquake) and the dimension of the focal region (fault). The theory underlying this procedure is given in Ref. [40]. It includes manifestly covariant equations.

Theory. The basic equations used in this paper relate (algebraically) the longitudinal displacement $\mathbf{v}_l$ (P wave) and the transverse displacement $\mathbf{v}_t$ (S wave), measured at a local site on Earth's surface, to the seismic-moment tensor M_{ij} and the duration T of the focal seismic activity. We assume that the other ingredients entering these relations, like Earth's density and wave velocities, are known. Also, we assume that the position of the focus is known, such that we know the unit vector $\mathbf{n}$ from the focus to the origin of the local frame. Consequently, the data include one parameter of the longitudinal displacement (its magnitude) and two parameters of the transverse displacement; this makes three known parameters. In general, the seismic-moment tensor M_{ij} has six components, which, together with the duration T, make seven unknowns. However, for a fault, the Kostrov representation holds for the seismic-moment tensor,[41, 42] which reduces the number of components from six to four; the energy conservation derived in Ref. [40], equates, in fact, one of these components to the earthquake duration (such that the seismic moment has only three independent components for a fault). It follows that we are left with four unknowns and three known parameters (equations). We need a fourth equation in order to solve the problem (*i.e.*, in order to determine the seismic-moment tensor). The fourth equation is provided by the covariance condition, which determines the problem.

Initial input. Data compatibility. We use a local reference frame with axes, denoted by 1, 2, 3, correspondig to the directions North-South, West-East and the local vertical, respectively. Let θ_0 and φ_0 be the latitude and the longitude of the origin of this local frame, respectively. We assume that the latitude θ_E and the longitude φ_E of the epicentre are also known. Then, we determine immediately the coordinates of the epicentre

$$x_1 = -R_0\theta \ , \ \ x_2 = R_0 \cos\theta_E \cdot \varphi \ \ , \tag{2}$$

where $\theta = \theta_E - \theta_0$, $\varphi = \varphi_E - \varphi_0$ (in radians, *e.g.* $\theta = \theta^\circ \cdot \frac{\pi}{180}$) and $R_0 = 6370km$ is Earth's mean radius. Usually, the depth H of the focus is also given by the seismic measurements, such that we might know the unit vector $\mathbf{n}$ directed from the focus to the origin of the local frame. Unfortunately, the measured longitudinal displacement $\mathbf{v}_l$ is not always along the vector $\mathbf{n}$, which raises a problem of compatibility of the data. This is why we prefer to estimate the depth H of the focus. The experimental determination of the depth of the focus may be more affected by errors than the experimental determination of the epicentral coordinates.

The displacements $\mathbf{v}_l$ and $\mathbf{v}_t$ should be measured from the P- and S-waves of the seismograms, respectively, each for all three directions, as the maximum value of the displacement (with its sign) on the same temporal side of the seismogram recordings (along the time axis); these recordings have a scissor-like (double-shock) characteristic pattern. The "same temporal side" means either up to the point where these patterns change sign, or away from that point. The "same side" of the P- and S-waves are indicated by arrows in Fig. 1. The displacements used here are the envelope of the zoomed out oscillatory curves of the P- and S-waves recorded by seismograms. Instead of the maximum values, mean values may also be used. In addition, the sign of the longitudinal components should be compatible with the position of the focus. For instance, for Vrancea earthquakes recorded at Bucharest, the sign of the longitudinal components should be either $(+, -, +)$ or $(-, +, -)$. We call this the sign rule. In practice, if the sign rule is not fulfilled the input data are useless.

Let $\mathbf{f} = (f_1,\, f_2,\, f_3)$ be the longitudinal displacement as read from the seismogram and let $\mathbf{g} = (f_1/f,\, f_2/f,\, f_3/f)$. Then, the coordinates of the epicentre should be given by $-Rg_1$, $-Rg_2$, where R is the distance to the focus; they should be as close as possible to the coordinates $x_{1,2}$, respectively. Therefore, we minimize the quadratic form $(Rg_1 + x_1)^2 + (Rg_2 + x_2)^2$ and get an estimate

$$R_1 = -\frac{g_1 x_1 + g_2 x_2}{g_1^2 + g_2^2} \tag{3}$$

for the focal distance, with a relative error

$$\chi_1 = 1 - \frac{(g_1 x_1 + g_2 x_2)^2}{(g_1^2 + g_2^2)(x_1^2 + x_2^2)} \ \ . \tag{4}$$

Making use of equation (3) we get an estimate

$$H_1 = -\sqrt{R_1^2 - (x_1^2 + x_2^2)} \tag{5}$$

for the depth of the focus.

Let $\mathbf{v}_t$ be the transverse displacement, measured as described above, and let $\mathbf{t} = \mathbf{v}_t / v_t$. It may happen that $\mathbf{f}$ and $\mathbf{v}_t$ are not perpendicular to each other; we define $\sin\phi = \mathbf{gt}$, and it may happen that $\phi \neq 0$. We define the vector

$$\mathbf{n} = \frac{1}{\cos\phi}(\mathbf{g} - \mathbf{t}\sin\phi) \quad , \tag{6}$$

which is perpendicualr to $\mathbf{t}$, and take the longitudinal displacement as

$$\mathbf{v}_l = f\mathbf{n} \quad . \tag{7}$$

We have now the possibility to get another estimate

$$R_2 = -\frac{n_1 x_1 + n_2 x_2}{n_1^2 + n_2^2} \tag{8}$$

of the focal distance and another estimate

$$H_2 = -\sqrt{R_2^2 - (x_1^2 + x_2^2)} \tag{9}$$

of the depth of the focus, with a relative error

$$\chi_2 = 1 - \frac{(n_1 x_1 + n_2 x_2)^2}{(n_1^2 + n_2^2)(x_1^2 + x_2^2)} \quad . \tag{10}$$

Finally, we use the mean values $R = (R_1 + R_2)/2$ and $H = (H_1 + H_2)/2$ for the focal distance and the depth of the focus. In practice, if the angle ϕ is too far from zero, the input data may be discarded since they lead to large errors.

Earthquake energy and magnitude. Focal volume, fault slip. According to Ref. [40] the reduced magnitude of the seismic moment is given by

$$M = (M_{ij}^2/2)^{1/2} = 4\pi\sqrt{2}\rho R^{3/2}\left(c_l v_l^2 + c_t v_t^2\right)^{1/2}\left(c_l^6 v_l^2 + c_t^6 v_t^2\right)^{1/4} \tag{11}$$

and the earthquake energy is

$$E = M/2 \tag{12}$$

(the magnitude of the seismic moment is $\overline{M} = \sqrt{2}M = (M_{ij}^2)^{1/2}$. Using the Gutenberg-Richter (Hanks-Kanamori) law

$$\lg E = 1.5 M_w + 15.65 \tag{13}$$

we derive the (moment) magnitude of the earthquake

$$M_w = \frac{1}{1.5}\left(\lg E - 15.65\right) \quad . \tag{14}$$

In these equations R is the focal distance and $\mathbf{v}_l$ is the longitudinal displacement (P-wave, equation (7)) as determined above; $\mathbf{v}_t$ is the transverse displacement (S-wave), as measured experimentally; ρ is Earth's mean density (we can take $\rho = 5g/cm^3$) and $c_{l,t}$ are the velocities of the longitudinal and transverse waves (we may take $c_l = 7km/s$ and $c_t = 3km/s$). All the equations are written for units cm,g,s.

Similarly, the focal volume is given by

$$V = \frac{M}{2\rho c_t^2} \quad , \tag{15}$$

whence we may infer the dimension of the focal region and the magnitude of the fault slip $l = V^{1/3}$.

Seismic-moment tensor. Focal strain, focal-activity duration. According to Ref. [40], the sesimic-moemnt tensor is given by

$$M_{ij} = \frac{M}{1 - m_4^2} \left[m_i n_j + n_i m_j - m_4 \left(m_i m_j + n_i n_j \right) \right] \quad , \tag{16}$$

where

$$\begin{gathered} m_i = -\frac{c_l^3 v_{li} + c_t^3 v_{ti}}{\left(c_l^6 v_l^2 + c_t^6 v_t^2 \right)^{1/2}} \quad , \\ m_4 = -\frac{c_l^3 v_l}{\left(c_l^6 v_l^2 + c_t^6 v_t^2 \right)^{1/2}} \end{gathered} \tag{17}$$

and $\mathbf{n}$ is given by equation (6). As discussed in **Introduction**, the components M_{ij} can be viewed as generalized force couples, while the vector $\mathbf{m}$ may be viewed as indicating the direction of a "force" acting in the focus; m_4 is a measure of the "force" acting along the observation radius (longitudinal "force"). We can check the traceless condition $M_{ii} = 0$ and the covariance condition $m_i^2 = 1$.

The focal strain is given by

$$u_{ij}^0 = \frac{M_{ij}}{2M} \quad , \tag{18}$$

where $u_{ij} = \frac{1}{2}(\partial_i v_j + \partial_j v_i)$ are the strain components for the displacement vector $\mathbf{v}$ (the superscript 0 stands for the focus). The duration of the seismic activity in the focal region is given by (Ref. [40])

$$T = (2R)^{1/2} \frac{\left(c_l v_l^2 + c_t v_t^2 \right)^{1/2}}{\left(c_l^6 v_l^2 + c_t^6 v_t^2 \right)^{1/4}} \quad ; \tag{19}$$

it is related to the focal volume by

$$V = \frac{4\pi R^2}{c_t^2 T} \left(c_l v_l^2 + c_t v_t^2 \right) \quad ; \tag{20}$$

hence we may estimate the rate of the focal strain u_{ij}^0/T and the rate of the focal slip l/T (during the seismic activity).

It is useful to have a quick and simple estimation of the order of magnitude of the various quantities introduced here. To this end we use a generic velocity c for the seismic waves and a generic vector $\mathbf{v}$ for the displacement in the far-field seismic waves. From the covariance equation $m^2 = 1$ we get immediately $cT \simeq \sqrt{2Rv}$, which provides an estimate of the duration T of the seismic activity in the focus in terms of the displacement measured at distance R. The focal volume can be estimated as $V \simeq \pi \left(2Rv \right)^{3/2} \simeq \pi (cT)^3$, as expected (dimension l of the focal region of the order cT; the rate of the focal slip is $l/T \simeq c$). The earthquake energy is $E \simeq \mu V \simeq M/2 \simeq 2\rho c^2 V$, where $\mu = 2\rho c^2$ is the Lame coefficient and M is the reduced magnitude $\left(M_{ij}^2 \right)^{1/2} = \sqrt{2} M$ of the seismic moment (and the magnitude of the vector $M_{ij} n_j$). The focal strain is of the order unity, as expected. The magnitude of the earthquake is given immediately by equation (14). In addition, we can see the relationship $\lg v = M_w + 10.43 - \lg\left[(2R)(2\pi\rho c^2)^{2/3}\right]$. Hence, we may see that the displacement measured at Bucharest for a Vrancea earthquake of magnitude $M_w = 7$ is of the order $v \simeq 30cm$.

Fault geometry. The geometry of the seismic activity in a fault is characterized by the normal $\mathbf{s}$ to the fault (unit vector) and the slip unit vector $\mathbf{a}$ lying on the fault; these two vectors are mutually orthogonal. According to Kostrov representation (and the covariance condition; see Ref. [40]), the seismic-moment tensor is given by

$$M_{ij} = M(s_i a_j + a_i s_j) \ . \tag{21}$$

We can see the conditions $M_{ii} = 0$ and $M_{ij}s_i s_j = 0$ (or $M_{ij}a_i a_j = 0$), which, together with the covariance condition $m_i^2 = 1$ (where $m_i = M_{ij}n_j/M$; and $m_4 = M_{ij}n_i n_j/M$), lead to three independent components of the tensor M_{ij}. From equation (21) we can see that, apart from the (simultaneous) symmetry operations $s \rightarrow -s$ and $a \rightarrow -a$, which indicate merely a reflection of the fault and the slip, there exists another symmetry given by $\mathbf{s} \longleftrightarrow \mathbf{a}$, which indicates an important uncertainty. Indeed, any fault slip is accompanied by another fault slip, along an orthogonal direction, as a consequence of mattter conservation. It follows that we are not able to make the difference between the direction of the fault and the direction of the slip, because, actually, we have another fault oriented along the slip, and, of course, another slip oriented along the original fault.

According to Ref. [40] the vectors $\mathbf{s}$ and $\mathbf{a}$ are given by

$$\begin{gathered} \mathbf{s} = \frac{\alpha}{\alpha^2-\beta^2}\mathbf{m} - \frac{\beta}{\alpha^2-\beta^2}\mathbf{n} \ \ , \\ \mathbf{a} = -\frac{\beta}{\alpha^2-\beta^2}\mathbf{m} + \frac{\alpha}{\alpha^2-\beta^2}\mathbf{n} \ \ , \end{gathered} \tag{22}$$

where

$$\begin{gathered} \alpha = \sqrt{\frac{1+\sqrt{1-m_4^2}}{2}} \ \ , \\ \beta = sgn(m_4)\sqrt{\frac{1-\sqrt{1-m_4^2}}{2}} \ \ ; \end{gathered} \tag{23}$$

the vector $\mathbf{n}$ is given by equation (6) and the vector $\mathbf{m}$ and the scalar m_4 are given by equation (17). These relations ensure the identity of equation (16) with equation (21). If we define two orthogonal coordinates $u = a_i x_i$ (along the slip) and $v = s_i x_i$ (along the normal to the fault), then the quadratic form $M_{ij}x_i x_j = const$ defines a hyperbola $uv = const/2M$; its asymptotes are directed along the normal to the fault $\mathbf{s}$ and the slip in the fault $\mathbf{a}$. We call it the seismic hyperbola. For high values of the reduced magnitude M of the seismic moment the seismic hyperbola is tight. Actually, for various *const* in $M_{ij}x_i x_j = const$ we get a hyperboloid directed along the third axis $\mathbf{s} \times \mathbf{a}$.

A similar hyperbola may be derived from equation (16) by using the coordinates $\xi = m_i x_i$ (along the vector $\mathbf{m}$) and $\eta = n_i x_i$ (along the vector $\mathbf{n}$); its equation is $2\xi\eta - m_4(\xi^2 + \eta^2) = const$. We recall that $\mathbf{m}$ indicates the direction of a "force" acting in the focus; the angle made by the vectors $\mathbf{m}$ and $\mathbf{n}$ is given by $\cos\chi = m_4$, the angle made by $\mathbf{n}$ and $\mathbf{s}$ (observation radius and the fault direction) is given by $\sin\psi = \sqrt{\left(1 + \sqrt{1-m_4^2}\right)/2}$ and the angle made by $\mathbf{n}$ and $\mathbf{a}$ (observation radius and the fault slip) is $\pi/2 - \psi$.

Explosions. For explosions, which are isotropic, the moment tensor is a scalar. We write it as

$M_{ij} = -M\delta_{ij}$. We have only a longitudinal displacement. The above formulae reduce to

$$M = 2\pi\rho c_l^2(2Rv_l)^{3/2} \ , \ V = \pi(2Rv_l)^{3/2} \ ,$$
$$T = \frac{\sqrt{2Rv_l}}{c_l} \ . \tag{24}$$

The "focal" region for explosions is a sphere. The minus sign in the definition of the moment tensor indicates the fact that the slip on a point of the surface of the "focal" sphere is opposite to the direction of the surface element at that point.

Also, we note that both a seismic shear faulting and an explosion produce a longitudinal displacement, such that their distinct contribution cannot be resolved (a superposition of a seismic shear faulting and an isotropic mechanism - the so-called "hybrid" mechanism - cannot be resolved).

Earthquake of 28.10.2018, Vrancea. The epicentre coordinates are $\theta_E = 45.61°$, $\varphi_E = 26.41°$ and the depth of the focus is $H = -147.8Km$ $(= x_3)$. We use the data from Cernavoda station with coordinates $\theta_0 = 44.3°$, $\varphi_0 = 28.3°$ (coordinates $x_1 = -145.642km$, $x_2 - 125.992km$). The position vector is

$$\boldsymbol{n} = (0.60,\ 0.52,\ 0.61) \ . \tag{25}$$

Within the accuracy used here the vector $\boldsymbol{v}_l$ is directed along the vector $\boldsymbol{n}$, with magnitude $v_l = 0.18cm$, so there is no need to estimate other depths and vectors $\boldsymbol{n}$. The sign rule for Cernavoda is $(+,+,+)$ (or $(-,-,-)$). The vector of the transverse displacement is

$$\boldsymbol{v}_t = (-0.30,\ 0.40,\ -0.08)cm \tag{26}$$

(magnitude $v_t = 0.51cm$) and the angle made by $\boldsymbol{v}_l(\boldsymbol{n})$ with $\boldsymbol{v}_t$ is $\simeq 92°$.

Making use of equations (11)-(15) we get the energy $E = 4.65 \times 10^{23}erg$, the magnitude of the seismic moment $\overline{M} = 1.30 \times 10^{24} erg \cdot cm$, the magnitude of the earthquake $M_w = 5.33$ and the focal volume $V = 9.6 \times 10^{11}cm^3$. The Institute for Earth's Physics, Magurele, announced the magnitude $M_w = 5.5$. We can see that the dimension of the focal volume (the focal slip) is $\simeq 100m$.

Making use of equations (16)-(20) we get the "force" vector

$$\boldsymbol{m} = (-0.46,\ -0.68,\ -0.56) \ , \ m_4 = -0.98 \ , \tag{27}$$

the seismic moment

$$(M_{ij}) = \begin{pmatrix} 1.4 & -7.5 & -1.6 \\ -7.5 & 1.6 & -4.8 \\ -1.6 & -4.8 & -2.8 \end{pmatrix} \times 10^{23} erg \cdot s \tag{28}$$

and the duration of the focal activity $T = 8.7 \times 10^{-3}s$; the focal strain is of the order $10^{-1}cm$, the rate of the focal strain is of the order $10cm/s$ and the rate of the focal slip is of the order $10^6cm/s$. The deviation of M_{ii} from zero in equation (28) is a measure of the error of these estimations.

Using equations (22) and (23), we get the parameters $\alpha = 0.78$, $\beta = -0.63$ and the fault and the slip vectors

$$\boldsymbol{s} = (0.09,\ -0.94,\ -0.26) \ ,$$
$$\boldsymbol{a} = (0.84,\ -0.09,\ 0.57) \ ; \tag{29}$$

these vectors pierce the Earth's surface at $\theta = 46.05°$, $\varphi = 33.38°$ $(\boldsymbol{s})$ and $\theta = 43.67°$, $\varphi = 26.18°$ $(\boldsymbol{a})$.

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Appendix. It may be of interest to determine the points where the vectors **s** and **a** pierce Earth's surface. For this it is necessary to express all the vectors in the reference frame of the Earth (a sphere). We have the vector which determines the origin of the local frame, the vector which determines the focus and the vector **s** (or **a**) with the origin in the focus. The point of interest on Earth's surface corresponds to a vector $\lambda\mathbf{s}$ (or $\lambda\mathbf{a}$), where λ has a well-determined value. We express this vector in Earth's frame and requires it to be on Earth's surface; this condition leads to the equation

$$\lambda^2 + 2\lambda\left[R_0 s_3 - R(\mathbf{ns})\right] - 2R_0 H = 0 \ ;$$

we need to choose for λ the smallest absolute value of the roots.

A simplified version of these calculations can be done for points close to the local observation point and the epicentre, such that we may approximate the Earth's surface by a plane surface. The corresponding equations are

$$H\tfrac{s_1}{s_3} + x_1 = -R_0\theta \ ,$$

$$H\tfrac{s_2}{s_3} + x_2 = R_0 \cos\theta' \cdot \varphi$$

($s_3 > 0$); the coordinates of the intersection point are $\theta' = \theta_0 + \theta$ and $\varphi' = \varphi_0 + \varphi$.

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