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On the Fermi sea-displacement operators as "exact Bose" operators

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Abstract

It is shown that the construction introduced recently by Setlur and Chang (see, for instance, Phys. Rev. **B57** 15 144 (1998)) for generalized Fermi sea-displacement operators contains undefined, ambiguous, and superfluous elements, which may lead to divergencies, and, in fact, these are not bosonic operators. As such, it is hard to see (and to accept) the claim made by these authors of solving the interaction problem in Fermi systems by means of these operators.

In a series of cca 6 recent preprints[1] and one printed paper[2] Setlur and Chang engaged themselves in delineating a general theory, aimed at solving exactly, or, at least, "exceedingly plausibly",[2] the interaction problem in both Fermi and Bose systems, irrespective of the interaction strength (or its sign), and in any spatial dimensions. Such a claim, by itself, would suffice to disqualify the authors, and to discredit their enterprise. Claiming that they draw largely from the work of Castro-Neto and Fradkin,[3] and that they generalize the concepts of Haldane,[4] Setlur and Chang attempt at reworking almost the whole body of the many-body theories in a personal manner, based on the central concept of bosonization.[5] In particular, the single-particle propagator is claimed to be computed "exactly for all wavelengths and energies", including "short-wavelength behaviour",[2] an assertion which is not proven, nor credible. The main point of their approach, that of constructing bosonic operators for Fermi systems, is shown here to be utterly in error.

For Bose systems Setlur and Chang[2] introduce condensate-displacement operators which satisfy Bose commutation relations. Similarly, sea-displacement operators are postulated for Fermi systems, satisfying Bose commutation relations, and it is assumed that products of Fermi operators have the same functional dependence on these operators as for the case of the Bose systems. Making use of the analogy with the Bose systems, the following relations are proposed for Fermi systems:

$$\begin{aligned} c^+_{k+q/2}c_{k-q/2} \;=\; & (\frac{N}{<N>})^{\frac{1}{2}}[\Lambda_k(q)a_k(-q)+a^+_k(q)\Lambda_k(-q)]+ \\ & +T_1(k,q)\sum_{q_1}a^+_{k+q/2-q_1/2}(q_1)a_{k-q_1/2}(q_1-q)- \\ & -T_2(k,q)\sum_{q_1}a^+_{k-q/2+q_1/2}(q_1)a_{k+q_1/2}(q_1-q) \quad , \end{aligned} \tag{1}$$

where c_k are Fermi operators (spin label is irrelevant here and, therefore, it is omitted), $a_k(q)$ are sea-dispacement operators,

$$[a_k(q), a^+_{k'}(q')] = \delta_{kk'}\delta_{qq'} \ , \ [a_k(q), a_{k'}(q')] = 0 \ , \tag{2}$$

$$a_k(0) = 0 \ , \tag{3}$$

and the coefficients T_1, T_2 and Λ are given by:

$$\begin{aligned} T_1(k,q) &= \sqrt{1-\overline{n}_{k+q/2}}\sqrt{1-\overline{n}_{k-q/2}} \ , \\ T_2(k,q) &= \sqrt{\overline{n}_{k+q/2}\overline{n}_{k-q/2}} \ , \\ \Lambda_k(q) &= \sqrt{\overline{n}_{k+q/2}(1-\overline{n}_{k-q/2})} \ ; \end{aligned} \tag{4}$$

n_k in the above formulas represents the Fermi occupation number (occupation number operator), $\overline{n}_k$ is its expectation value on the ground-state, N stands for the operator of the total number of particles and $\langle N \rangle$ denotes the average number of paticles. As one can see, eqs. (1) and (2) provide a bosonic representation of the particle-density operators for Fermi systems. It is claimed that the occupation number itself has a bosonic representation in this theory, given by

$$n_k = n^\beta(k)\frac{N}{< N >} + \sum_q a^+_{k-q/2}(q)a_{k-q/2}(q) - \sum_q a^+_{k+q/2}(q)a_{k+q/2}(q) \ , \tag{5}$$

where

$$n^\beta(k) = \frac{1}{\exp(\beta(\epsilon_k - \mu)) + 1} \tag{6}$$

is the Fermi distribution.

The "exact bosonic" character of the Fermi sea-displacement operators would be embodied in the ansatz[2]

$$a_k(q) = \frac{1}{\sqrt{n_{k-q/2}}} c^+_{k-q/2} M(k,q) c_{k+q/2} \ , \tag{7}$$

where the operator $M(k,q)$ has to be determined in such a way as to ensure the Bose commutation relations required by (2). In the limit of the random-phase approximation (RPA) eq. (7) is written as

$$a_k(q) = \frac{1}{\sqrt{n_{k-q/2}}} c^+_{k-q/2} \left(\frac{n^\beta(k-q/2)}{< N >}\right)^{1/2} e^{i\theta(k,q)} c_{k+q/2} \ , \tag{8}$$

where the phase $\theta(k,q)$ is a functional of the number operator. The authors give nowhere the phase $\theta(k,q)$, and, as such, the central point of their work remain undefined.

Equations (7) and (8) raise several difficulties. First, we note that the Fermi number operator n_k has the idempotency property $n_k^2 = n_k$, and, therefore, $\sqrt{n_{k-q/2}}$ in (7) and (8) might be taken simply as being equal with $n_{k-q/2}$. As such, this element in the construction given by (7) or (8) is superfluous. If a formal proof would still be required, we note here that, indeed, a general operatorial function $f(A)$ may formally be represented by the associated Taylor series

$$f(A) = \sum_{m=0} \frac{f^{(m)}(a)}{m!}(A-a)^m \ , \tag{9}$$

for instance; and for $A = n_k$ and $a = 1$ one obtains from (9)

$$f(n_k) = f(1) + (1 - n_k)(f(0) - f(1)) \quad , \tag{10}$$

which, for the particular case $f = \sqrt{x}$, leads to $\sqrt{n_k} = n_k$. Moreover, the factor $1/\sqrt{n_{k-q/2}}$ in (7) and (8) implies, as it is written, a division by zero, since the fermion occupation number may have a vanishing eigenvalue, and, consequently, this factor may lead to divergencies. Therefore, a certain sense must be attached to this writing, but the authors do not care to say which one; as such, this remains an ambiguous element in their construction. If, for instance, $n_{k-q/2}$ in this factor is replaced by $n_{k-q/2} + \varepsilon I$, where I is the identity operator, and the limit $\varepsilon \to 0$ is taken at the end of the calculations, one obtains

$$\frac{1}{\sqrt{n_{k-q/2}}} \to \frac{1}{n_{k-q/2} + \varepsilon I} = \frac{1}{\varepsilon}(I - \frac{1}{\varepsilon + 1} n_{k-q/2}) \quad , \tag{11}$$

and

$$\frac{1}{\sqrt{n_{k-q/2}}} c^+_{k-q/2} \to \frac{1}{\varepsilon}(I - \frac{1}{\varepsilon + 1} n_{k-q/2}) c^+_{k-q/2} = \frac{1}{\varepsilon + 1} c^+_{k-q/2} \to c^+_{k-q/2} \quad , \tag{12}$$

as expected, *i.e.* the factor $1/\sqrt{n_{k-q/2}}$ in (7) and (8) is superfluous. Of course, the same result is obtained working with the function $f = 1/\sqrt{x + \varepsilon}$, and using the expansion (9). Then, equation (8) becomes

$$a_k(q) = c^+_{k-q/2} (\frac{n^\beta(k - q/2)}{< N >})^{1/2} e^{i\theta(k,q)} c_{k+q/2} \quad , \tag{13}$$

and using the fact that $\theta(k, q)$ is a functional of the number operator, as declared by the authors, one obtains straightforwardly

$$[a_k(q), a^+_k(q)] = \frac{n^\beta(k - q/2)}{< N >} (n_{k-q/2} - n_{k+q/2}) \quad . \tag{14}$$

Obviously, this is not a bosonic commutation relation as required in (2), and, consequently, it is hard to see any consistency, both in the approach, the computations, and in the intentions of these authors. Of course, future publications, which would "bend the rules"[2] in order to "capture what one is looking for",[2] may try to clarify such points, but we doubt much that they would succeed. These authors might try to suggest that (14) would become boson-like commutations relations when averaged over the Fermi sea; if so, we draw the attention that this would be at variance with their own claim that the new Fermi-sea displacement operators are "no longer restricted to be close to the Fermi surface".[2] Moreover, the sea-displacement operators defined by (13) are only consistent with (5) for

$$\frac{n_k}{< N >} \left[\sum_{k_1} n_{k_1} n^\beta(k_1) - N n^\beta(k) \right] = 0 \quad , \tag{15}$$

which requires $n^\beta(k_1) = n^\beta(k) = constant$, *i.e.* the absence of the Fermi surface. We can not refrain ourselves from emphasizing the apparent "consistency" of such a conclusion: indeed, if the fermions are described entirely and exactly in terms of bosons, there would be no Fermi surface at all, since, indeed, bosons have no Fermi surface.

The above considerations are not restricted to the RPA limit. Indeed, making use of (12) the general ansatz expressed in (7) becomes

$$a_k(q) = c^+_{k-q/2} M(k, q) c_{k+q/2} \quad ; \tag{16}$$

let $|v>=|1,1,1,...,1,0_{k-q/2},1,...1,0,0,0,....0,1_{k'\neq k+q/2},0,0,0,...>$ be a state vector in the space of the occupation numbers, *i.e.* an empty fermion state at $k-q/2$ below the Fermi surface, and an occupied fermion state at $k^{'}\neq k+q/2$ above the Fermi surface; then, one obtains $<v|[a_k(q),a_k^+(q)]|v>=0$, which, certainly, is at variance with the bosonic character of the sea-displacement operators. This shows again that the theory discussed here is utterly wrong.

In conclusion, one may say that the bosonic construction proposed by Setlur and Chang for Fermi sea-displacement operators[1],[2] is plagued with blatant errors, both at the level of the mathematical and physical consistency, and at the level of the professional requirements a scientific text should fulfill and obey. We can hardly see the necessity of such publications.

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[5] A relevant series of references are included in Refs. 1 and 2 above.

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