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Integrable systems, Mathematical Physics and Statistical Mechanics

M. Apostol
Department of Theoretical Physics Institute of Atomic Physics, Magurele-Bucharest MG-6, POBox MG-35, Romania
email:apoma@theory.nipne.ro

In a recent paper,[1] the lack of visibility of the integrable models in statistical mechanics, and, in general, of mathematical physics, is decried once again. The integrable models in statistical physics include Ising models, Heisenberg models, delta-interacting gases, the 6- and 8-vertex models, the Hubbard models, the Potts models, the fractional (or exclusion)-statistics models, etc; formerly, they were called exactly, or rigorously soluble (or solvable) models. Both physics and mathematics have a meaning, while the mathematical physics has not; or, as far as it has, it is theoretical physics. The lack of meaning of the mathematical physics generates its lack of visibility. The exact, or rigurous results of mathematical physics are meaningless. This amounts, obviously, to saying that we do not know what mathematical physics is. As a matter of fact, the name itself of "mathematical physics" makes no sense; we have atomic physics, for instance, which is that physics which deals with atoms, or nuclear physics, *i.e.* the physics of atomic nuclei; would "mathematical physics" mean the physics which deals with mathematics?. The practitioners of mathematical physics say no, of course, and, as far as we know about physics and mathematics, such a "physics" would be impossible. There may exist, for educational purposes, a discipline called the equations of mathematical physics; these are equations derived in physics or mathematics, and they may form a chapter of either theoretical physics, or mathematics; but, again, the practitioners of mathematical physics deny that they would deal with the equations of mathematical physics. Of course, things which have a meaning, like physics and mathematics, are much easier to be defined, *i.e.* to say what they are, because we understand them; the meaning being understanding; things which are meaningless resist any attempt of being defined; this latter case is the unfortunate case of mathematical physics.

The meaning is the ideas; physics and mathematics have ideas. The main idea of physics is motion; we perceive motion in various ways, and another idea of physics is the measurement concept; both relativity and quantum mechanics came out from the way we conceive the measurement process. Our various perceptions of motion make physics an empirical science; what we think about these perceptions are other ideas of physics, and they produce the various branches of physics; all of them originate in subjective representations of numbers, and at this point physics meets mathematics; and is called theoretical physics. The main idea of mathematics is numbers; we have various ways of representing numbers, all being subjective representations, and these ways produce various branches of mathematics. We say that we understand when we get a glimpse of the idea, and then we say that what we are looking at has a meaning. We have ideas of physics or mathematics, then we follow paths which lead also to ideas of physics and mathematics; if these paths connect the same ideas, then we say that they are correct; if not, we say that the paths are incorrect.

Space and time are mechanics, matter waves are quantum mechanics, chaotical motion is statistical physics, the absolute motion of light is electromagnetism and relativity; the space multiplicity is geometry, the numbers multiplicity is algebra, the numbers continuum is calculus, the numbers structure is number theory; etc. The irreducible ideas behind all of these representations are relations between numbers, these relations being themselves numbers, in their turn. Mathematical physics has no such ideas, or, as far as it has, these are ideas of physics or mathematics. Almost all of the integrable models in statistical physics are defined through particular mathematical relations, which either are not convertible into physical ideas, or are contrary to the general ideas of motion; the resulting mathematical objects are therefore meaningless. Most of these models turn around a certain equation, or transformation, called the starr-triangle equation, which deals with numerical relationships between objects deprived of their physical meaning.

Perhaps one of the most typical example of mathematical physics is the exact solution given by Onsager, and by Yang, to the two-dimesional Ising model; however, the critical temperature was already at that time prety well localized, either by mean-field methods, or by Kramers' estimations (where one of the first versions of starr-triangle transformation appears); the critical exponents were also given quite satisfactorily at that time by the Landau theory of the phase transitions. The subsequent renormalization group is important for devising the method of treating the fluctuations near a phase transition, by emphasizing the ideas of scaling and universality (*i.e.* through advancing the understanding of the Ginsburg-Landau functional), and not as much as for computing "exactly" the critical exponents. An exact computation of a physical phenomenon is not only impossible, but not even desirable; because such a problem is posed in fact in other terms, the terms of the understanding; and computations are only needed (and possible) in order to understand; even in engineering, and technological design, the "exact" computations are carried up to the fullfilment of the original requirements, which is precisely the understanding of the problem; this understanding providing also the possibility of performing such computations. In fact, it is precisely the lack of such an understanding which allows a particular computation to be performed exactly, the exact solution being nothing but a rewrite of the problem. The solutions of a second-order algebraic equation written with radicals are, in fact, the equation cast into another form; the mathematical idea behind is that such an equation has indeed two and only two solutions; which is something else. The Onsager solution tells nothing about the main ideas of phase transitions, namely the interplay between energy and number of statistical configurations (entropy), the nature of the condensed phase and the order parameter, the thermodynamic non-analiticity of the partition function, the scaling of the fluctuations and the correlation length, the universal quartic form of the Ginsburg-Landau functional, the "effective" hamiltonians.

Newton was troubled because he knows that God is perfect and there were long-standing rumors that the planetary orbits made by God would not be perfect circles, nor circles with circles, but rather some lumps; God must have had some other plans, and the universal attraction restored the perfection of God and of the natural world; such a perfection is science, and the harmony of the equations is theoretical physics. This is the kind of ideas we have in physics or mathematics.

Mathematical physics of statistical mechanics claims to have problems; problems can only arise from ideas, and mathematical physics could not possibly have problems; it can only have desires. A list of such "problems" has recently been reviewed.[2] I comment on it briefly here.

It is claimed[3] that we need "to know how to predict long range order"; we already know it, by mean-field and by Landau functional of the order parameter; the various applications of these ideas are still lacking perhaps, to a certain extent, like the effect of dimensionality, the classical gas-liquid or gas, liquid-solid, transitions; as regarding the long-range the corelations brought about by long-range forces are still to be expressed clearly, which is something else. It is also claimed in Ref.3 that the Second Law of Thermodynamics would not be an absolute law, and this would make a problem; of course it is not absolute, it subsists on the chaotical motion, and the problem would be how the chaotical motion, *i.e.* the statistical physics is possible; by itself is the answer. The hope to derive the chaotical motion from mechanics, which is statistical mechanics, which would hopefully lead, among other things, to the increase of entropy, is in vain; mechanics and statistical physics refer to different things. The infringement upon irreversibility of some dynamical systems is also said to be a problem in Ref.3; on the contrary, it is not. Various cases of illustrating the chaos, bistability, etc, may be attractive, but not problematic. "To reformulate the microscopic laws of physics to include probability and time symmetry breaking"[4] is already done, in some sense; in other sense it would be impossible by the very terms of the problem. A statistical mechanics attempt is unreasonable.[5],[6] The relation betwen Bose-Einstein condensation and superfluidity is also viewed in Ref.3 as unsatisfactory, due to the presence of interaction; interaction, however, makes the superfluidity of bosons which tend to condense, and superfluidity and Bose-Einstein condensation are different things, though related. "Where does the line between classical and quantum mechanics really come in" asks Ref.3; well, we know; it is all related with $\hbar$, the correspondence principle and the work of Feynman. Ref.3 asks also why the classical $\int dpdq \cdot \rho \ln \rho$ has to be replaced by the corresponding *trace* formula in quantum mechanics; this is not a question. Finally, the quantum electrodynamics (either relativist or non-relativist) is touched upon in Ref.3, as a totally unsatisfactory theory; however, the mass and charge renormalization deals with the infinities, and the possible divergence of the perturbation series is dealt with by quasi-particles; after all, the virtual states are still there, in the theory. The origin, and possible significance of the infinite quantities may possibly be indeed a problem.

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