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**Some notes on Comment on "Breakdown of Bohr's correspondence principle" J. Theor. Phys. 48 1 (2000)**

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1. The quasi-classical approximation has certain well-defined conditions of validity, as those given in the first paragraph of my Comment (J. Theor. Phys. **48** 1 (2000)). Providing these conditions are satisfied, the quasi-classical approximation is expressed by Bohr-Sommerfeld's quantization rules given by eq. (1) in my Comment. These rules say the integral of the action over the classical trajectory equals Planck's constant $h$ multiplied by $v+\gamma$, wher $v$ is a high quantum number and $\gamma$ is a correcting phase (usually called the "quantum defect"). The condition of validity requires high values of $v$, *i.e.* a large action in comparison to Planck's constant. If this condition is satisfied over a large part of the trajectory and it is violated over a small part of the trajectory, such that the latter is negligibly small in comparison with the former, then the quasi-classical approximation is valid over the whole trajectory, in view of the fact that the action is an integral over the trajectory. This is almost a trivial point.

2. By definition, the correcting phase $\gamma$ is 1/2 at most, or $\pi$, in phase units. If the quantum number $v$ is high enough, *i.e.* if the quasi-classical approximation holds, this phase defect brings only a small, correcting contribution. The quasi-classical approximation may, therefore, be valid, irrespective of the value of the quantum defect. In fact, for central potential $\gamma = 1/2$, and the quasi-classical approximation may work very well, as for Coulomb potential, for instance. As long as the quasi-classical approximation works it can not be broken down by the quantum defect, even if the latter approaches $\pi$. This is another almost trivial point.

3. It is well-known that the quasi-classical approximation breaks down near the turning points of the trajectory. In the second paragraph of my Comment the extent of this breaking-down is estimated, and it is found that for a sufficiently large potential constant $C_n$ this region is sufficiently small to be neglected. Essential for this estimation is the existence of a finite threshold energy $E_0$ for such short-range potentials, a point which is overlooked in the commented paper. The condition of validity of the quasi-classical approximation transcribes into an inequality between $C_n$ and $E_0$, as given in the second paragraph of my Comment ($\left|r-r_t\right|/r_t \ll 1$ or $C_n \left|E_0\right|^{(n-2)/2} \gg 1$).

4. Providing this condition is satisfied, the energy levels of the top spectrum may be obtained within the quasi-classical approximation by a series expansion in powers of energy $E$. As shown in the second paragraph of my Comment I limit myself to the linear approximation to this expansion, as the commented paper gives no indication as to whether the validity conditions of the quasi-classical approximation have been checked out or not for the numerical data employed. Once such an analysis be performed the series expansion in powers of $E$ can be carried out to the necessary

order, and the agreement with the quantum numerical calculations would thereby be obtained. From the numerical data given in paper (Bo Gao, Phys. Rev. Lett. **83** 4225 (1999)) however I would suspect that the validity conditions for the quasi-classical approximation are not met for the top spectrum, so that checking out the coincidence of the numerical results in this case may be pointless. The discrepancy pointed in the paper, between quantum numerical results for the top spectrum and a "quasi-classical" formula which in fact does not apply to the top spectrum, is irrelevant. A comparison with the proper quasi-classical formula may also indicate a discrepancy, as the conditions of applicability of such a formula would not be fulfilled. This would not mean of course that "Bohr's correspondence principle breaks down", as the commented paper claims erroneously, but, simply, the conditions of validity for the correspondence principle are not met by the numerical input data in that particular case analyzed by the paper.

5. In my opinion the erroneous conclusion of the paper originates in a prejudgment, formed probably on the basis of the Coulomb potential, that Bohr's correspondence principle holds unconditionally for the top spectrum. The situation for the short-range potentials is different, and it offers the opportunity of confirming the validity of the quantum mechanics, and of the quasi-classical approximation. The difference consists in the finite threshold energy and in the "fall-on-the-centre" phenomenon.

6. If the potentials $-C_n/r^n, n > 2$ are extended to $r \to 0$, as the commented paper seems to do, the quasi-classical approximation is valid for lower-energy levels, due to the "fall-on-the-centre" phenomenon. This is another point overlooked by the paper (as the "ultraviolet" cut-off is, needed for finite numerical results). This point is shown in the third paragraph of my Comment, where I derive precisely that quasi-classical formula for approximating the lower-energy levels which is the same as the one used by the commented paper for comparison with the numerical results. This formula was in fact derived before in Ref.3 in my Comment (for $n = 3$), and it is confirmed in the commented paper by comparison with quantum numerical results. One can therefore see that, in spite of its claim, the paper does, in fact, confirm the Bohr's correspondence principle.

7. Both the quasi-classical formula for lower-energy levels and that for the top spectrum given in my Comment can straightforwardly be derived by simple calculus, as I carefully indicate in the second and third paragraphs of the Comment. I have refrained to give excessive mathematical details for the sake of the space economy. However, a longer version of the Comment is available upon request (J. Theor. Phys. **46** 1 (2000); **47** 1 (2000)).