Journal of Theoretical Physics

Founded and Edited by M. Apostol **76** (2002)

ISSN 1453-4428

Electric flow through a ferromagnet-superconductor junction

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Abstract

It is investigated the possibility of controlling the electric flow through a ferromagnet-superconductor junction by spin polarization ("field-induced superconducting transistor"-FIST effect). The ferromagnetic and superconducting properties are briefly reviewed, and the ferromagnet-superconductor junction is analyzed. The formation of a perfect contact at such junctions is characterized, and reasons are given that such a contact may support a ballistic regime of transport for the ferromagnetic sample, while the superconducting sample being subject to a diffusive transport. For such a ballistic regime it is shown that the conductivity of the junction increases monotonically with increasing magnetization, including both positive or negative jumps, giving thus the possibility of controlling the flow through magnetization.

1 Introduction

We all want the modern electronic devices be as small as possible in size, while supporting at the same time voltages and electric flows comparable with those at the macroscopic scale. Unfortunately, this trend towards miniaturization raises a problem, because usual conductors diminish the electric resistance while reducing the size. This is why we prefer semiconductors and potential barriers like metal oxides at metal-metal or metal-semiconductor junctions, or inversion layers at semiconducting junctions, for such miniatural electronic devices. Superconductors exhibit natural potential barriers at such junctions, hence the interest in investigating their properties in miniatural electronic devices.

The thermal and electric transport performed by the electron quasi-particles at a conductor-superconductor junction exhibits certain peculiarities, in comparison with the transport between two normal conductors, as a consequence of the presence of the superconducting gap in the quasi-particle spectrum. The quasi-particles are reflected by the superconducting gap, such that a temperature drop occurs at the junction, as well a corresponding counter-flow of heat and charge. This is known as the Andreev reflection, though its resemblance with Seebeck and Peltier effects is striking; a similar behaviour is present for an opposite thermal flow, passing through a superconductor-conductor junction. The passage of an electric flow through a conductor-superconductor junction is accompanied by a voltage drop at the junction and a reflected electric flow propagating backward into the conductor; an opposite electric flow, passing from the superconductor into the conductor, implies a similar voltage drop.

The passage of the electron quasi-particles in such thermal or electric flows into, or from, a superconductor is thought to be affected by the superconducting correlations between the quasi-particles spins. Indeed, usually, a superconductor favours the antiparallel spins, hence the idea that the corresponding jumps into temperature or voltage, as well as the thermal or electric flows themselves, might be controlled, in principle, by a spin polarization. Such a spin polarization occurs naturally into a ferromagnet, hence the investigation of the Andreev reflection at a ferromagnet-superconductor junction; varying the magnetization, by a temperature change slightly below the magnetic critical temperature, and much below the superconducting critical temperature, one might control the flows into, or from, a superconductor, much the same as in a transistor, the magnetization playing the role of a gate voltage; this would be a "field-induced superconducting transistor (FIST)", the transistor effect being induced by the spin-polarization field; all the same, it would be a new device in the spintronics field. It is shown herein that the dependence of the flow on the magnetization resides in the conduction of the ferromagnetic sample, the control being effective in the ballistic regime of transport for the ferromagnetic sample and the diffusive transport regime for the superconducting sample.

One of the major issues in transport phenomena at a junction is the role of the interface. Usually, a more or less extended contact develops at an interface, due to the mutual atomic diffusion of the solids into each other; for an extended contact a "third solid" appears then, in-between the two partners of the junction, with its own contribution to the transport coefficients; however, new junctions can be defined between the original solids and the "third" one, which, now, are almost perfect contacts; the role of such an ideal, perfect contact is therefore essential in describing a junction, it giving rise to an ideal Kapitza resistance; except for this additional, small contribution, the transport is carried through such a junction as for almost equal Fermi levels. In addition, certain matching conditions must be fulfilled at the junction, which imposes certain limitations upon a practical realization of the FIST. The matching conditions are possible for an extended contact, due to the slow spatial variations along it; however, an extended contact may diminish the efficiency of the FIST effect. The presence of an additional potential barrier at the ferromagnet-superconductor is not excluded for the FIST effect, the matching conditions being also fulfilled in this case, though it contributes its own resistance.

2 Ferromagnet

We adopt a Fermi liquid picture for the charge carriers in the ferromagnet;[1] the charge carriers are assumed to be electrons, with an isotropic single-particle energy spectrum $\varepsilon(\mathbf{k})$ labelled by the wavevector $\mathbf{k}$ in the normal (non-ferromagnetic) state; their number is given by $N = Vk_F^3/3\pi^2$, where k_F denotes the Fermi wavevector and V is the volume of the sample; the quasi-particles have a Fermi velocity

$$v_n = \partial\varepsilon/\hbar\partial k\,|_{k=k_F} = \hbar k_F/m^* \quad , \tag{1}$$

where m^* is their effective mass (and $\hbar$ is Planck's constant); the Fermi level is defined by $\mu_n = \varepsilon(k_F)$ (which defines the Fermi surface by fixing μ_n from the number of particles). The function $\varepsilon(\mathbf{k})$ can be derived, within certain limits, from the quasi-classical description of matter aggregation,[2] including the so-called quantal corrections which give discrete energy levels or energy bands from an original quasi-free-particle picture for the electrons; in general, the label $\mathbf{k}$ may not be a wavevector, but we focus here mainly on conducting solids where $\mathbf{k}$ is a wavevector (or so-called a pseudo-wavevector, for crystalline solids); however, the knowledge of the function $\varepsilon(\mathbf{k})$ is not very useful (leaving aside that its knowledge has also an inherent uncertainty), except for

[1] M. Apostol, **The Electron Liquid**, *apoma*, MB (2000) (a).

[2] L. C. Cune and M. Apostol, **Metallic Binding**, *apoma*, MB (2000).

its derivative at the Fermi surface which is related to the quasi-particle velocity according to (1); it is also worth noting that the second-order term in the $\mathbf{k}$-expansion of $\varepsilon(\mathbf{k})$ at the Fermi surface is uncertain due to the interaction effects, and it controls in fact the quasi-particles lifetime.

Below a critical temperature T_m the ferromagnetic state begins to set up; it is characterized by a temperature-dependent gap Δ_m in the single-particle energy spectrum, which reads now

$$\begin{aligned} \varepsilon_1(\mathbf{k}) &= -\Delta_m/2 + \varepsilon(k) \ , \\ \varepsilon_2(\mathbf{k}) &= \Delta_m/2 + \varepsilon(k) \ , \end{aligned} \tag{2}$$

as corresponding to spin up (label 1) and spin down (label 2), respectively. Such a ferromagnetic energy spectrum can be derived within a mean-field theory for exchange interaction of the Hartree-Fock quasi-particles. The number of electrons is given now by

$$N = Vk_{F1}^3/6\pi^2 + Vk_{F2}^3/6\pi^2 \ , \tag{3}$$

and the magnetization reads

$$M = \mu_B(Vk_{F1}^3/6\pi^2 - Vk_{F2}^3/6\pi^2) \ , \tag{4}$$

where $\mu_B = e\hbar/2mc$ is Bohr's magneton (with usual notations $-e$ is the electron charge, m is the electron mass and c denotes the velocity of light).[3] It is convenient to introduce a reduced magnetization defined as $m = M/\mu_B N$, which leads to

$$\begin{aligned} k_{F1} &= k_F(1+m)^{1/3} \ , \\ k_{F2} &= k_F(1-m)^{1/3} \end{aligned} \tag{5}$$

for the two Fermi wavevectors in (3) and (4); equations (3) and (4) can also be recast as

$$\begin{aligned} N &= (Vk_F^3/6\pi^2)[(1+m)+(1-m)] \ , \\ M &= \mu_B(Vk_F^3/6\pi^2)[(1+m)-(1-m)] \ ; \end{aligned} \tag{6}$$

obviously, the relative magnetization varies between 0 and 1, $0 < m < 1$. In the ferromagnetic state there are two types of quasi-particles, corresponding to spin up and spin down, moving with velocities

$$\begin{aligned} v_{F1,2} = v_{1,2} = \partial\varepsilon_{1,2}/\hbar\partial k \left|_{k=k_{F1,2}} \right. = \hbar k_{F1,2}/m^* = \ , \\ = v_n(1 \pm m)^{1/3} \ ; \end{aligned} \tag{7}$$

this is the main point through which the dependence on magnetization is introduced in the thermal or electric flows through the ferromagnet-superconductor junction,[4] together with the m-dependence of the Fermi wavevectors $k_{F1,2}$ given by (5). The Fermi level of the ferromagnetic state is given by

$$\mu_m = -\Delta_m/2 + \varepsilon(k_{F1}) = \Delta_m/2 + \varepsilon(k_{F2}) \ ; \tag{8}$$

hence,

$$\Delta_m = \varepsilon(k_F(1+m)^{1/3}) - \varepsilon(k_F(1-m)^{1/3}) \ ; \tag{9}$$

[3] It is assumed that the gyromagnetic factor is slightly renormalized, as usually, so that we may use the magnetic momentum of the free electrons.

[4] Higher-order corrections to the effective mass as due to the interaction effects on the two ferromagnetic branches of energy levels may be neglected.

this equation determines the temperature dependence of the magnetization m. Indeed, the ferromagnetic gap has a typical dependence $\Delta_m = \Delta_{m0}(1-T/T_m)^{1/2}$ on temperature T close to T_m; for lower temperatures its temperature slope is vanishing, as for a typical mean-field theory. Since $k_{F1,2}$ have a slow dependence on magnetization (except for $k_{F2} = k_F(1-m)^{1/3}$ for $m \sim 1$), we may use the expansion

$$\Delta_m = \frac{2}{3}\hbar v_n k_F m \tag{10}$$

for equation (9); similarly, the Fermi level reads

$$\mu_m = -\Delta_m/2 + \mu_n + \frac{1}{3}\hbar v_n k_F m + O(m^2) = \mu_n + O(m^2) \ , \tag{11}$$

whence one can see that it does not change appreciably in the ferromagnetic state. These m-expansions can be used for small values of m; usually, the ferromagnetic gap Δ_m is smaller than $(2/3)\hbar v_n k_F$, so that the magnetization acquires indeed small values; however, if Δ_m exceeds $(2/3)\hbar v_n k_F$ below a certain (low) temperature then the magnetization stays at unity for vanishing temperatures; there, the expansions (10) and (11) are not valid anymore. It is also worth noting that the Fermi level μ_n has a well-known T^2-correction, which contributes to μ_m together with the temperature dependence of the magnetization; however, both these temperature contributions are small and they may be neglected. Typically, the magnetization m acquires small values for large values of the product $\hbar v_n k_F$ (which may be taken as a measure of the conduction bandwidth) and, viceversa, it acquires higher values for small values of $\hbar v_n k_F$, so that the ferromagnetic gap Δ_m is relatively small in comparison with the Fermi energy (without quantal corrections); this behaviour is consistent with the exchange character of the magnetic interactions, which affects mainly the single-electron states close to the Fermi surface, within a certain, well-determined range.

3 Superconductor

Let $c_{\mathbf{k}}$ be the destruction operator of a quasi-particle state in a normal conductor; it obeys Heisenberg's equation

$$i\hbar\partial c_{\mathbf{k}}/\partial t = \varepsilon(\mathbf{k})c_{\mathbf{k}} = [\mu_n + \hbar\mathbf{v}_n(\mathbf{k}-\mathbf{k}_F)]c_{\mathbf{k}} \ , \tag{12}$$

or, introducing the field operator $\psi(\mathbf{r}) = (1/\sqrt{V})\sum_{\mathbf{k}} c_{\mathbf{k}}e^{i\mathbf{kr}}$ for $\mathbf{k}$ close to $\mathbf{k}_F$,

$$i\hbar\partial\psi/\partial t = (\mu_n - \hbar\mathbf{v}_n\mathbf{k}_F - i\hbar\mathbf{v}_n\partial/\partial\mathbf{r})\psi \ ; \tag{13}$$

it is worth recalling here that the quasi-particles with the energy levels $\varepsilon(\mathbf{k})$ are not independent particles, except for the vicinity of the Fermi surface where their lifetime is infinite. In addition they are wavepackets of plane waves with wavevectors close to each $\mathbf{k}$-wavevector.

Various other elementary excitations can couple to electron quasi-particles; the effect of such a coupling can be written as an electron-electron effective (residual) interaction

$$H_{e-e} = \frac{1}{2}\int d\mathbf{r}d\mathbf{r}' \cdot g(\mathbf{r}-\mathbf{r}')\psi^+_\alpha(\mathbf{r})\psi^+_\beta(\mathbf{r}')\psi_\beta(\mathbf{r}')\psi_\alpha(\mathbf{r}) \ , \tag{14}$$

where α,β are spin labels and $g(\mathbf{r}-\mathbf{r}')$ is a potential (here chosen as spin-independent for simplicity); equation (13) reads now

$$\begin{gathered} i\hbar\partial\psi_\alpha/\partial t = (\mu_n - \hbar\mathbf{v}_n\mathbf{k}_F - i\hbar\mathbf{v}_n\partial/\partial\mathbf{r})\psi_\alpha + \\ + \int d\mathbf{r}' \cdot g(\mathbf{r}-\mathbf{r}')\psi^+_\beta(\mathbf{r}')\psi_\beta(\mathbf{r}')\psi_\alpha(\mathbf{r}) \ ; \end{gathered} \tag{15}$$

such an interaction may lead to superconductivity, by a macroscopic occupation $<\psi_\alpha(\mathbf{r})\psi_\beta(\mathbf{r})>\neq 0$ of the pair states.[5] The pair wavefunction $<\psi_\alpha(\mathbf{r})\psi_\beta(\mathbf{r})>$ breaks the gauge symmetry $\psi\rightarrow\psi e^{i\theta}$, as for a phase-coherent off-diagonal long-range order; actually, the pair wavefunction possesses a crystalline symmetry too, as well as spin-singlet or -triplet, and time reversal symmetries; for a spin triplet the orbital symmetry is odd, while for a spin singlet it is even, according to the parity of the state under space inversion; the s-wave and d-wave symmetries are included among the latter. Here we assume a δ-type interaction $g(\mathbf{r}-\mathbf{r}')=g\delta(\mathbf{r}-\mathbf{r}')$, which makes the pair wavefunction a spin singlet $<\psi_{-\alpha}(\mathbf{r})\psi_\alpha(\mathbf{r})>$; we define $F_\alpha=g<\psi_{-\alpha}(\mathbf{r})\psi_\alpha(\mathbf{r})>$. According to its definition $F_{-\alpha}=-F_\alpha$, while $F^*_{-\alpha}=F_\alpha$ by time reversal symmetry; it follows that we may define the superconducting gap parameter $\Delta_\alpha=\Delta^*_\alpha=-\Delta_{-\alpha}$ (>0) through $F_\alpha=i\Delta_\alpha$. In addition, we include the basic time-dependence $\psi_\alpha\sim e^{-i\mu_n t/\hbar}$ in equation (15), such as the superconducting gap be time-independent; equation (15) describes then the (first-order) perturbations to the superconducting state, *i.e.* its elementary excitations; this amounts to subtracting $\mu_n N$ from the hamiltonian in writing down the equations of motion, as the number N of the elementary excitations is not conserved; equation (15) becomes then

$$i\hbar\partial\psi_\alpha/\partial t=(-\hbar\mathbf{v}_n\mathbf{k}_F-i\hbar\mathbf{v}_n\partial/\partial\mathbf{r})\psi_\alpha+i\Delta_\alpha(\mathbf{r})\psi^+_{-\alpha}(\mathbf{r}) \ ; \tag{16}$$

in addition we assume a constant $\Delta_\alpha=\Delta_\alpha(\mathbf{r})$ as for a s-wave pair state,[6] which is typical for an electron-phonon mechanism of superconductivity; equation (16) leads to

$$\begin{aligned} i\hbar\partial c_{\mathbf{k}\alpha}/\partial t&=\hbar v_n(k-k_F)c_{\mathbf{k}\alpha}+i\Delta_\alpha c^+_{-\mathbf{k}-\alpha} \ , \\ -i\hbar\partial c^+_{-\mathbf{k}-\alpha}/\partial t&=\hbar v_n(k-k_F)c^+_{-\mathbf{k}-\alpha}+i\Delta_\alpha c_{\mathbf{k}\alpha} \end{aligned} \tag{17}$$

for $\mathbf{k}$ along $\mathbf{v}_n$, which are solved for the well-known superconducting spectrum

$$\varepsilon_\pm(\mathbf{k})=\mu_n\pm\sqrt{\Delta^2_\alpha+\hbar^2v^2_n(k-k_F)^2} \ , \tag{18}$$

with (the original) $c_{\mathbf{k}\alpha}\sim e^{-i(\mu_n+\hbar\omega)t/\hbar}$, $c^+_{-\mathbf{k}-\alpha}\sim e^{i(\mu_n-\hbar\omega)t/\hbar}$, and $\hbar\omega=\pm\sqrt{\Delta^2_\alpha+\hbar^2v^2_n(k-k_F)^2}$; the lower branch joints smoothly the rest of the original energy spectrum, so that the superconducting Fermi level is given by[7]

$$\mu_s=\mu_n-\Delta_\alpha \ . \tag{19}$$

It is convenient to measure the wavevectors with respect to the Fermi wavevector, *i.e.* $\varepsilon_\pm(\mathbf{k})=\mu_n\pm\sqrt{\Delta^2_\alpha+\hbar^2v^2_nk^2}$, so that the solutions to equation (17) read

$$\begin{aligned} c_{\mathbf{k}\alpha}&=u_kb_{\mathbf{k}\alpha}+iv_kb^+_{-\mathbf{k}-\alpha} \ , \\ c_{-\mathbf{k}-\alpha}&=u_kb_{-\mathbf{k}-\alpha}-iv_kb^+_{\mathbf{k}\alpha} \ , \end{aligned} \tag{20}$$

where $u_k=|\cos\theta_k|$, $v_k=|\sin\theta_k|$, $\tan\theta_k=-(E_k-\hbar v_nk)/\Delta_\alpha$, $E_k=\hbar\omega=\sqrt{\Delta^2_\alpha+\hbar^2v^2_nk^2}$, or

$$\begin{aligned} u^2_k&=\tfrac{1}{2}(1+\hbar v_nk/E_k) \ , \\ v^2_k&=\tfrac{1}{2}(1-\hbar v_nk/E_k) \ , \end{aligned} \tag{21}$$

[5] L. P. Gorkov, ZhETF **34** 735 (1958) (Sov. Phys.-JETP **7** 505 (1958)).

[6] The high-temperature superconducting cuprate oxides seem to possess a d-wave pairing, arising probably from an electron-lattice interaction with antiferromagnetic fluctuations; see, for instance, C. C. Tsuei and J. R. Kirtley, Revs. Mod. Phys. **72** 969 (2000).

[7] It is worth noting that the formal chemical potential $\partial E/\partial N$ is μ_n; indeed, adding a pair to the ground-state the energy increases by $2\Delta_\alpha$ with respect to the Fermi energy $\mu_s=\mu_n-\Delta_\alpha$, *i.e.* Δ_α per particle (a $b_{\mathbf{k}\alpha}$-quanta of energy is $\hbar\omega$); here one can see part of the pairs preserving their original fermion character, while by their macroscopic occupation they resemble more an ensemble of bosons.

for the energy branch $\varepsilon(\mathbf{k}) = \mu_n + sgn(k)E_k$, and $u_k v_k = \Delta_\alpha / 2E_k$. The self-consistency condition $\Delta_\alpha = -ig < \psi_{-\alpha}(\mathbf{r})\psi_\alpha(\mathbf{r}) >$leads to the well-known equation

$$1 = -\frac{gk_F^2}{2\pi^2} \int dk \frac{\tanh \beta E_k}{2E_k} \ ; \tag{22}$$

hence, one may see that interaction must be attractive, *i.e.* $g < 0$, in this case; one obtains the well-known critical temperature

$$T_c \simeq \hbar v_n k_c e^{-1/Dg} \ , \tag{23}$$

where $D = k_F^2 / 2\pi^2 \hbar v_n$ is the density of states (per spin) at the Fermi surface, k_c is a wavevector cutoff,[8] and the sign of the interaction has been changed; similarly, one obtains the temperature dependence of the gap $\Delta_\alpha = \Delta_{\alpha 0}(1 - T/T_c)^{1/2}$ for temperatures close to the critical temperature, the gap $\Delta_{\alpha 0}$ being given by $\Delta_{\alpha 0} \simeq 2\hbar v_n k_c e^{-1/Dg}$. We may neglect the temperature dependence of the superconducting gap and Fermi level (19), assuming the temperature be sufficiently low for superconductivity be well developed.

4 Ferromagnet-Superconductor Junction

According to the quasi-classical description of matter aggregation[9] the cohesion of a solid is governed by a self-consistent potential $\varphi(\mathbf{r})$; its simplest form is given by

$$\varphi(\mathbf{r}) = \sum_i \frac{z_i^*}{|\mathbf{r} - \mathbf{R}_i|} e^{-q|\mathbf{r} - \mathbf{R}_i|} \ , \tag{24}$$

where z_i^*denote effective ionic charges (in atomic units), $\mathbf{R}_i$ are the ionic positions and q is a screening wavevector; the screening wavevector is estimated as $q \simeq 0.77 z^{*1/3}$ (in atomic units), where $z^* = \overline{z}_i^*$is the average effective charge; one may introduce an average inter-ionic separation a, and the product qa is estimated as $c = qa \simeq 2.73$; the electron density n is related to the self-consistent potential through $4\pi n = q^2 \varphi$; the average potential as given by (24) is $\varphi = 4\pi z^* / a^3 q^2$. This quasi-classical description is refined in the next step by so-called quantal corrections, which lead, among others, to a shift of the Fermi level toward negative values (at this level of approximation the Fermi level is placed at zero energy, while the chemical potential of the electrons is $-\varphi$).

Let us consider a semi-infinite solid with a plane free surface at $x = 0$; the corresponding average potential as given by (24) is

$$\begin{aligned} \varphi(x) &= \tfrac{4\pi z^*}{a^3 q^2}(1 - \tfrac{1}{2}e^{qx}) \ , \ x < 0 \ , \\ \varphi(x) &= \tfrac{2\pi z^*}{a^3 q^2} e^{-qx} \ , \ x > 0 \ ; \end{aligned} \tag{25}$$

one can see that there exists a change

$$\begin{aligned} \delta\varphi(x) &= -\tfrac{2\pi z^*}{a^3 q^2} e^{qx} \ , \ x < 0 \ , \\ \delta\varphi(x) &= \tfrac{2\pi z^*}{a^3 q^2} e^{-qx} \ , \ x > 0 \end{aligned} \tag{26}$$

[8] The scale energy $\hbar v_n k_c$ is of the order of the Debye energy $\hbar\omega_D$ for a phonon-electron superconducting interaction.

[9] L. C. Cune and M. Apostol, *loc cit.*

in the self-consistent potential at the surface, in comparison with the potential in the bulk, so that a corresponding change $\delta n(x) = q^2\delta\varphi(x)/4\pi$ in the electron density appears at the free surface; the electrons spill over the surface and give rise to a charge double layer; the surface is depleted by $z^*/2a^3q$ electrons per unit area; the electric field arising from the sharp surface is compensated by the dipole field of the surface charge redistribution. The energy (per unit area of the surface) associated with this double layer can be estimated as

$$\begin{gathered}\delta E = -\tfrac{1}{2}\int dx\cdot\delta\varphi\delta n = -\tfrac{q^2}{8\pi}\int dx\cdot(\delta\varphi)^2 = \\ = \tfrac{q^2}{4\pi}\int dx\cdot x\delta\varphi(\partial\delta\varphi/\partial x) = -\int dx\cdot x\delta n E = -\tfrac{\pi z^{*2}}{2a^6q^3}\ ,\end{gathered} \tag{27}$$

and one can see that it originates in the dipole field of intensity $E = -\partial\delta\varphi/\partial x$; it is a surface energy. It is also worth noting that the surface double layer extends over distances of the order of the atomic distances a ($\sim 1/q$), and the surface ions relax by $\delta a \sim 1/4q \sim 0.1a$, for the surface energy given above. The surface energy (27) is a second-order contribution in the potential (and electron density) change, and, consequently, it is comparable to the uncertainy in quasi-particle energy; it contributes therefore to the surface-scattering lifetime of the quasi-particles. Indeed, the surface energy (27) can also be written as $\delta E = -\pi n^2/2q^3$, where $n = z^*/a^3$ is the electron concentration, or $\delta\varepsilon = \pi n/2q^3d$ an energy per electron, where d denotes the length of the sample; it can be compared with the Fermi energy $\mu \sim \varphi$ of the electron liquid, leading to $\delta\varepsilon/\mu = (1/8c^2)(a/d)$, and a surface-scattering lifetime $\tau_s \sim (\hbar/\mu)(d/a)$; this is Casimir's finite-size (boundary scattering) lifetime of the electron quasi-particles.[10] Typically, for conductors, the electron-electron uncertainty in electron energy is of the order of $\delta\varepsilon \sim (T/\mu)^2\mu$ (or $(\Delta\varepsilon/\mu)^2\mu$), while the electron-phonon uncertainty in electron energy can be represented as $\delta\varepsilon \simeq T/F$, $F = (M/m)(\hbar\omega_D/\mu)^2$, where M is the ionic mass and ω_D is the Debye frequency.[11] Except for very low temperatures, or atomic-size samples, the boundary scattering lifetime is very long, and, therefore, it contributes little to transport. It is also worth noting the work function of the solid

$$\begin{gathered}W = -\varphi(+\infty) + \varphi(-\infty) = -\int dx\cdot\partial\delta\varphi/\partial x = \\ = \int dx\cdot x(\partial^2\delta\varphi/\partial x^2) = -4\pi\int dx\cdot x\delta n = \\ = q^2\int dx\cdot x\delta\varphi = 4\pi z^*/a^3q^2 = \varphi\ ,\end{gathered} \tag{28}$$

as expected, where Poisson's equation $\partial^2\delta\varphi/\partial x^2 = 4\pi\delta n = q^2\delta\varphi$ is employed. A similar exponential decay at the surface is suffered by the quantal corrections to the electron energy levels, in particular by the energy band structure.

Let us consider now two distinct solids labelled by 1 and 2, respectively, with a plane interface at $x = 0$; solid 1 extends from $x = -\infty$ to $x = 0$ and solid 2 extends from $x = 0$ to $x = +\infty$. The Fermi energies, *i.e.* the extension in energy from the bottom of the energy bands to the top of the Fermi seas (the top not necessarily placed at zero energy) are denoted by $\mu_{1,2}$; the bottom of the energy bands are placed at $-\varphi_{1,2}$. When put in contact the interface ions are separated by a potential barrier of width $\sim a$ and height $\sim z^*\varphi$, where a is an average inter-ionic separation and $z^*\varphi$ denotes an average potential energy; if the two solids are similar, *i.e.* the difference Δz^* in their effective charges is very small, they form up a perfect contact; otherwise, the ions are perturbed by $\sim z^*\Delta\varphi \sim z^*\Delta z^*e^2/a$, and tunnel through across the top of the barrier; the

[10] H. B. G. Casimir, Physica **5** 495 (1938).
[11] M. Apostol, **Transport Theory**, *apoma*, MB (2001) (b).

well-known transmission coefficient

$$T^2 = \frac{4k^2\kappa^2}{(k^2+\kappa^2)^2 \sinh^2 a\kappa + 4k^2\kappa^2} \tag{29}$$

of a rectangular barrier, where $\kappa^2 \ll k^2 = (2M/\hbar^2) z^* \Delta\varphi \sim (2M/\hbar^2) z^* \Delta z^* e^2/a$, becomes $T^2 = 4/(a^2k^2+4)$, and, since $a^2k^2 \sim (M/m) z^* \Delta z^* (a/a_H)$, one gets

$$T^2 \simeq \frac{m}{M} \frac{1}{z^* \Delta z^*} \frac{a_H}{a} \ , \tag{30}$$

where M is the ionic mass and $a_H = \hbar^2/me^2$ is Bohr's radius. The distance covered by the ion is[12] $\Lambda_c \sim -a/\ln R$, where R is the reflection coefficient, $R^2 = 1 - T^2$; one obtains

$$\Lambda_c \sim a \frac{M}{m} z^* \Delta z^* (a/a_H) \ ; \tag{31}$$

this length Λ_c is a measure of the contact width, but the two solids are not at equilibrium, and it is in fact a diffusion length of one solid into another (with a decreasing velocity); the diffusion velocity

$$v = \hbar k/M \sim \sqrt{m/M} \cdot \sqrt{z^* \Delta z^* (a/a_H)} \cdot v_F \tag{32}$$

is much lower than the Fermi velocity v_F of the electrons,[13] but the diffusion takes a longer time, so that the width of the contact is much larger than the lattice constant. Typical values for Λ_c are of the order of $10^2 - 10^3$
A , while the mean-free path of the electron quasi-particles, as given by $\Lambda \sim v_F \hbar\mu/T^2$, for instance, (for electron-electron interaction in conductors[14]) are of the order of $10^3 - 10^4$
A at room temperature. One can see that Λ_c is shorter than Λ, and at low temperatures Λ may increase appreciably.[15] Over the contact width Λ_c the electron energy levels vary smoothly; for instance, the self-consistent potential across the interface reads

$$\begin{aligned} \varphi(x) &= \varphi_1 + \tfrac{1}{2}\Delta\varphi e^{x/\Lambda_c} \ , \ x < 0 \ , \\ \varphi(x) &= \varphi_2 - \tfrac{1}{2}\Delta\varphi e^{-x/\Lambda_c} \ , \ x > 0 \ , \end{aligned} \tag{33}$$

and one can see that the relative work function $-\int dx \cdot (\partial\varphi/\partial x) = -\Delta\varphi = \varphi_1 - \varphi_2$ is the contact potential between the two solids, as expected. For large contact widths, the interface brings its own contribution (as a "third solid" in-between the junction of the two), for instance to transport coefficients.[16]

We assume a perfect interface between two conductors, and focus on the electron quasi-particles transport; the difference $\Delta\varphi$ between the two chemical potentials is small in comparison with

[12] M. Apostol, J. Theor. Phys. **74** (2001) (c).

[13] It may be increased by external perturbations, like an electric field, for instance, or raising the temperature, which also helps bringing the two solids in "atomic" contact.

[14] And similarly for electron-phonon interaction (M. Apostol, *loc cit* (b)).

[15] In classical semiconductors the contact width is much narrow, of the order of 10
A , as a consequence of the drastic reduction in the effective charges z^*(and their differences), while the mean-free path of the charge carriers is longer (~ 100
A); see, for instance, M. Apostol, *loc cit* (b).

[16] It is worth noting in this connection that the contacts discussed here are those appearing naturally and freely between two solids, and not contacts realized by a limited deposition or growth of an additional, external solid in-between, like metal-oxide-metal, semi- or superconductor, where tunneling currents are measured through the oxide potential barrier (see, for instance, I. Giaever, Revs. Mod. Phys. **46** 245 (1974)).

the cohesion scale φ, and placed at the bottom of the bands; the quantal corrections bring even smaller contributions; it follows that the difference $\Delta\mu$ between the two Fermi energies (*i.e.* the extent in energy from the bottom of the bands to the Fermi surface) is small in comparison with the Fermi energy μ, so that the quasi-particles have in fact a common Fermi level μ and a change $\Delta\mu$ in energy on passing across the interface.[17] It follows that the electron energy uncertainty $\hbar/\tau \sim (\delta\mu)^2/\mu$ (or $\hbar/\tau \sim T^2/\mu$) suffers a change $\Delta(\hbar/\tau) \sim ((\delta\mu)^2/\mu)(\Delta\mu/\mu)^2 = (\hbar/\tau)(\Delta\mu/\mu)^2$, and a similar change occurs for the electron-phonon uncertainty in the quasi-particle energy; therefore, the quasi-particle lifetime τ suffers a relative change according to

$$\Delta(1/\tau)/(1/\tau) = (\Delta\mu/\mu)^2 \tag{34}$$

on passing through the interface. All the transport coefficients are affected by a similar relative change due to the presence of the interface; in particular the electric conductivity σ undergoes a decrease $\Delta\sigma/\sigma = -(\Delta\mu/\mu)^2$, and the electric resistivity ρ increases by $\Delta\rho/\rho = (\Delta\mu/\mu)^2$; this is Kapitza's contact resistance;[18] a relative voltage drop $\Delta U/U = (\Delta\mu/\mu)^2$ occurs at the interface.

The relative jump $(\Delta\mu/\mu)^2$ at the interface affects all the transport properties of the quasi-particles, in particular their lifetime τ and mean-free path Λ.[19] The fraction $\tau_f = (\Delta\mu/\mu)^2\tau$ is the flip time of the quasi-particles on passing through the interface, and $\Lambda_f = (\Delta\mu/\mu)^2\Lambda$ is the corresponding flip path, or penetration length. Accordingly, at the ferromagnet-conductor interface the quasi-particle spin flips over the length Λ_f, so the magnetization is gradually destroyed over the length Λ_f into the ferromagnet, and it penetrates similarly over a distance Λ_f into the normal conductor (the difference in the two Fermi velocities in the magnetic state bears no relevance in estimating the penetration length; a more suggestive measure of two competing flip lengths $\Lambda_{1,2}$ is $(\Lambda_1\Lambda_2)^{1/2}$). At the superconductor-normal conductor interface the superconducting gap extends up to Λ_f into the normal conductor, and is destroyed gradually over the same length into the superconductor; the Fermi velocity in the superconductor is that corresponding to the normal state of the superconductor, while the quasi-particle lifetime is formally affected by the superconducting gap.[20] The same description applies also to a conductor-insulator interface, by noting that the insulating gap is a "quantal correction" in terms of the present approach, so the insulating gap is destroyed over Λ_f length at the interface, and penetrates over a similar length into the conductor.[21]It follows, according to the present description, that the charge carriers in a (perfect contact) ferromagnet-superconductor junction move close to the same common Fermi level μ, with an almost common (normal state) Fermi velocity, and possess an additional, small contribution to their lifetime due to the presence of the interface; both the magnetization and the superconductiong gap vanishes over a penetration length Λ_f across the interface, as determined above. It is easy to see that for a perfect contact Λ_f is comparable with Λ_c.

5 Andreev Reflection

We focus first on the superconducting equations (16) and (17), where we drop out the label n for the Fermi velocity $\mathbf{v}$, and the superconducting gap Δ_α is assumed constant and positive; in

[17] It is worth noting that such "perfect" contacts appears in fact at the separation between each of the two solids and their common, extended contact (the "third solid"), as described above.

[18] P. L. Kapitza, ZhETF **11** 1 (1941).

[19] It is also worth noting that the frequency $\Delta(1/\tau)$ gives the number of transitions per unit time for a superconducting pair from one superconductor into another in Josephson junctions, *i.e.* $i\hbar\partial\psi_{1,2}/\partial t = \hbar\Delta(1/\tau)\psi_{2,1}$, where $\psi_{1,2}$ are the condensate wavefunctions.

[20] See, for instance, R. J. Schrieffer, **Theory of Superconductivity**, *Benjamin*, NY (1964).

[21] At the interface between a conductor and a semiconductor (of narrow band) an extended contact is built up (except for a limited growth or deposition).

addition we introduce the one-particle amplitudes

$$\varphi_\alpha = \langle 0 |\psi_\alpha| 1\alpha\rangle \ , \ \chi_\alpha = \left\langle 0 \left|\psi^+_{-\alpha}\right| 1\alpha\right\rangle \ , \tag{35}$$

where $|1\alpha\rangle$ is an excited one-particle state; the amplitude φ_α is the wavefunction of a $\mathbf{k},\alpha$-quasi-particle, while χ_α represents a $-\mathbf{k},-\alpha$-quasi-hole in a superconducting pair; indeed, for instance,

$$\varphi_\alpha = \langle 0 |\psi_\alpha| 1\alpha\rangle = \frac{1}{\sqrt{V}}\sum e^{i\mathbf{k}'\mathbf{r}}\left\langle 0 \left|\mathbf{c}_{\mathbf{k}'\alpha'}\mathbf{c}^+_{\mathbf{k}\alpha}\right| 0\right\rangle = \frac{1}{\sqrt{V}}e^{i\mathbf{kr}} \ , \tag{36}$$

which is the wavefunction of a quasi-particle, and similarly for χ_α for the superconducting state; the connection of the amplitudes above with the canonical transform (20) is obvious; equation (16) and its mate read now

$$\begin{aligned} i\hbar\partial\varphi_\alpha/\partial t &= (-\hbar\mathbf{v}\mathbf{k}_F - i\hbar\mathbf{v}\partial/\partial\mathbf{r})\varphi_\alpha + i\Delta_\alpha\chi_\alpha \ , \\ -i\hbar\partial\chi_\alpha/\partial t &= (-\hbar\mathbf{v}\mathbf{k}_F - i\hbar\mathbf{v}\partial/\partial\mathbf{r})\chi_\alpha + i\Delta_\alpha\varphi_\alpha \ , \end{aligned} \tag{37}$$

and it is easy to check up the continuity equation

$$\partial(|\varphi_\alpha|^2 + |\chi_\alpha|^2)/\partial t + \mathbf{v}\partial(|\varphi_\alpha|^2 - |\chi_\alpha|^2)/\partial\mathbf{r} = 0 \tag{38}$$

for each spin orientation α; however, $\Delta_{-\alpha} = -\Delta_\alpha$ and, similarly, $\chi_{-\alpha} = -\chi_\alpha$ according to its definition, so that equations (37) are the same for each spin orientation and we may drop out the label α for the superconducting gap and amplitudes. Equations (37) are Gorkov's equations;[22] they tell that a quasi-particle in a superconductor acquires two distinct states, one as a quasi-particle, another as a quasi-hole in a superconducting pair; according to (38), the localization probability $|\varphi|^2 + |\chi|^2$ of a quasi-particle in the superconducting state changes in time according to the divergence of the current $\mathbf{j} = \mathbf{v}(|\varphi|^2 - |\chi|^2)$. The current $\mathbf{j}$ consists of two contributions, $\mathbf{v}|\varphi|^2$ which flows along the velocity, and $-\mathbf{v}|\chi|^2$ which flows in the opposite direction; this latter contribution is the Andreev reflection;[23] though it comes from holes, and, at first sight it may appear as enhancing the net flow, one can see that, on the contrary, it is precisely the opposite case, it diminishes the net flow, because the amplitudes of both quasi-particles and quasi-holes are less than unity, and the quasi-holes move in the same direction as the quasi-particles; actually, one can see, by making use of (20), that $\varphi \sim u_k^2$ and $\chi \sim iu_kv_k$, and the localization probability goes like u_k^2, hence the quasi-particles truly encounter a potential barrier on their attempt of entering a superconductor, and, consequently, they are reflected by the superconductor gap, as well as transmitted through;[24]since the superconducting gap is very small in comparison with the Fermi energy, at sufficiently low temperatures the quasi-particle lifetime is long enough to allow for the Andreev reflection. It is worth emphasizing that a $-\mathbf{k},-\alpha$-quasi-hole is equivalent to a $\mathbf{k},\alpha$-quasi-particle propagating backwards in time, hence the counter-flow associated with χ_α and the Andreev reflection. Equations (37) also read

$$\begin{aligned} \hbar(\omega + \mathbf{v}\mathbf{k}_F + i\mathbf{v}\partial/\partial\mathbf{r})\varphi &= i\Delta\chi \ , \\ \hbar(\omega - \mathbf{v}\mathbf{k}_F - i\mathbf{v}\partial/\partial\mathbf{r})\chi &= -i\Delta\varphi \ , \end{aligned} \tag{39}$$

[22] L. P. Gorkov, *loc cit.*

[23] A. F. Andreev, ZhETF **46** 1823 (1964) (Sov. Phys.-JETP **19** 1228 (1964)).

[24] A thin conductor-superconductor-conductor sandwich in the ballistic regime would exhibit interference patterns or pulse-like transport of the flow, near the edge of the gap (irrespectively of above or below), over a characteristic transmission time (see, for instance, M. Apostol, *loc cit* (c)).

for $\varphi, \chi \sim e^{-i\omega t}$; in addition, we remove the $\mathbf{vk}_F$-term in (39) by introducing

$$\xi = e^{-i\mathbf{k_F r}}\varphi \ , \ \eta = e^{-i\mathbf{k_F r}}\chi \ , \tag{40}$$

so that (39) become

$$\begin{aligned} \hbar(\omega + i\mathbf{v}\partial/\partial\mathbf{r})\xi = i\Delta\eta \ , \\ \hbar(\omega - i\mathbf{v}\partial/\partial\mathbf{r})\eta = -i\Delta\xi \ ; \end{aligned} \tag{41}$$

for $\xi, \eta \sim e^{i\mathbf{kr}}$ one can check up the superconducting spectrum $\hbar\omega = \pm\sqrt{\Delta^2 + \hbar^2(\mathbf{vk})^2}$, while the reduced wavefunctions are given by

$$\begin{aligned} \xi = \tfrac{C_\alpha}{\sqrt{2}}\sqrt{1 + \mathbf{vk}/\omega}e^{i\mathbf{kr}} \ , \\ \eta = \tfrac{-iC_\alpha}{\sqrt{2}}\sqrt{1 - \mathbf{vk}/\omega}e^{i\mathbf{kr}} \ , \end{aligned} \tag{42}$$

where C_α is a constant, $\hbar\mathbf{vk} = \pm\sqrt{(\hbar\omega)^2 - \Delta^2}$and $\hbar\omega > \Delta$ (otherwise the quasi-particle does not propagate, and the wavefunctions decay exponentially with the distance); ξ is the (reduced) wavefunction of a quasi-particle of momentum $\hbar\mathbf{k}$, while η represents a quasi-hole of momentum $-\hbar\mathbf{k}$ and spin $-\alpha$, *i.e.* a "reflected" quasi-hole or a reflected "quasi-particle". It is worth noting that the "reflected" quasi-holes (or quasi-particles) change the whole wavevector $\mathbf{k}$, not only one component; this shows that the Andreev reflection is not on the interface, but on the superconductor as a whole. The wavevectors $\mathbf{k}$ in (42) are small in comparison with the Fermi wavevector $\mathbf{k}_F$ (where the velocity $\mathbf{v}$ is calculated), so that the wavefunctions ξ, η vary slowly in space. The constant C_α bears temporarily a spin label, for the sake of generality, though a spin imbalance destroys usually the superconductivity. Before passing to the ferromagnet-superconductor junction we note that the transmitted (tunneling) current in the superconductor is

$$\mathbf{j}_{t\alpha} = \mathbf{v}(|\xi|^2 - |\eta|^2) = |C_\alpha|^2\,\mathbf{v}(\mathbf{vk}/\omega) \ ; \tag{43}$$

in addition, we also note that (42) are consistent with (20), as expected.

We may pass now to the Andreev reflection in a ferromagnet-superconductor junction; according to the discussion in the preceding section the Fermi energy in ferromagnet is taken as being equal to the Fermi energy in superconductor, as for a perfect contact, and Kapitza's resistance is neglected; under these circumstances equations (37) hold for the ferromagnet by simply droping out the superconducting-gap contribution; obviously, the remaining part depends on the spin orientation, through both the Fermi velocity and Fermi wavevector; in addition, χ vanishes for the non-superconducting sample, (indeed, $\eta \to 0$ in (42) for $\Delta \to 0$, as expected), so that we may write down (39) and (41) as

$$[\omega + \mathbf{v}_{1,2}(\mathbf{k}_{F1,2} - \mathbf{k}_F) + i\mathbf{v}_{1,2}\partial/\partial\mathbf{r}]\xi_{1,2} = 0 \ , \tag{44}$$

where the velocities $\mathbf{v}_{1,2} = \mathbf{v}(1 \pm m)^{1/3}$ and the Fermi wavevectors $\mathbf{k}_{F1,2} = (1 \pm m)^{1/3}$ correspond to spin up and down, respectively, as defined in (7) and (5), m being the reduced magnetization. In addition, we may note that the term $\mathbf{v}_{1,2}(\mathbf{k}_{F1,2} - \mathbf{k}_F) = \pm(1/3)vk_F m \sim \Delta_m$ is small according to the discussion in section 2, *i.e.* it is comparable to Δ with respect to the Fermi energy; consequently it is immaterial in (44); it follows that the corresponding equations (39) and (41) for the ferromagnet reduce to

$$(\omega + i\mathbf{v}_{1,2}\partial/\partial\mathbf{r}]\xi_{1,2} = 0 \ , \tag{45}$$

whose solution is

$$\xi_{1,2} = A_{1,2} e^{i\mathbf{k}_{1,2}\mathbf{r}} \ , \tag{46}$$

for

$$\mathbf{v}_{1,2}\mathbf{k}_{1,2} = \mathbf{v}(1 \pm m)^{1/3}\mathbf{k}_{1,2} = \omega \ ; \tag{47}$$

one can notice in (47) that for m close to unity the wavevector k_2 must acquire large values (the minimum value of ω is $\Delta/\hbar$), which raises problems (a similar situation would have been encountered in fact if the term $\mathbf{v}_{1,2}(\mathbf{k}_{F1,2} - \mathbf{k}_F)$ would have been large in (44)); actually, several restrictions are put on the quasi-particles wavevectors by the requirement that the excitation energy be $\hbar\omega$ (restrictions arising from the geometric orientation with respect to the velocity $\mathbf{v}$, for instance, in this respect), but the essential one is the wavevectors $\mathbf{k}$ be small in comparison with the Fermi wavevector $\mathbf{k}_F$; in this respect the ferromagnet equation (45) looks more as the asymptotic form of the superconductor equations (41), so that the Andreev reflection proceeds in the same manner in the opposite direction, *i.e.* from the superconductor to the ferromagnet, in particular; in this connection, it is worth noting that the mutual positions of the two samples need not be specified, the Andreev reflection proceeding in fact on the asymptotic superconducting boundaries. Therefore, we must keep in mind that only the first-order spatial derivative has been kept in Schrodinger's equation, as corresponding to the linearized spectrum of the quasi-particles, and, while slowly-varying wavefunctions ensure, to this accuracy, the continuity of their first-order derivative, on the contrary, wavefunctions varying rapidly in space do not do that anymore; this is why, the Andreev reflection must be viewed with caution for magnetization values close to unity, where large quasi-particles wavevectors are implied; the lifetime of the quasi-particles is largely diminished in this case at the ferromagnet-superconductor boundary, and the corresponding transmission coefficient is diminished; as a consequence, fluxes may not flow anymore through the junction, giving rise to superheating, for instance, which may damage the junction and change the problem. However, this is true for spin-down quasi-particle fluid only, whose density of states diminishes correspondingly for $m \sim 1$, so that its contribution to the transmission coefficient is not significant in this region. One may estimate the occurrence of this anomalous situation from $\hbar v k_F (1-m)^{1/3}(k/k_F) \sim \Delta$, which leads to $m/(1-m)^{1/3} \leq \Delta_m/\Delta$.

The continuity condition of the wavefunctions ξ given by (42) and (46) leads to

$$A_{1,2} = \frac{C_{1,2}}{\sqrt{2}}\sqrt{1 + \mathbf{v}\mathbf{k}/\omega} \ , \tag{48}$$

for a boundary placed arbitrarily at $x = 0$ (it is worth noting that the components perpendicular to $\mathbf{v}$ of the small wavevectors $\mathbf{k}$ are not affected by equations); on the other hand, the incoming current is given by

$$\mathbf{j}_i = \mathbf{v}_1 \left|A_1\right|^2 + \mathbf{v}_2 \left|A_2\right|^2 = \mathbf{v}[(1+m)^{1/3}\left|A_1\right|^2 + (1-m)^{1/3}\left|A_2\right|^2] \ ; \tag{49}$$

making use of (43) we may define the transmission coefficient

$$\begin{gathered} w = (j_{t1} + j_{t2})/j_i = \frac{|C_1|^2 + |C_2|^2}{(1+m)^{1/3}|A_1|^2 + (1-m)^{1/3}|A_2|^2}(\mathbf{v}\mathbf{k}/\omega) = \\ = \frac{2(|A_1|^2 + |A_2|^2)}{(1+m)^{1/3}|A_1|^2 + (1-m)^{1/3}|A_2|^2}\frac{\mathbf{v}\mathbf{k}/\omega}{1 + \mathbf{v}\mathbf{k}/\omega} \ ; \end{gathered} \tag{50}$$

the asymptotic spin amplitudes in the superconductor are equal, *i.e.* $\left|A_1\right|^2 = \left|A_2\right|^2$ (and $\left|C_1\right|^2 = \left|C_2\right|^2$), so we obtain

$$w = \frac{4}{(1+m)^{1/3} + (1-m)^{1/3}}\frac{\mathbf{v}\mathbf{k}/\omega}{1 + \mathbf{v}\mathbf{k}/\omega} \ , \tag{51}$$

or

$$w = \frac{2}{(1+m)^{1/3} + (1-m)^{1/3}} w_0 \ , \tag{52}$$

where

$$w_0 = 2\frac{\mathbf{vk}/\omega}{1+\mathbf{vk}/\omega} \tag{53}$$

is the transmission coefficient for zero magnetization; within the present approximation

$$w_0 \simeq 2\sqrt{2}\sqrt{\hbar\omega/\Delta - 1} \ . \tag{54}$$

One can see that the transmission coefficient in the Andreev reflection increases slowly with increasing magnetization,

$$w = (1 + m^2/9)w_0 \tag{55}$$

for small values of m. For higher magnetization the Andreev reflection may be suppressed for the spin-down quasi-particle fluid. It is also worth noting that the increase in the transmission coefficient with increasing magnetization is due to this slow, spin-down quasi-particles ($(1-m)^{1/3}$-component in (52)), which go through mainly by diffraction; their performance is limited for higher magnetization, as discusssed above; the faster, spin-up quasi-particles ($(1+m)^{1/3}$-component in (52)) are in fact scattered by colliding the potential barrier, thus contributing in the opposite, decreasing, direction to the transmission coefficient.

6 Electric Resistance of the Junction

For a voltage drop U

$$-\frac{\partial n}{\partial \varepsilon} e^2 U \frac{2 \cdot d\mathbf{p}}{(2\pi\hbar)^3} \tag{56}$$

charge is transported per unit volume by a quasi-particle, where n denotes the Fermi distribution; during the quasi-particles lifetime τ the charge flux (charge per unit area) is

$$-\frac{\partial n}{\partial \varepsilon} e^2 U v_x \tau \frac{2 \cdot d\mathbf{p}}{(2\pi\hbar)^3} \ , \tag{57}$$

while the total flow (charge per unit area and unit time) is

$$j = -\frac{2e^2}{(2\pi\hbar)^3} \int d\mathbf{p} \cdot \frac{\partial n}{\partial \varepsilon} v_x^2 \tau (\partial U/\partial x) \ ; \tag{58}$$

from $j = \sigma E$, where $E = -\partial U/\partial x$ is the electric field, we obtain the electric conductivity

$$\sigma = \frac{e^2}{3\pi^2\hbar} k_F^2 v\tau \ . \tag{59}$$

In the derivation given above the statistical equilibrium is assumed, as well a mean-free path much shorter than the length of the sample, a low, uniform electric field, and a lifetime free of finite-size contributions or other geometric effects. [25] It is easy to see that for a mean-free path $\Lambda = v\tau$ comparable with the sample length equation (58) leads to an electric current

$$I = \frac{e^2}{3\pi^2\hbar} k_F^2 A \cdot U \ , \tag{60}$$

[25] For details see M. Apostol, *loc cit* (b).

through area A, whence the quanta e^2/h of electric conductance can be inferred. Actually, in such a ballistic regime of transport the lifetime τ does not appear anymore in (57), and the angle integration gives 1/2 instead of 2/3 in (58); we obtain therefore

$$j = \frac{e^2 k_F^2}{4\pi^2 \hbar} U \ , \tag{61}$$

i.e. an electric resistance

$$R = \frac{4\pi^2 \hbar}{e^2 k_F^2} \tag{62}$$

for unit area. It is worth noting that in a ballistic transport regime the resistance may depend on the voltage drop, in some cases; indeed, for a normal conductor we have obviously $\hbar \mathbf{v} \delta \mathbf{k}_F = -eU$, and the current $j = (-ek_F^2/2\pi^2) \int du (\mathbf{v} \delta \mathbf{k}_F) = (e^2 k_F^2 / 4\pi^2 \hbar) U$, hence the ballistic resistance (62) again; for a superconductor the current is reduced by $\mathbf{vk}/\omega$, according to (43), where $\hbar\omega = -eU$; one obtains $R_s^{-1} = R^{-1}\sqrt{e^2U^2 - \Delta^2}/eU$, which is the typical behaviour for the tunneling resistance in superconductors;[26] one may also note that though equilibrium may suffer in a ballistic transport regime, the quasi-particle lifetime diminishes for higher voltages. The presence of the $eU/\sqrt{e^2U^2 - \Delta^2}$-factor in the superconducting resistance is very important, because the ballistic resistances are extremely low; indeed, typical values for R given by (62) are $R \simeq 10^{-25} \Omega m^2$, so the voltage U has to be very close to the superconducting gap in order to get a practicable device; it is worth noting that such a reduction factor in the conductivity comes from the Andreev reflection in the superconductor, which shows again that the superconductor behaves like a genuine potential barrier; since it may be difficult to finely tune the voltage precisely just above the superconducting gap, a convenient reduction in the conductivity may also be achieved by an additional tunneling barrier interposed between ferromagnet and superconductor; we note that such a barrier does not change anything essentially in the Andreev reflection as derived before, it just act like an additional resistance at the junction; in particular, the transmission of the quasi-particles through a potential barrier is instantaneous, preserving their energy and velocity.[27] Such potential barriers are usually made of an oxide layer grown up at the interface, within a limited depth; oxide layers can perform extended contacts, but a limited growth produces thin layers with sharp separation interfaces; indeed, the atom tunneling through such a potential barrier is extremely slow, so that the characteristic contact length Λ_c is irrelevant, the ferromagnet-oxide-superconductor separation being much sharper this time. Unfortunately, the ballistic transport in superconductor is a non-equilibrium transport, so the quasi-particle spin-flip in the superconductor is less likely in this case; consequently, the associated magnetic mean-field of spin polarization may destroy the superconductivity, and spoil thereby the consistency of the envisaged device; this is why it still looks preferably to have a diffusive transport in the superconducting sample. We also note that precisely for the same reason a reciprocal situation, where the magnetization would be destroyed by the superconducting correlations of the quasi-particles penetrating the ferromagnetic sample would not take place, so the ballistic transport is possible in the feromagnet.

Turning now back to (59) one can see that the electric conductivity of a ferromagnet does not depend essentially on magnetization; indeed, the dependence on the magnetization comes through the velocity v and Fermi wavevector k_F in (59), which gives $(1/2)(v_1 k_{F1}^2 + v_2 k_{F2}^2) = (1/2) v k_F^2 (1 + m + 1 - m) = v k_F^2$; a slow magnetization dependence may be included in the lifetime, but its contribution is uncertain. This point is supported by the fact that flows are proportional to

[26] See, for instance, I. Giaever, *loc cit.*, as well as L. Esaki, Revs. Mod. Phys. **46** 237 (1974), and references therein; a supercurrent may also appear through the tunneling barrier between two superconductors for zero voltage, as it is well-known (B. Josephson, Revs. Mod. Phys. **46** 251 (1974)).

[27] See, for instance, M. Apostol, *loc cit* (c).

density of states $\sim k_F^2/v$ multiplied by velocity v multiplied by mean-free path $v\tau$ in the diffusive regime, hence their $\sim k_F^3$ proportionality to density, and the independence of magnetization.

The electric conductivity corresponding to the tunneling current in a superconductor can be derived in a similar way; however, the flow involves now the quantal probability beside the statistical one, *i.e.* it is given by

$$j = -\frac{2e^2}{(2\pi\hbar)^3}\int d\mathbf{p}\cdot\frac{n}{T}v_x^2\tau[|\varphi|^2 - |\chi|^2](\partial U/\partial x) \ , \tag{63}$$

where the temperature is so small in comparison with the superconducting gap that we may use $n = e^{-\hbar\omega/T}$, $\hbar\omega > \Delta$, for the Fermi distribution; the wavefunctions φ and χ are those given by (40) and (42) for $|C_\alpha|^2 = 1$; one can see that

$$|\varphi|^2 - |\chi|^2 = \mathbf{vk}/\omega \simeq \sqrt{2}\sqrt{(\hbar\omega - \Delta)/\Delta} \tag{64}$$

is much lesser than unity (which corresponds to a normal conductor), as due to the Andreev reflection. Making use of (63) and (64) one can compute the tunneling electric conductivity of a superconductor as

$$\begin{aligned}\sigma_s = \frac{e^2}{3\pi^2\hbar}\frac{\sqrt{2}k_F^2 v\tau}{T}\int_\Delta^\infty d\xi\cdot\sqrt{\frac{\xi-\Delta}{\Delta}}e^{-\xi/T} = \\ = \frac{e^2}{3\pi^2\hbar}k_F^2 v\tau\sqrt{\pi T/2\Delta}e^{-\Delta/T} \ ;\end{aligned} \tag{65}$$

the lifetime is not affected too much in the supeconducting state, in comparison to the lifetime in the normal state, at least for an effective electron-phonon collision regime; consequently, one may write

$$\sigma_s = \sigma\sqrt{\pi T/2\Delta}e^{-\Delta/T} \tag{66}$$

for the tunneling conductivity of the superconducting state, where σ is the electric conductivity of the normal state. One can note in (66) a drastic reduction in the electric conductivity, in comparison with the normal state, as a consequence of the Andreev reflection. In addition, from

$$\begin{aligned} j = \sigma\frac{U-U_0}{l_f} \ , \\ j = \sigma_s\frac{U_0}{l_s} \ , \end{aligned} \tag{67}$$

for a feromagnet-superconductor junction, where $l_{f,s}$ denote the lengths of the ferromagnet and superconducting samples, respectively, one obtains the electric resistance of the junction

$$R_j = l_f/\sigma + l_s/\sigma_s = R + R_s \tag{68}$$

for unit area, whence one can see that it is independent of magnetization; U_0 denotes the voltage drop at the junction. However, the superconducting resistance $R_s = l_s/\sigma_s$ is very high in comparison with the normal resistance $R = l_{f,s}/\sigma$. In particular

$$R_s = l_s/\sigma_s = R\sqrt{2\Delta/\pi T}e^{\Delta/T} \tag{69}$$

is the additional, large electric resistance due to the Andreev reflection in the superconductor, similar to the one computed originally for a thermal flow.[28] Indeed, the thermal conductivity due to electron quasi-particles can be computed in a similar way as before, replacing the charge $-e$ with the energy ε or $\hbar\omega$; above a certain temperature, but still much below the superconducting

[28] A. F. Andreev, *loc cit.*

gap, the electron quasi-particles bring the main contribution to heat transport. It is also worth noting in (67) that the voltage drop is continuous at the junction, but its space derivative, *i.e.* the electric field, is not.

We turn now the attention to a ferromagnet-superconductor junction where the ferromagnet is in the ballistic or quasi-ballistic regime. We assume that the superconducting sample is in the diffusive regime, *i.e.*

$$j = \frac{1}{R}\sqrt{\pi T/2\Delta}e^{-\Delta/T}U_0 \ , \tag{70}$$

where $R = 3\pi^2\hbar l_s/e^2k_F^2\Lambda$ as given by (65); very likely, the Andreev reduction factor in (70) and the diffusive factor l_s/Λ in R increase sufficiently the superconducting sample resistance, such as to make the device practicable. Let us assume that the temperature is sufficiently low and the ferromagnetic sample is sufficiently thin that the length l_f be much shorter than the mean-free path $\Lambda = v\tau$ in the normal state of the feromagnet, $l_f < \Lambda$; increasing the magnetization the spin-up electron fluid increases its mean-free path $\Lambda_1 = \Lambda(1+m)^{1/3}$, so that it transports in the ballistic regime; therefore, we may write down

$$j_1 = \frac{e^2k_{F1}^2}{8\pi^2\hbar}(U-U_0) = \frac{e^2k_F^2}{8\pi^2\hbar}(1+m)^{2/3}(U-U_0) \ , \tag{71}$$

according to the discussion above; the spin-down electron fluid decreases its mean-free path $\Lambda_2 = \Lambda(1-m)^{1/3}$ on increasing magnetization; up to a threshold magnetization $m_t = 1-(l_f/\Lambda)^3$it is still in the ballistic regime, so that

$$j_2 = \frac{e^2k_{F2}^2}{8\pi^2\hbar}(U-U_0) = \frac{e^2k_F^2}{8\pi^2\hbar}(1-m)^{2/3}(U-U_0) \ ; \tag{72}$$

it follows

$$j = j_1 + j_2 = \frac{e^2k_F^2}{8\pi^2\hbar}[(1+m)^{2/3} + (1-m)^{2/3}](U-U_0) \ , \tag{73}$$

which means a resistance

$$R_f = R\frac{2}{(1+m)^{2/3}+(1-m)^{2/3}} \ , \ m < m_t \ , \tag{74}$$

where $R = 4\pi^2\hbar/e^2k_F^2$ as given above; for $m > m_t$ the man-free path Λ_2 gets shorter than the length l_f of the sample and the spin-down fluid flows in the diffusive regime; in this case

$$\begin{aligned} j_2 &= \tfrac{e^2}{6\pi^2\hbar}k_{F2}^2v_2\tau\tfrac{U-U_0}{l_f} = \tfrac{e^2k_F^2}{6\pi^2\hbar}\tfrac{v\tau}{l_f}(1-m)(U-U_0) = \\ &= \tfrac{e^2k_F^2}{6\pi^2\hbar}\tfrac{\Lambda}{l_f}(1-m)(U-U_0) = \tfrac{2}{3}\tfrac{1}{R}\tfrac{1-m}{(1-m_t)^{1/3}}(U-U_0) \ ; \end{aligned} \tag{75}$$

it follows the resistance

$$R_f = R\frac{2}{(1+m)^{2/3}+\frac{4}{3}\frac{1-m}{(1-m_t)^{1/3}}} \ , \ m > m_t \tag{76}$$

for the ferromagnetic sample; the two resistances given by (74) and (76) are discontinuous at the threshold magnetization $m = m_t$, as a consequence of the distinct numerical factors in the ballistic and diffusive conductivities; this negative jump in the resistance is in fact round-off (by geometric effects, for instance), and it may be viewed as a negative resistance for magnetization values close to magnetization threshold; apart from this jump the resistance R_f exhibits a monotonous increase with magnetization over the entire range $0 < m < 1$; in addition, as discussed in the previous section, the Andreev reflection may greatly be diminished for values of the magnetization m close to unity for the spin-down quasi-particle fluid (in the sense that the corresponding electric flow may drastically be reduced), but its contribution to the conductivity is small for $m \sim 1$. We note two limiting behaviours for R_f, namely $R_f \sim R(1+m^2/9)$ for $m \sim 0$ (which is similar to the behaviour of the transmission coefficient w as given by (55)), and $R_f \sim 2^{1/3}R\{1+\frac{1}{3}[\frac{2^{4/3}}{(1-m_t)^{1/3}}-1](m-1)\}$ for $m \sim 1$. We note also that the resistance of the junction is $R_j = R_f + R_s$, and it depends on magnetization through R_f; increasing the magnetization the electric flow through the junction may be diminished, as one can see from (74) and (76), or, it may be increased in the region of the jump, just as for a transistor; this is the FIST effect; one can notice that the controlling effect comes entirely from the ferromagnetic sample, whose magnetization acts like a "gate voltage" for the transistor; in principle, the effect holds for a ferromagnetic-normal conductor junction too, only the Andreev reduction factor being absent now, the length of the normal sample must be large, which is not convenient; the Andreev reflection in the superconducting sample reduces the electric flow very much, in comparison with a normal conductor, so that it may effectively be controlled by the magnetization of the ferromagnetic sample. It is also worth noting that magnetization may be changed by varying the temperature of the ferromagnetic sample, and one may worry about changing on this occasion the lifetime τ too, which was assumed constant above; however, the change in the lifetime is much smaller than the change in the magnetization, or the change in the superconducting gap, for temperatures close to the magnetic critical temperature T_m, but much lower than the superconducting critical temperature T_c. For mean-free paths longer than the width of the sample, one may worry about fluctuations that are inherent to such a quasi-two-dimensional ensemble; however, the fluctuations time for a quasi-particle goes like $\hbar\mu^{1/2}/T^{3/2}$, and one can see that it is much shorter than the lifetime $\sim \hbar\mu/T^2$ and the equilibrium time $\hbar/T$, which means that fluctuations, both quantal and statistical, do not impede upon the ballistic transport.[29]

For $\Lambda < l_f < 2^{1/3}\Lambda$ there exists another threshold $m_t = (l_f/\Lambda)^3 - 1$ below which both spin-up and spin-down fluids flow diffusively, while for $m > m_t$ the spin-up fluid flows ballistically; the ferromagnetic resistance is then given by

$$R_f = \frac{3}{4}R(1+m_t)^{1/3}\ ,\ m < m_t \tag{77}$$

in the former case, and

$$R_f = \frac{3}{4}R(1+m_t)^{1/3}\frac{2}{1-m+\frac{3}{4}(1+m_t)^{1/3}(1+m)^{2/3}}\ ,\ m > m_t \tag{78}$$

in the latter, where R is the same as above; for small values of the magnetization the resistance is constant (and close to the value R corresponding to $l_f < \Lambda$), while for higher values of magnetization it increases up to the same value $2^{1/3}R$ as above; at the threshold it has a positive jump, in contrast to the case $l_f < \Lambda$, where the jump is negative. The reduced ferromagnetic resistance is shown in Fig.1 for the two cases.

7 Concluding Remarks

The physical conditions of the FIST effect require certain limitations, connected especially with the matching conditions of the Andreev reflection at the ferromagnet-superconductor junction.

[29]See, for instance, M. Apostol, *loc cit* (b).

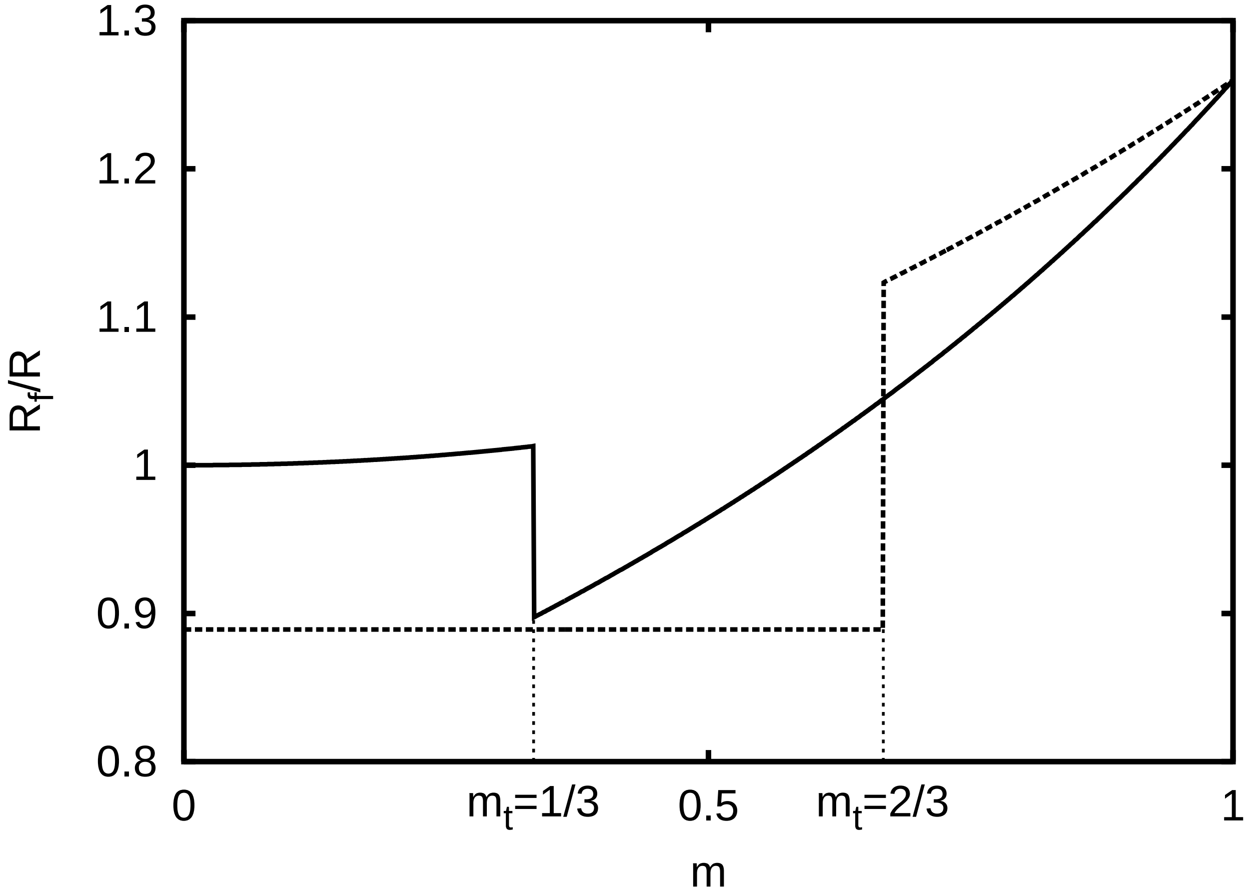

Figure 1: Reduced resistance of a ferromagnetic sample *vs* magnetization in the ballistic (solid line) and quasi-ballistic (dotted line) regime for two arbitrary values of the threshold magnetization m_t.

We summarize here briefly the main physical picture of the effect. Suppose two distinct solids in contact, sharing an interface. If the two solids are very dissimilar they diffuse largely into one another, and an extended contact is built up at the interface. Such a contact acts like a "third solid" in-between the former two, with its own properties. Along such an extended contact the physical properties vary slowly, and the FIST effect would be possible in principle, with a limited efficiency however, especially due to the limitations such an extended contact would put on the ballistic regime of transport in the ferromagnetic sample. New junctions may be defined between each of the two original solids and the third one, which exhibit perfect contacts; hence, the essential role played by perfect contacts in FIST junctions. Indeed, if the two solids are similar they diffuse into each other over a rather limited scale length Λ_c, which contributes to the quasi-particle lifetime. Such a contribution corresponds in fact to the slight difference in the Fermi energies, which brings an uncertainty in the quasi-particles energy, leading to a small Kapitza's resistance. Otherwise, the Fermi energies may be taken the same in the two solids. In particular, typical products like vk_F^2 which enter transport coefficients in the diffusive regime aqcuire similar values, as do the quasi-particle lifetimes. Both the ferromagnetic and superconducting gap do not change appreciably this picture. In particular, the quasi-particle lifetime is similar with the one in normal state, as the residual interactions are effective for the ground-states of such condensed phases, while preserving the same effect for the elementary excitations as in the normal state. In the diffusive regime the transport through a ferromagnet-superconductor junction is not affected by spin polarization. On the contrary, it depends on magnetization in the ballistic regime of transport, through the conductivity of the ferromagnetic sample. However, the conductivities in the ballistic regime are high, so that, in order to be controllable, the transport needs a higher resistance in this regime. This is provided by the Andreev reflection in the superconductor in the diffusive regime. A ballistic transport regime for the superconducting sample may prove to be inconsistent. The ballistic regime is favoured by a perfect contact and low temperatures, such as the quasi-particle mean-free path Λ be longer than, or comparable with the length of the sample, and, of course much longer than the characteristic contact length Λ_c. The change in magnetization may be performed by slight changes in temperature just below the magnetic critical temperature, but much lower than the superconducting critical temperature. Under this circumstance the change in Λ is small, and may be neglected. The spin flip and superconducting-pair decay take place over a scale length Λ_f which is comparable with Λ_c, so that one may still have a sharp ferromagnetic-superconductor junction in the ballistic regime for the ferromagnetic sample. In addition, for high values of the magnetization ($m \to 1$) the spin-down fluid of quasi-particles in the ferromagnet ceases to fulfill the matching conditions, leading thus to a high decrease in the corresponding lifetime; however, the two spin fluids of quasi-particle act like two conductors coupled in parallel, and the spin-up contribution dominates the junction resistance.

8 A critique of Some Previous Investigations

Blonder and Tinkham[1] studied the electric flow through a conductor-superconductor junction (Cu-Nb) based on their own theoretical model.[2] These studies do not go further than classical tunneling experiments of Giaever's epoch. The so-called Sharvin resistance[3] of a micro-bridge is nothing but Casimir's resistance in a disguised form.

Deutscher and Feinberg[4] make a fantastic discussion about a conductor (ferromagnet)-superconductor-conductor (ferromagnet) device, that bears no relevance whatsoever on real things. This fantasy gets wrong the Andreev reflection, the ferromagnetism, the superconductivity, etc. The work seems to be based on previous, equally enigmatic and obscure, work by Soulen et al.[5], which claims to measure the magnetization by using a feromagnet-superconductor junction.

In a recent paper Merill and Si[6] discuss the "spin injection into s- and d-wave superconductors"; however, such a thing does not exist to a large extent; in addition, these authors make confusions about supercurrents, single-particle currents, Andreev reflection, etc.

An interesting phase-coherent transport in a superconductor-conductor-superconductor mesoscopic structure has recently been analyzed,[7] which, however, brings not very much in addition to Josephson-type interferometers.

A much more rigurous experiment is reported by Worledge and Geballe.[8] They claim to having deposited 50Å of Al on 20Å of $SrTiO_3$ (STO) on 1000Å of $La_{0.67}Sr_{0.33}MnO_3$ (LSMO) on 300Å of $YBa_2Cu_3O_7$ (YBaCu) on a STO substrate. Al is either normal or superconducting, STO acts like a tunneling barrier, LSMO seems to be highly spin-polarized while YBaCu is superconducting. Very likely everything is in the ballistic regime, and spin-polarized injection currents destroy soon the superconductivity. The experiment measures typically the differential conductivity, which consists of $\sim (d/dU)\sqrt{e^2U^2-\Delta^2}+f(m)$, where the function $f(m)$ may be the ferromagnetic conductivity as given in one of the preceding sections; at zero voltage a certain supercurrent may also appear between the two superconductors; measuring this differential curent-voltage characteristics one may infer the value of m; the "de-superconductivization" of the superconducting sample, as performed by orbital depairing, Zeeman spliting-up of the pairs and spin-orbit interaction,[9] is a particular feature of the differential characteristic which deserves special attention, but it is not worth investigating.

A related work puts forward recently[10] a superconducting transistor made of a stack of three superconducting films, the intermediate one being itself a double layer; by applying a magnetic field the inner superconductor swings into a normal conductor, leading to an increase of current.

Jedema et al[11] have analyzed recently a ferromagnet-superconductor "contact spin resistance" with serious confusions about Andreev reflection and other things, and, consequently, inconsistent results.

A series of periodical peaks at the superconductor-conductor-superconductor junction for a Josephson-like tunneling transport has recently been suggested, as associated with Andreev reflection.[12]

Spin-polarized currents seem to be created in quantum wells by polarized light.[13]

Spin-polarized current-voltage characteristics are claimed to have recently been calculated for a ferromagnet-superconductor junction;[14] however, the results of this author do not go much beyond the classical $j \sim \sqrt{e^2U^2-\Delta^2}$ with variations as those coming from additional tunneling barriers.

A ferromagnet-superconductor-ferromagnet transistor for spin-polarized currents was originally proposed by Johnson[15], by varying the superconducting gap with a magnetic field.

Spin-polarized currents that flip at the interface, injected from a ferromagnet (Ni-Fe permalloy) into a normal (paramagnetic) conductor (Al), have originally been demonstrated by Johnson and Silsbee.[16]

9 Some Additional Notes

It seems that the notion of "transferring a superconducting pair into a normal conductor, or ferromagnet" is present with some authors in spintronics, in order to investigate spin correlations; however, unless an extended contact is present, in which case the transfer is continuously smooth from a pair to quasi-particles, a pair can only be transferred by giving it at least 2Δ, which means one Δ at least per each quasi-particle, which amounts to Andreev reflection for quasi-particles with an excitation energy $\hbar\omega > \Delta$. Pairs can only be transferred between two superconductors, within the coherence length at the junction, in which case we have the well-known Josephson interference.

Orbital depairing, Zeeman spliting up of the pairs and spin-orbit scattering are thought probably to contribute essentially to destroying the superconductivity by a spin-polarized injected current. However, this is not quite true. First, the lifetime brought about by such interactions is comparable with, if not longer than the usual eletron-electron and electron-phonon interactions; in addition, in the diffusive regime of transport the spin polarization is flipped out shortly after pasing through the junction, by the differences in the Fermi energies (which are larger than the lifetime interactions), for equilibrium;[30] in the ballistic regime the spin-polarized current gives rise to a high magnetic field, which destroys the superconductivity above certain, relatively low, values of the current. In addition, if it brings the superconducting sample in the intermediate state, then superconducting vortices occur and the normal regions act like pinholes for the transport.

It is perhaps worthwhile estimating in this connection the magnetic field induced by a spin-polarized current. It is easy to see that the electric flow may be written as $j \sim e(k_F^2/\hbar)(eU)(\Lambda/l)$ with usual notations, or $j \sim env(eU/\hbar vk_F)(\Lambda/l)$; one may also take μ for $\hbar vk_F$, though we know that usually the latter may be pretty larger than the former; it follows that $n' \sim n(eU/\hbar vk_F)(\Lambda/l)$ electrons are transported by the flow per unit volume.[31] They have a magnetization $M = n'\mu_B m$ per unit volume, if the current is spin-polarized, so that, from $MH = H^2/8\pi$, it follows a magnetic field $H \sim n'\mu_B m$. Now, $\mu_B \cong 9 \cdot 10^{-21}$erg/Gs, and we may estimate a magnetic field $H \sim 10^3(eU/\hbar vk_F)(\Lambda/l)m$ Gs; in the ballistic regime such a field may be high enough to reach the critical value for superconductivity.

Indeed, there are $\Delta n \sim k_F^2 \Delta k_F \sim k_F^2 \Delta/\hbar v \sim n(\Delta/\hbar vk_F)$ pairs per unit volume, whose energy $\Delta n \cdot \Delta$ is comparable with the critical magnetic energy $H_c^2/8\pi$; hence a typical value $H_c \sim 10^2 - 10^3$Gs for the critical field.[32]

Let $n_p = \Delta n = n(\Delta/\hbar vk_F)$; a longitudinal displacement field $\mathbf{u}$ produces a change $\delta n_p = -n_p div\mathbf{u}$ in the pair density; their energy $E_p = n_p\Delta = (\hbar vk_F/n)n_p^2$ per unit volume changes by $\delta E_p = (\hbar vk_F/n)n_p^2)(div\mathbf{u})^2$; together with the kinetic energy $n \cdot m(\partial\mathbf{u}/\partial t)^2$ per unit volume one obtains the sound waves $\omega = sk$ propagating in the pair fluid with velocity $s = \Delta/\sqrt{\hbar vk_F m} = \Delta/\hbar k_F$;[33] hence supercurrents lower than $en_p s = env(\Delta/\hbar vk_F)^2$.

Furthermore, it is also worth recalling here a few basic questions related to superconductivity. First, as it is well-known, the superconducting gap is an uncertainty in the Fermi energy, as regards the formation or destruction of the superconductivity, so that $\Delta \sim \hbar v/\xi$, where $\xi \sim a(\hbar vk_F/\Delta)$ is the coherence length of the superconducting pairs (and the pair density $\Delta n \sim 1/a^2\xi$). It is over this coherence length where the superconducting pairs are continuously created and destroyed in a superconductor; this process suffers a proximity effect at the interface of a superconductor with a normal conductor, or a ferromagnet, where it competes with the stronger uncertainty arising from the difference in the Fermi energies; this is why the destruction of the superconducting pairs

[30] The exchange interaction is always present.

[31] It is worth noting that the charge en' is transported per unit volume under the voltage drop Uv/l per unit time, which makes an energy $en'vU/l = jU/l = jE$ per unit volume and unit time, as expected, where E is the electric field.

[32] Similarly, the Curie field (mean-field) of a ferromagnet can be estimated from $\Delta_m \sim \mu_B H$, hence typical values $H \sim 10^5$Gs, which are much higher than characteristic magnetic fields in a superconductor.

[33] They are the superconducting phasons.

proceeds over the characteristic length Λ_f, which is typically much shorter than the coherence length. On the other hand, at the interface between two superconductors, it is precisely such coherence lengths over which the pairs are delocalized, making possible the flow of the Josephson's currents. A similar discussion holds also for the ferromagnet, and the spin-flip length Λ_f of the spins at the interface between a ferromagnet and a normal conductor, or superconductor.

Secondly, let us assume that the sample has a finite thickness d, so that the energy spectrum is $\hbar^2 k_\parallel^2/2m + \hbar^2 n^2/2md^2$, where $k_\parallel$ is the in-plane wavevector and n denotes here the transversal quantal number; it is easy to see that the Fermi energy is $\mu \sim \varepsilon_0 n_c^2$, where $\varepsilon_0 = \hbar^2/2md^2$ is the transversal localization energy and the cutoff $n_c \sim d/a$ (for $n_c \gg 1$); the superconductivity tries to acquire a two-dimensional character, as corresponding to the n-branches of the spectrum, which makes it unstable against fluctuations; in order to preserve its three-dimensional character the condition $\Delta\varepsilon \sim \varepsilon_0 n \ll \Delta$ should be satisfied for n as large as n_c; it follows $d \gg a(\mu/\Delta)$, which is a characteristic length comparable with the coherence length.

Actually, for such thin samples the superconductivity is only partially destroyed, because the above condition $\Delta\varepsilon \sim \varepsilon_0 n \ll \Delta$ is satisfied up to $n_s \sim \Delta/\varepsilon_0$; the number of states affected by superconductivity can easily be computed from $N_s \sim A\sum_n^{n_s} k_\parallel^2 \sim (A/a^2)(\Delta/\varepsilon_0)$, where A is the area of the sample; one obtains cylindrical superconducting domains of a radius comparable with the coherence length, covering the area $A' = A(\Delta/\varepsilon_0) \sim A(d^2/a\xi) \ll A$, where $\xi \sim a(\mu/\Delta)$ has been used for the coherence length; it is easy to see that their number is $N_d \sim (d^2/a\xi^3)A = (ad^2/\xi^3)N_e$, where N_e is the number of electrons per unit area.

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Acknowledgments

This work has been caried out within the Swiss National Science Foundation SCOPES Programme, Grant #7BUPJ062407.00/1-FCST