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On the superfluid spectrum of He^4

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Abstract

The experimental superfluid spectrum of He^4 obtained by neutron scattering experiments is derived from Bogoljubov theory by using a screened Coulomb potential with an oscillatory tail.

It is well-known that Bogoljubov's theory[1] has been introduced with the aim of providing a "microscopic" derivation of the superfluid spectrum of He^4 proposed by Landau, *i.e.* a derivation of the sound-like, vortex and roton excitations by using atomic-like potentials. Although this goal has been achieved qualitatively even in early studies,[2] it was not until recently that problem of fitting the experimental superfluid spectrum has been quantitatively discussed by using atomic-like potentials.[3] We provide herein such a quantitative reproduction of the well-known superfluid spectrum of He^4 obtained by neutron scattering experiments,[4] by using a typical atomic-like potential consisting of a screened Coulomb potential with an oscillatory tail.

The potential reads

$$v(r) = A\frac{e^{-br}}{r}(a_0 + a_1 r + a_2 r^2 + a_3 r^3 + a_4 r^4 + a_5 r^5) \ , \tag{1}$$

where a_i $(i = 0, ...5)$ and b are fitting parameters and A is a scale factor; indeed, the atomic potentials behave like $1/r$ at very short distances, while exhibiting an oscillatory exponential decay at long distances. The potential given by (1) is suggested by a similar atomic potential derived recently by the authors for atomic clusters.[5] The Fourier transform of (1) is given by

$$v(q) = \frac{4\pi A}{q}\sum_{i=0}^{5} a_i \frac{i! \sin\left[(i+1)\arctan\left(\frac{q}{b}\right)\right]}{(b^2+q^2)^{(i+1)/2}} \tag{2}$$

which is inserted into the Bogoljubov spectrum

$$\varepsilon(q) = \{(\hbar^2 n/m)v(q)q^2 + (\hbar^2 q^2/2m)^2\}^{1/2} \ , \tag{3}$$

where n $(= 1/(3.7\text{Å})^3)$ is the density of liquid He^4 and m denotes the mass of He^4 atoms ($\hbar$ is Planck's constant).

The experimental curve from Ref. 4 is shown as solid line in Fig. 1; the function $\varepsilon(q)$ given by (3) is fitted to this experimental curve in the following way: the parameter a_0 is chosen such that, for very short distances, the potential (1) behave like Z^2/r, where $Z = 2$ is the atomic number; the

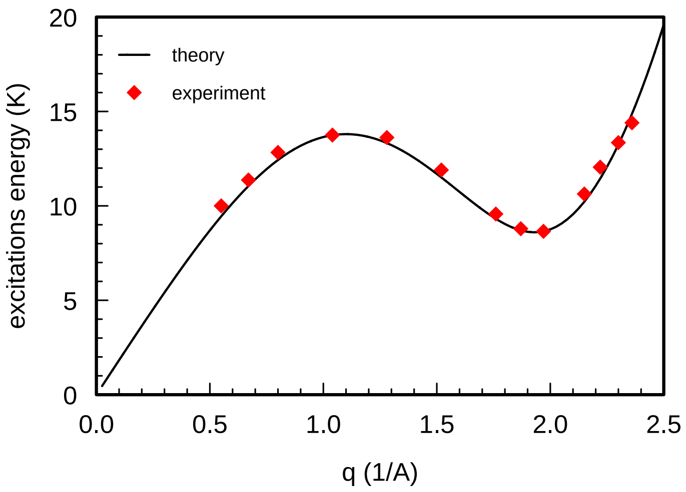

Figure 1: Excitation spectrum fitted with Bogoljubov theory; the experimental values are taken from Ref. [4]

parameters a_i, $i > 0$, are determinated as functions of b by imposing the maximum and minimum values in the excitation spectrum[4] (maximum value for $q = 1.1\mathring{A}^{-1}$, $\varepsilon(q) = 13.8$K and minimum value for $q = 1.93\mathring{A}^{-1}$, $\varepsilon(q) = 8.6$K) and the phonon behavior for small q, $\varepsilon(q \to 0) \simeq u \cdot \hbar q$, where $u = 240$m/s is the sound velocity; finally, the parameter b is determined by minimizing the χ^2-test to the fit. The best fit ($\chi^2 = 1.5\%$) is obtained for $A = 10^6$K, $b = 3.58\mathring{A}^{-1}$, a_0 =0.668545 Å, a_1 =-4.791583, a_2 =9.638526$\mathring{A}^{-1}$, a_3 =-7.679544$\mathring{A}^{-2}$, a_4 = 2.579594$\mathring{A}^{-3}$, a_5 =-0.302344$\mathring{A}^{-4}$. The sound velocity is $u = \sqrt{nv(q)/m} = 240$m/s, and the two extrema in Fig. 1 are placed correctly at $q = 1.1(\mathring{A})^{-1}$, $\varepsilon = 13.8$K and $q = 1.93(\mathring{A})^{-1}$, $\varepsilon = 8.6$K. A similar fit for a hard-core potential $v(r) = v$ for $r < a$ and $v(r) = 0$ for $r > a$ is less favourable; as expected, a screened Coulomb potential with oscillatory tail provides a superior fitting to the experimental data. In addition, the spectrum exhibits slight translational shifts for small changes in the parameters, as indicated by the experimental data in admixtures of He^4-He^3, and He^4 under pressure (see, for instance, Ref. 3).

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