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Correlation functions and next-earthquake distribution in seismic hazard

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Introduction. Recent progress has been recorded in investigating the mechanism of producing earthquakes, and in the statistical patterns exhibited by their magnitude, location and temporal distributions. These investigations have been carried out with the explicit purpose of devising new, more confident tools for assessing seismic risk and hazard. The main object of interest identified in such studies is the next-earthquake (or waiting time, or expectation time, or inter-occurrence time, or pair) distribution, or its generalizations known as statistical correlation functions.[1]

The basic relationship

$$1 + t/t_0 = (1 + E/E_0)^r \tag{1}$$

between time t of accumulating seismic energy E in a seismic focus has been established,[2] where r is an exponent which depends on the geometry of the focus and the mechanism of accumulating energy. The threshold time $t_0 = T/N_0$ is the mean time of N_0 earthquakes produced in time T, or the parameter of the seismicity rate t_0^{-1}. The energy E_0 is the threshold energy in the Gutenberg-Richter equation $E/E_0 = e^{bM}$, where $b = 3.5$ and M denotes the (moment) magnitude.[1]

Accumulating time t in (1) is in fact the mean recurrence time of earthquakes, distributed therefore by $1/t^2$, which leads straighforwardly to the Gutenberg-Richter magnitude distribution $\beta e^{-\beta M} dM$, where $\beta = br$, or the excedence rate (or recurrence law)

$$N_{ex}/T = t_0^{-1} e^{-\beta M} \tag{2}$$

of the number of earthquakes with magnitude greater than M per unit time.

The quantity given in equation (2) above identifies a scale time, or a seismicity rate of the earthquakes with magnitude greater than M.

The two parameters t_0^{-1} (seismicity rate) and β (the slope of the logarithmic recurrence law), and, implicitly, the parameter r, are well documented for various seismic regions, time intervals and ranges of magnitude. For a localized, point-like focus, and a uniform mechanism of accumulation seismic energy parameter r acquires the value $r = 1/3$, corresponding to $\beta = 1.17$. Statistical analysis of recurrence law (and its other various, equivalent forms) indicates $-\ln t_0 = 12.65$ (t_0 measured in years, $\beta = 1.38$ and $r = 0.39$ for a worldwide population of earthquakes with magnitude in the range $5.8 < M < 7.3$. Similarly, $-\ln t_0 = 17.25$, $\beta = 2.3$ and $r = 0.66$ for Southern California. A recent analysis for 1999 earthquakes with magnitude $M > 3$ recorded in Vrancea between 1974 and 2004 gives the average values[3] $-\ln t_0 = 9.68$, $\beta = 1.89$ and $r = 0.54$ (a direct fit to the logarithmic recurrence law (2) gives $-\ln t_0 = 8.99$, $\beta = 1.76$ and $r = 0.50$).

[1]One can hardly appraise Richter's genius in 1935, in definining the magnitude M as the decimal logarithm of the maximum displacement d in microns recorded by a special seismograph placed somewhere at 100km from the epicentre. It makes the energy $E \sim R^3$ be measured by its flux $E_s \sim R^2$ as $E_s \sim d$, hence $\lg E \sim (3/2) \lg E_s \sim (3/2)M$, which leads to $b = 2.3 \cdot (3/2)$ in $\ln(E/E_0) = bM$.

In general, parameter r is the inverse dimension of the focal zone, for a uniform mechanism of accumulating energy, so that a value close to $r \sim 0.5$, as obtained for Vrancea, indicates a seismic fault geometry of the focal zone, in agreement with geophysical data on depth and geographical distribution of earthquake epicentres in this region.

The identification of the mean recurrence time and the seismicity rate is of utmost importance in guiding our knowledge. Earthquakes with magnitude greater than M are characterized, for instance, by a mean recurrence time $t_r = t_0 e^{\beta M}$, a relationship which may guide our search for predicting large seismic events. Although large deviations are usually associated with this mean periodicity, of the order of $(\sqrt{2}-1)t_r$ (originating in the Poisson-like distribution $\sim e^{-t/t_r}$), it seems that the big seismic events are, generally, to be expected in accordance with this rhytmicity.

Over short time the seismic activity is more complex. First, seisms that accompany the main shocks may occur, like aftershocks and foreshocks. They are produced by a self-replication mechanism,[4] which leads to the well-known Omori's law $\sim 1/\tau$ for their rate, where τ is the time measured with respect to the main shock. Their original, generating distribution is exponential, and holds, together with corresponding Omori's law, for various other variables characterizing this seismic activity, like the difference in magnitude, or the inverse of the released energy. On this basis, Bath's empirical law was established (the average highest aftershock is by 1.2 lower in magnitude than the main shock), as well as the rate of the released energy $E \sim 1/\tau$.[4]

Omori's law is, however, only a particular aspect of the seismic activity over short term. It requires a determined relationship between the magnitude and time elapsed from the main shock, which is not warranted always by the empirical data. In fact, the distinction between main shocks and accompanying seismic events is unsharp, any seism being at the same time an aftershock, a main shock on its own, or the foreshock announcing a main shock. In fact, magnitude and time are decoupled, as independent statistical variables, and the seismic activity is governed by the seismicity rate.

The seimic activity over short term is described by the next-earthquake distribution function

$$D(\tau) = dN/Nd\tau = \frac{1}{N}\sum_i \delta(t_{i+1} - t_i - \tau) \ , \tag{3}$$

where N is the number total of earthquakes and t_i is the moment of occurrence of the i-th earthquake in a temporal series. This function must depend on a time scale, and the only one available is given by the frequency $R = t_0^{-1} e^{-\beta M}$ of $N = N_{ex}$ earthquakes with magnitude exceeding the value M. For reasons of normalization we must have $D(\tau) = Rf(R\tau)$, where f is a universal function. This scaling relationship leads to inverse power laws (as elementary solutions for its Laplace transform), as well as superpositions of such elementary solutions for its general solution.[5]

On the other hand, for small values of τ we have $D(\tau) \sim t \sim E^r \sim 1/\tau^r$, such that $D(\tau) \sim R/(R\tau)^r$ in this limit. We obtains thereby a clustering effect, given by the fact that large energies are released in short time, governed by a power law with exponent r different from unity (since earthquakes are distributed in time with dimension one while their energy is accumulated in space, with a higher dimension). Aftershocks do contribute to this exponent r, as governed by exponent unity, such that r in $D(\tau)$ should be rather considered a fitting exponent. Moreover, large τ in $D(\tau)$ are rare events, so that it is natural to assume a Poisson-like distribution $D(\tau) \sim e^{-R\tau/B}$ for large values of τ, and, therefore,

$$D(\tau) = C \cdot R \frac{1}{(R\tau)^r} \cdot e^{-R\tau/B} \tag{4}$$

for the next-earthquake distribution,[6] where exponent r is a fitting parameter. This is an Euler's gamma function distribution, and $CB^{1-r}\Gamma(1-r)=1$. parameter B originates in the exact time-energy relatonship (1), that leads actually to an increased $R=t_0^{-1}(1+e^{bM})^r$ as compared with $R=t_0^{-1}e^{-\beta M}$, which is corrected by factor, say, $2^r=B$ for vanishing M, *i.e.* vanishing energies, *i.e.* large τ.

It is worth emphasizing the status of fitting parameter for exponent r in (4). Rigorously speaking, the exponent r in the limit $R\tau \to 0$ in (4) is the same as parameter r obtained from the recurrence law (0.5 for Vrancea, for instance, or 0.66 for California). The aftershocks are produced by a distinct mechanism of self-replication, which acts independently of the seimicity rate, so they do contribute a similar power law in this limit, with exponent unity. It follows that r in (4) may acquire in fact even higher values in the limit $R\tau \to 0$ (and deviations from scaling). On the other hand, equation (4) gives an increased weight to data corresponding to large values of $R\tau$, which makes the fitting exponent r acquire lower values. Therefore, we expect a lower value for r from fitting pair distribution data, in comparison with the value obtained for r by fitting the recurrence law.

Indeed, the pair distribution $D(\tau)$ has been analysed for a worldwide population of earthquakes with parameters $r=0.33$, $B=1.58$ and $C=1/2$.[7] A similar analysis was carried out for 1999 earthquakes recorded in Vrancea with magnitude $M>3$ between 1974 and 2004, with parameters $r=0.25$, $B=1.17$ and $C=0.71$.[6] The results were used to estimate the seismic hazard for Vrancea, for $M>5$, for instance. It is obtained cca 0.8% a probability of having two earthquakes in Vrancea with $M>5$ in the same day, then a gradual, but slow, decreasing of probability. For instance, after 60 days, we experience 0.2% a probability of having another earthquake with $M>5$ in Vrancea, approaching slowly the probability unity in about one year and a quarter, which is practically the mean recurrence time for such earthquakes.

The pair distribution $D(\tau)$ can be further generalized, to statistical correlation functions of the form $P(M,t;M_0)$, giving the probability of having an earthquake of magnitude M at time t following the occurrence of an earthquake of magnitude M_0.[8] Such correlations functions have been analyzed for Vrancea, with the result of being able to use them for short-term prediction of small earthquakes, with a good confidence level because the statistics is good, but they turned out to be practically useless for greater earthquakes, where the statistics is poor, and the confidence level is very low. Obviously, their version corresponding to earthquakes with magnitude greater than M and, respectively, M_0 are also relevant, and, in this context, such versions with $M=M_0$ are in fact, precisely, the pair distribution $D(t)$ discussed above.

This nearly completes our basic quantitative knowledge of eartqhuakes, their focal mechanism and their statistical distributions.

The present project aims at further developing this knowledge according to the following objectives.

Objectives and relevance

1. Extension of statistics, both for Vrancea and other regions, in order to include more refined distributions with respect to other various variables, like location, depth, ranges of magnitude, etc. The relevance consists in possibly revealing new statistical patterns in earthquake occurrence, which may be useful in assessing seimic hazard.

2. Application of the programme described above for analyzing the seismic activity to other regions of seismic interest. In particular, the application of the mean recurrence time is desirable for analyzing the succession of big seismic events, the analysis of the recurrence law for determining the basic parameters t_0^{-1}, β and r, and the study of the pair distribution over short time.

3. For rich statistics, the correlation function $P(M,t;M_0)$ of earthquakes with magnitude greater than M occurring at time t after the occurrence of an earthquake with magnitude greater than M_0 is particularly interesting, as a direct generalization of the pair distribution function (corresponding to $M = M_0$). It is conceivable that the scale time is $R = t_0^{-1}e^{-\beta(M+M_0)/2}$ here, and such a hypothesis is worth investigating.

Such a research programme should be conducted in conjunction with the related one, described in Ref. 9.

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