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Citation: *J. Appl. Phys.* **112**, 024905 (2012); doi: 10.1063/1.4737610

View online: <http://dx.doi.org/10.1063/1.4737610>

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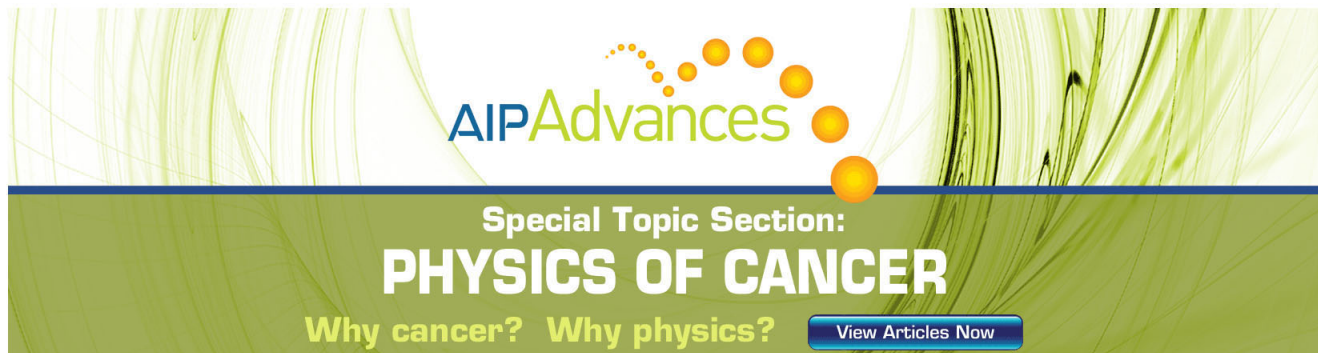
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## A generalization of the dipolar force

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(Received 3 May 2012; accepted 20 June 2012; published online 20 July 2012)

The static dipolar force is generalized to time-dependent classical distributions of dipoles and electromagnetic fields. This force may exhibit a remarkable resonance character for induced dipoles, related to the pole structure of the polarizabilities. The resonance phenomenon is illustrated for two macroscopic polarizable bodies, with mutually induced polarizations, using the well-known Lorentz-Drude model for the dielectric response with optical dispersion and a characteristic (resonance) frequency. Specifically, the calculations are performed for distances much longer than the dimension of the bodies (“point-like” bodies), but shorter than the characteristic wavelength (sub-wavelength, stationary, near-field regime). The polarizations are induced via a localized external field acting upon only one body. The force is practically vanishing for distinct substances and acquires a non-vanishing value for identical substances. It falls off as the 7-th power of the distance, being reminiscent of the van der Waals-London force. The conditions of validity of this resonance phenomenon are emphasized. Particular cases corresponding to independent external fields or two isolated, interacting bodies (closed system) are also analyzed, with similar conclusions regarding the resonance character of the force. © 2012 American Institute of Physics. [<http://dx.doi.org/10.1063/1.4737610>]

### I. INTRODUCTION

The interaction of light with matter in the sub-wavelength regime (*i.e.*, for distances much shorter than the relevant wavelengths) is of much interest today in the context of nano-plasmonics, metamolecules and metamaterials, enjoying a wide range of applications.<sup>1–8</sup> In general, the electromagnetic interaction acquires various forms, depending on the particular conditions of motion of the electric charges and currents. As it is well known, classical particles and electromagnetic fields are subjected to the ponderomotive Lorentz force and Maxwell stress forces. Quantum mechanical perturbation theory leads to van der Waals-London or Casimir forces acting between atoms and molecules.<sup>9–15</sup> The vacuum zero-point fluctuations, associated with the motion of the electric polarization, generate similar forces in matter.<sup>16–22</sup> Usually, such forces are related to the multipolar expansion of the electromagnetic interaction, especially the dipolar interaction. Particular importance is given to the resonant character of such forces, in various contexts.<sup>23,24</sup> A resonant energy transfer is well known for the dipolar interaction of donor-acceptor molecules (Forster mechanism),<sup>25–28</sup> and resonant polarizabilities have been identified for aromatic molecules placed between two metallic clusters.<sup>29</sup>

Long-range forces acting between polarizable bodies (dipole oscillators) in intense optical fields have been identified,<sup>7,30–32</sup> exhibiting, typically, characteristic spatial oscillations, related to the wavelength of the external fields. These optical forces may lead to bound states and ordered crystalline structures, generating a genuine “optical matter.”<sup>31</sup> Usually, the derivation of these forces do not exploit the

resonant character associated with the pole structure of the dielectric response function (polarizability). We get here the optical force as a particular case of our general approach.

We present here a generalization of the static dipolar force arising from the classical time-dependent polarization of two macroscopic bodies. The force exhibits a remarkable resonance character, in the sense that it acquires finite values for identical substances and is practically vanishing for distinct substances. In order to illustrate this resonance phenomenon we employ the well-known electric susceptibility (dielectric response function), which exhibits optical dispersion associated, in general, with a set of characteristic frequencies (Lorentz-Drude model<sup>33–37</sup>). The calculations are carried out in the sub-wavelength (stationary) regime (near-field zone) for two “point-like” polarizable bodies, each with one characteristic frequency (and leaving aside the spatial dispersion). The distances are much larger than the dimension of the bodies, but smaller than the characteristic wavelengths. The polarization is driven by a localized, external electric field acting upon only one body. Particular cases corresponding to independent external fields or two isolated, interacting bodies (closed system) are also analyzed, with similar conclusions regarding the resonance character of the force.

### II. DYNAMIC DIPOLAR FORCE

We consider two polarizable bodies, one placed at  $\mathbf{R}$  (denoted by  $a$ ) and another placed at the origin (denoted by  $b$ ). All the quantities pertaining to these two bodies are labelled by  $a$  and, respectively,  $b$ . The bodies have charge and current densities  $\rho_{a,b}(\mathbf{r}, t)$  and  $\mathbf{j}_{a,b}(\mathbf{r}, t)$ , which generate electric and magnetic fields  $\mathbf{E}_{a,b}(\mathbf{r}, t)$  and  $\mathbf{H}_{a,b}(\mathbf{r}, t)$ . In general, the quantities which belong to body  $a$  (placed at  $\mathbf{R}$ )

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depend parametrically on the position  $\mathbf{R}$ ; for brevity, we omit occasionally to write explicitly this dependence. The Lorentz force acting upon body  $a$  on behalf of body  $b$  is given by

$$\mathbf{F} = \int d\mathbf{r} \rho_a(\mathbf{r}, t) \mathbf{E}_b(\mathbf{r}, t) + \frac{1}{c} \int d\mathbf{r} \mathbf{j}_a(\mathbf{r}, t) \times \mathbf{H}_b(\mathbf{r}, t). \quad (1)$$

It is convenient to use Fourier transforms (wavevector  $\mathbf{k}$  and frequency  $\omega$ ) and to introduce the electromagnetic potentials  $\Phi$  and  $\mathbf{A}$ . The fields can then be written as

$$\begin{aligned} \mathbf{E}_b(\omega, \mathbf{k}) &= i \frac{\omega}{c} \mathbf{A}_b(\omega, \mathbf{k}) - i \mathbf{k} \Phi_b(\omega, \mathbf{k}), \\ \mathbf{H}_b(\omega, \mathbf{k}) &= i \mathbf{k} \times \mathbf{A}_b(\omega, \mathbf{k}). \end{aligned} \quad (2)$$

As usually, we average the force given by Eq. (1) over the time  $T$ , much longer than the period of the relevant frequencies, according to the recipe  $(1/T) \int dt$ . Such an averaging procedure implies an infrared frequency cutoff  $\Delta\omega \simeq 1/T$ . Using the continuity equation  $\omega \rho_a(\omega, \mathbf{k}) - \mathbf{k} \cdot \mathbf{j}_a(\omega, \mathbf{k}) = 0$ , we get

$$\begin{aligned} \mathbf{F} &= \frac{i}{(2\pi)^4 T} \int d\omega d\mathbf{k} \cdot \mathbf{k} \left[ \frac{1}{c} \mathbf{j}_a^*(\omega, \mathbf{k}) \mathbf{A}_b(\omega, \mathbf{k}) \right. \\ &\quad \left. - \rho_a^*(\omega, \mathbf{k}) \Phi_b(\omega, \mathbf{k}) \right]. \end{aligned} \quad (3)$$

As it is well known, the electromagnetic potentials are given by

$$\begin{aligned} \Phi_b(\mathbf{r}, t) &= \int d\mathbf{r}' \frac{\rho_b(\mathbf{r}', t - |\mathbf{r} - \mathbf{r}'|/c)}{|\mathbf{r} - \mathbf{r}'|}, \\ \mathbf{A}_b(\mathbf{r}, t) &= \frac{1}{c} \int d\mathbf{r}' \frac{\mathbf{j}_b(\mathbf{r}', t - |\mathbf{r} - \mathbf{r}'|/c)}{|\mathbf{r} - \mathbf{r}'|}, \end{aligned} \quad (4)$$

or, by taking the Fourier transforms,

$$\begin{aligned} \Phi_b(\omega, \mathbf{k}) &= \rho_b(\omega, \mathbf{k}) G(\omega, k), \\ \mathbf{A}_b(\omega, \mathbf{k}) &= \frac{1}{c} \mathbf{j}_b(\omega, \mathbf{k}) G(\omega, k), \end{aligned} \quad (5)$$

where

$$G(\omega, k) = \frac{4\pi}{k^2 - \omega^2/c^2 - i \text{sgn}(\omega) 0^+} \quad (6)$$

is the Green function of the (retarded) Coulomb interaction. We can check easily the Lorenz gauge  $\mathbf{k} \cdot \mathbf{A}(\omega, \mathbf{k}) - (\omega/c) \Phi(\omega, \mathbf{k}) = 0$  on these equations (related to the continuity equation for the distributions labelled by  $b$ ). Inserting Eqs. (5) into Eq. (3), the force becomes

$$\begin{aligned} \mathbf{F} &= \frac{i}{(2\pi)^4 T} \int d\omega d\mathbf{k} \cdot \mathbf{k} \left[ \frac{1}{c^2} \mathbf{j}_a^*(\omega, \mathbf{k}) \mathbf{j}_b(\omega, \mathbf{k}) \right. \\ &\quad \left. - \rho_a^*(\omega, \mathbf{k}) \rho_b(\omega, \mathbf{k}) \right] G(\omega, k). \end{aligned} \quad (7)$$

We can see that the force acting on body  $a$  on behalf of body  $b$  is the same (and of opposite direction) as the force

acting upon body  $b$  on behalf of body  $a$ , as expected (the law of action and reaction).

Further on, we assume that the two bodies are ‘‘point-like’’, *i.e.*, their dimensions are much smaller than any distance of interest (this amounts to a short-wavelength cutoff  $\Delta\mathbf{k}$ ). Specifically, we assume that their polarizations (dipole moment of the unit volume) are given by

$$\mathbf{p}_a = v_a \mathbf{P}_a(t, \mathbf{R}) \delta(\mathbf{r} - \mathbf{R}), \quad \mathbf{p}_b = v_b \mathbf{P}_b(t, \mathbf{R}) \delta(\mathbf{r}), \quad (8)$$

where  $v_{a,b}$  are the volumes of the two bodies and  $\mathbf{P}_{a,b}(t, \mathbf{R})$  are the polarization vectors (dipole moments per unit volume). For generality, we assume that the polarization vector  $\mathbf{P}_b(t, \mathbf{R})$  of body  $b$  (placed at the origin) may depend on  $\mathbf{R}$ . Using  $\rho_{a,b} = -\text{div} \mathbf{p}_{a,b}$  and the continuity equation, we get the polarization charge and current distributions

$$\begin{aligned} \rho_a(\mathbf{r}, t) &= -v_a [\mathbf{P}_a(t, \mathbf{R}) \text{grad}] \delta(\mathbf{r} - \mathbf{R}), \\ \mathbf{j}_a(\mathbf{r}, t) &= v_a \frac{\partial \mathbf{P}_a(t, \mathbf{R})}{\partial t} \delta(\mathbf{r} - \mathbf{R}) \end{aligned} \quad (9)$$

and

$$\begin{aligned} \rho_b(\mathbf{r}, t) &= -v_b [\mathbf{P}_b(t, \mathbf{R}) \text{grad}] \delta(\mathbf{r}), \\ \mathbf{j}_b(\mathbf{r}, t) &= v_b \frac{\partial \mathbf{P}_b(t, \mathbf{R})}{\partial t} \delta(\mathbf{r}). \end{aligned} \quad (10)$$

For the Fourier transforms we get

$$\begin{aligned} \rho_a(\omega, \mathbf{k}; \mathbf{R}) &= -i v_a [\mathbf{k} \mathbf{P}_a(\omega; \mathbf{R})] e^{-i\mathbf{k}\mathbf{R}}, \\ \mathbf{j}_a(\omega, \mathbf{k}; \mathbf{R}) &= -i v_a \omega \mathbf{P}_a(\omega; \mathbf{R}) e^{-i\mathbf{k}\mathbf{R}} \end{aligned} \quad (11)$$

and

$$\begin{aligned} \rho_b(\omega, \mathbf{k}; \mathbf{R}) &= -i v_b [\mathbf{k} \mathbf{P}_b(\omega; \mathbf{R})], \\ \mathbf{j}_b(\omega, \mathbf{k}; \mathbf{R}) &= -i v_b \omega \mathbf{P}_b(\omega; \mathbf{R}). \end{aligned} \quad (12)$$

Introducing these charge and current densities in Eq. (7) we can write the force as

$$\begin{aligned} \mathbf{F} &= \frac{i v_a v_b}{(2\pi)^4 T} \int d\omega d\mathbf{k} \cdot \mathbf{k} \left\{ \frac{\omega^2}{c^2} \mathbf{P}_a^*(\omega, \mathbf{R}) \mathbf{P}_b(\omega, \mathbf{R}) \right. \\ &\quad \left. - [\mathbf{k} \mathbf{P}_a^*(\omega, \mathbf{R})][\mathbf{k} \mathbf{P}_b(\omega, \mathbf{R})] \right\} e^{i\mathbf{k}\mathbf{R}} G(\omega, k). \end{aligned} \quad (13)$$

The  $\mathbf{k}$ -factors in Eq. (13) can be replaced by the derivatives of the exponential  $e^{i\mathbf{k}\mathbf{R}}$  with respect to  $\mathbf{R}$ ; the polarizations  $\mathbf{P}_{a,b}(\omega, \mathbf{R})$  can be related to the external fields  $\mathbf{E}_{a,b}^{\text{ext}}(\omega, \mathbf{R})$  by means of the (scalar) polarizabilities,

$$\mathbf{P}_{a,b}(\omega, \mathbf{R}) = \alpha_{a,b}(\omega) \mathbf{E}_{a,b}^{\text{ext}}(\omega, \mathbf{R}); \quad (14)$$

and the integration over  $\mathbf{k}$  of  $e^{i\mathbf{k}\mathbf{R}} G(\omega, k)$  (performed in the upper  $k$ -half-plane) is the spherical wave

$$\frac{1}{(2\pi)^3} \int d\mathbf{k} e^{i\mathbf{k}\mathbf{R}} G(\omega, k) = \frac{e^{i\omega R}}{R}; \quad (15)$$

the force given by Eq. (13) becomes

$$\mathbf{F} = \frac{v_a v_b}{2\pi T} \int d\omega \left[ \frac{\omega^2}{c^2} \mathbf{P}_a^*(\omega, \mathbf{R}) \mathbf{P}_b(\omega, \mathbf{R}) + P_{ai}^*(\omega, \mathbf{R}) P_{bj}(\omega, \mathbf{R}) \frac{\partial^2}{\partial R_i \partial R_j} \right] \frac{\partial}{\partial \mathbf{R}} \frac{e^{i\omega R}}{R}, \quad (16)$$

or

$$\mathbf{F} = \frac{v_a v_b}{2\pi T} \int d\omega \alpha_a^*(\omega) \alpha_b(\omega) E_{ai}^{ext*}(\omega, \mathbf{R}) E_{bj}^{ext}(\omega, \mathbf{R}) \cdot \left( \frac{\omega^2}{c^2} \delta_{ij} + \frac{\partial^2}{\partial R_i \partial R_j} \right) \frac{\partial}{\partial \mathbf{R}} \frac{e^{i\omega R}}{R}, \quad (17)$$

where summations are performed over the components labels  $i$  and  $j$ . We can see that the force given by these equations is a generalization of the static dipolar force (it contains frequency-dependent factors); we may call it a dynamic dipolar force. Indeed, in the static limit  $\omega \rightarrow 0$  and using (static) polarizations  $\mathbf{P}_{a,b}$ , we can check easily that this force is proportional to  $\mathbf{F} = -\partial U / \partial \mathbf{R}$ , where

$$U = \frac{\mathbf{P}_a \mathbf{P}_b}{R^3} - \frac{3(\mathbf{P}_a \mathbf{R})(\mathbf{P}_b \mathbf{R})}{R^5} \quad (18)$$

is the static dipolar interaction energy.

It is also worth noting that Eq. (17) leads to the well-known optical force in the wave-zone  $\omega R/c \gg 1$ . Indeed, in this case we may retain only the spatial derivatives of the exponential in Eq. (17). Assuming for simplicity a monochromatic external field  $\mathbf{E}^{ext}$  parallel with  $\mathbf{R}$ , acting upon the both bodies, we get immediately from Eq. (17) the interaction energy of the two bodies  $U \sim v_a v_b (\omega/c)^2 |E^{ext}|^2 (\cos \omega R/c)/R$ , which is the typical result corresponding to the optical force.<sup>30–32</sup>

It is worth noting that the dipolar force can also be obtained by starting with the interaction Lagrangian

$$L_{int} = - \int d\mathbf{r} \rho_a(\mathbf{r}, t) \Phi_b(\mathbf{r}, t) + \frac{1}{c} \int d\mathbf{r} \mathbf{j}_a(\mathbf{r}, t) \mathbf{A}_b(\mathbf{r}, t), \quad (19)$$

corresponding to the charge and current densities of the body  $a$  placed in the external field generated by body  $b$ . Following the procedure described above we get

$$L_{int} = \frac{v_a v_b}{2\pi T} \int d\omega \left[ \frac{\omega^2}{c^2} \mathbf{P}_a^*(\omega, \mathbf{R}) \mathbf{P}_b(\omega, \mathbf{R}) + P_{ai}^*(\omega, \mathbf{R}) P_{bj}(\omega, \mathbf{R}) \frac{\partial^2}{\partial R_i \partial R_j} \right] \frac{e^{i\omega R}}{R}. \quad (20)$$

The force derived from such an interaction Lagrangian is given by  $\mathbf{F}' = \partial L_{int} / \partial \mathbf{R}$ . This force includes the effect of the external fields, through the  $\mathbf{R}$ -dependence of the polarizations (external fields). In order to get the force  $\mathbf{F}$ , these effects must be subtracted from the force  $\mathbf{F}'$ . If the polarizations (external fields) do not depend on  $\mathbf{R}$ , the two forces  $\mathbf{F}$  and  $\mathbf{F}'$  coincide. If the bodies are mutually polarized (as we discuss in the Sec. III), the interaction becomes in fact a self-interaction and a factor 1/2 must be included in the interaction Lagrangian.

We can see from Eq. (20) that the same result is obtained by starting with charge and current densities of the body  $b$  placed in the external field generated by body  $a$ , as expected. In view of the symmetry  $a \leftrightarrow b$  ( $\mathbf{R} \rightarrow -\mathbf{R}$ ), the law of action and reaction is fulfilled.

In general, except for the static contribution, the  $\omega$ -integral in Eq. (17) has a finite value, so that the force is vanishing in the limit  $T \rightarrow \infty$ . However, for a pole structure of the polarizabilities  $\alpha_{a,b}(\omega)$ , especially for identical substances  $a = b$  ( $\alpha_a(\omega) = \alpha_b(\omega)$ ), the integration over  $\omega$  may give a singular (indefinitely increasing) contribution, which, together with the vanishing factor  $1/T$ , may lead to a finite value for the force. Such a circumstance has an obvious resonance character, and we may call the corresponding force a resonant dipolar force. In order to illustrate this resonance phenomenon, we make further on a few simplifying assumptions.

### III. MUTUALLY POLARIZED BODIES

We assume that an external electric field  $\mathbf{E}_a^{ext}(t) = \mathbf{E}(t)$  is attached to the body  $a$ , which falls off sufficiently rapidly with the distance, such that it acts only upon the body  $a$  and not upon body  $b$ . Consequently,  $\mathbf{E}(t)$  does not depend on  $\mathbf{R}$  (it depends only on the time  $t$ ); similarly, the polarization  $\mathbf{P}_a(t)$  induced in body  $a$  depends only on the time  $t$  and does not depend on the position  $\mathbf{R}$ . In order to simplify the calculations we assume that the external field  $\mathbf{E}(t)$  is of the form  $\mathbf{E}(t) = \mathbf{E} \Delta t \delta(t)$ , where  $\Delta t$  is a characteristic duration of the external field pulse; for numerical estimations it can be taken of the order of an ultraviolet frequency cutoff. Then, the Fourier transform of the external field is  $\mathbf{E}(\omega) = \mathbf{E} \Delta t$ .

Further, we assume that the external field for body  $b$  is the field  $\mathbf{E}_a(\mathbf{r} = 0, t)$  generated by body  $a$ . This field can be computed most conveniently by making use of the Fourier transforms and equations of the type (2), (5) and (6). The integration over  $\mathbf{k}$  can be performed straightforwardly, as in Eq. (15), leading to a spherical wave. The intervening  $\mathbf{k}$ -factors can be handled by noticing that  $\rho_a(\omega, \mathbf{k}; \mathbf{R})$  and  $\mathbf{j}_a(\omega, \mathbf{k}; \mathbf{R})$  contain the factor  $e^{-i\mathbf{k}\mathbf{R}}$ , according to Eq. (11). Therefore, the  $\mathbf{k}$ -factors are replaced by suitable derivatives of the form  $\partial / \partial \mathbf{R}$ , as we did in deriving Eq. (17) for the force. By straightforward calculations we get

$$\mathbf{E}_a(\omega, \mathbf{r} = 0; \mathbf{R}) = v_a \alpha_a(\omega) \left[ \frac{\omega^2}{c^2} \mathbf{E}(\omega) + \left( \mathbf{E}(\omega) \frac{\partial}{\partial \mathbf{R}} \right) \frac{\partial}{\partial \mathbf{R}} \right] \frac{e^{i\omega R}}{R}, \quad (21)$$

or

$$\mathbf{E}_a(\omega, \mathbf{r} = 0; \mathbf{R}) = v_a \alpha_a(\omega) \left\{ \frac{\omega^2}{c^2} \left[ \mathbf{E}(\omega) - \frac{(\mathbf{E}(\omega) \mathbf{R}) \mathbf{R}}{R^2} \right] + \frac{1}{R} \left( i \frac{\omega}{c} - \frac{1}{R} \right) \left[ \mathbf{E}(\omega) - 3 \frac{(\mathbf{E}(\omega) \mathbf{R}) \mathbf{R}}{R^2} \right] \right\} \frac{e^{i\omega R}}{R}. \quad (22)$$

The external field acting upon body  $b$  is  $\mathbf{E}_b^{ext}(\omega, \mathbf{R}) = \mathbf{E}_a(\omega, \mathbf{r} = 0; \mathbf{R})$ .

We turn now to establishing a model for the polarizabilities  $\alpha_{a,b}(\omega)$ .

In general, leaving aside the spatial dispersion, the polarization  $\mathbf{P}(\omega; \mathbf{R})$  is related to an external electric field  $\mathbf{E}^{ext}(\omega; \mathbf{R})$  by

$$\mathbf{P}(\omega; \mathbf{R}) = \chi(\omega)\mathbf{E}_t(\omega; \mathbf{R}) = \chi(\omega)[\mathbf{E}^{ext}(\omega; \mathbf{R}) + \mathbf{E}^{int}(\omega; \mathbf{R})], \quad (23)$$

where  $\chi(\omega)$  is the electric susceptibility,  $\mathbf{E}^{int}(\omega; \mathbf{R})$  is the internal electric field (generated by the polarization charges and currents) and  $\mathbf{E}_t = \mathbf{E}^{ext} + \mathbf{E}^{int}$  is the total electric field. We can view the polarization of matter as consisting of a slight displacement  $\mathbf{u}(\mathbf{r}, t)$  of the mobile charges, say electrons, with respect to their neutralizing, (quasi-) rigid background of positive (ionic) charges. Under these circumstances, a density imbalance  $\delta n = -\text{div}\mathbf{u}$  occurs, which gives rise to a polarization charge density  $\rho = e\text{div}\mathbf{u}$  and a polarization current density  $\mathbf{j} = -e\text{div}\mathbf{u}/\partial t$ , where  $n$  is the particle density and  $-e$  is the particle (electron) charge. We can see that the polarization is given by  $\mathbf{P} = -en\mathbf{u}$ . This representation of the electric polarization turned out to be very useful in dealing with electromagnetic phenomena in macroscopic bodies, including reflection, refraction, surface plasmons, and polaritons in a half-space (semi-infinite solid),<sup>38,39</sup> a slab,<sup>40</sup> electromagnetic coupling between two half-spaces<sup>41</sup> or the Mie scattering of the electromagnetic field by a sphere.<sup>42</sup> The displacement  $\mathbf{u}$  obeys Newton's law of motion. Let us assume a characteristic restoring (elastic) force acting locally, with a characteristic frequency  $\omega_c$ . For a homogeneous piece of matter, leaving aside the spatial dispersion and relativistic and magnetic effects, Newton's law reads

$$m\ddot{\mathbf{u}} = -e\mathbf{E}_t - m\omega_c^2\mathbf{u} - m\gamma\dot{\mathbf{u}}, \quad (24)$$

where  $m$  is the particle mass and a damping term is included ( $\gamma \ll \omega_c$ ). This equation can be transformed straightforwardly into an equation for the polarization  $\mathbf{P}$ , leading to  $\mathbf{P}(\omega) = \chi(\omega)\mathbf{E}_t(\omega)$  and the well-known electric susceptibility

$$\chi(\omega) = -\frac{1}{4\pi} \frac{\omega_p^2}{\omega^2 - \omega_c^2 + i\omega\gamma}, \quad (25)$$

where  $\omega_p = \sqrt{4\pi ne^2/m}$  is the plasma frequency. This is the well-known Lorentz-Drude (plasma) model of polarizable matter,<sup>33-37</sup> which assumes a homogeneous, isotropic matter, without spatial dispersion, represented by a field of harmonic oscillators of frequency  $\omega_c$ . It leads to the well-known Lydane-Sachs-Teller dielectric function  $\varepsilon(\omega) = 1 + 4\pi\chi(\omega)$ , including both longitudinal ( $\omega_L = \sqrt{\omega_c^2 + \omega_p^2}$ ) and transverse modes ( $\omega_T = \omega_c$ ).<sup>43</sup> For conductors  $\omega_c = 0$ , for dielectrics  $\omega_c \neq 0$ . We use Eq. (25) for the dielectric response and consider  $\omega_p$ ,  $\omega_c$  and  $\gamma$  as parameters characterizing each particular substance. The model can be generalized by including the spatial dispersion, several characteristic frequencies  $\omega_c$ , or by adding an external magnetic field, etc. It is worth noting the absence of the magnetic part of the Lorentz force in

Eq. (24), according to the non-relativistic motion of the slight displacement  $\mathbf{u}$ . It is easy to see that, apart from relativistic contributions, it would introduce non-linearities in Eq. (24), which are beyond our assumption of a small displacement  $u$ . Using spatial Fourier transforms, this approximation can be formulated as  $\mathbf{k}\mathbf{u}(\mathbf{k}) \ll 1$ , where  $\mathbf{k}$  is the wavevector.

According to Eqs. (2) and (4), the mean value of the internal fields  $\mathbf{E}_{a,b}^{int}$ , generated by the polarization charges  $\rho_{a,b}$  and currents  $\mathbf{j}_{a,b}$ , can be viewed as being proportional to the volumes  $v_{a,b}$  of the bodies. We consider these volumes sufficiently small (point-like bodies), such as we may neglect the internal fields  $\mathbf{E}_{a,b}^{int}$  in Eq. (23), which becomes

$$\mathbf{P}_a(\omega) \simeq \chi_a(\omega)\mathbf{E}(\omega), \quad \mathbf{P}_b(\omega, \mathbf{R}) \simeq \chi_b(\omega)\mathbf{E}_a(\omega, \mathbf{r} = 0; \mathbf{R}). \quad (26)$$

This approximation is valid for distances and wavelengths much larger than the size of the bodies, such that we may consider the bodies as being point-like. It amounts to use the electric susceptibility  $\chi(\omega)$  for the polarizability  $\alpha(\omega)$ . In general, the results presented here are valid as long as the polarizability has a pole structure as the dielectric susceptibility given by Eq. (25). Such a Lorentzian structure of the polarizability is well documented for large molecules, molecular aggregates and clusters.<sup>29,44-47</sup>

It is worth noting that, in general, the process of mutual polarization described here is an iterative process, *i.e.*, in the same manner as the field of body  $a$  polarizes body  $b$ , the field of body  $b$  polarizes in turn body  $a$ , and so on. Applying iteratively Eqs. (26), we can see easily that this succession of mutual polarizations amounts to an iteration factor  $\sim v_a v_b \chi_a \chi_b$  (and a geometric series with this ratio). Since the volumes  $v_{a,b}$  are considered as being very small, we may neglect the iterative nature of the polarization process.

We insert now the external fields  $\mathbf{E}(\omega)$ ,  $\mathbf{E}_a(\omega, \mathbf{r} = 0; \mathbf{R})$  (Eq. (21)) and the electric susceptibilities  $\chi_{a,b}(\omega)$  given by Eq. (25) in Eq. (17) for the dipolar force. For brevity, we omit the label  $c$  in  $\omega_{ca,b}$  and write simply  $\omega_{a,b}$ . It is easy to see that Eq. (17) implies an integration with respect to  $\omega$  of functions containing factors of the form

$$\frac{\omega^n}{(\omega^2 - \omega_a^2)^2 + \omega^2\gamma_a^2} \cdot \frac{1}{\omega^2 - \omega_b^2 + i\omega\gamma_b} e^{2i\omega R/c}, \quad (27)$$

where  $n$  is a positive integer (the integration is performed in the upper half-plane). We can see that the main contribution to these integrals comes from poles of the type  $\simeq \pm\omega_{a,b} + i\gamma_{a,b}/2$ . The oscillating factor  $e^{\pm 2i\omega_{a,b}R/c}$  can reduce appreciably the value of these integrals. We limit ourselves to  $\omega_{a,b}R/c \ll 1$ , *i.e.*,  $R \ll \lambda_{a,b}$ , where  $\lambda_{a,b} = c/\omega_{a,b}$  are the wavelengths of the characteristic frequencies  $\omega_{a,b}$  (much longer than the linear dimensions of the bodies). We call it the sub-wavelength regime (near-field zone, or stationary regime). Under these circumstances, we can omit the oscillating factor  $e^{\pm 2i\omega R/c}$  in Eq. (27), as well as terms containing factors of the form  $(\omega/c)^n$ , which give  $1/\lambda_{a,b}^n \ll 1/R^n$ . This approximation amounts to use

$$\mathbf{F} = \frac{v_a v_b}{2\pi T} \int d\omega \left[ \mathbf{P}_b(\omega, \mathbf{R}) \frac{\partial}{\partial \mathbf{R}} \right] \left[ \mathbf{P}_a^*(\omega) \frac{\partial}{\partial \mathbf{R}} \right] \frac{\partial}{\partial \mathbf{R}} \frac{1}{R} \quad (28)$$

for the force given by Eq. (16) and the field

$$\begin{aligned} \mathbf{E}_a(\omega, \mathbf{r} = 0; \mathbf{R}) &= v_a \chi_a(\omega) \left( \mathbf{E}(\omega) \frac{\partial}{\partial \mathbf{R}} \right) \\ \frac{\partial}{\partial \mathbf{R}} \frac{1}{R} &= -v_a \chi_a(\omega) \left( \mathbf{E}(\omega) \frac{\partial}{\partial \mathbf{R}} \right) \frac{\mathbf{R}}{R^3}, \end{aligned} \quad (29)$$

given by Eq. (21). We get easily

$$\mathbf{F} = \frac{3v_a^2 v_b \Delta t^2}{2\pi(4\pi)^3 T R^8} \omega_{pa}^4 \omega_{pb}^2 I_0 \left\{ \left[ E^2 + 4 \frac{(\mathbf{E}\mathbf{R})^2}{R^2} \right] \mathbf{R} - (\mathbf{E}\mathbf{R})\mathbf{E} \right\} \quad (30)$$

where

$$\begin{aligned} I_0 &= \int d\omega \frac{1}{(\omega^2 - \omega_a^2)^2 + \omega^2 \gamma_a^2} \operatorname{Re} \left( \frac{1}{\omega^2 - \omega_b^2 + i\omega \gamma_b} \right) \\ &= \int d\omega \frac{\omega^2 - \omega_b^2}{[(\omega^2 - \omega_a^2)^2 + \omega^2 \gamma_a^2][(\omega^2 - \omega_b^2)^2 + \omega^2 \gamma_b^2]}. \end{aligned} \quad (31)$$

As we see in Sec. IV, this integral has a resonance character.

It is worth noting that the same result for the force given by Eq. (30) is obtained from  $\mathbf{F} = \partial L_{int}/\partial \mathbf{R}$ , by using the interaction Lagrangian given by Eq. (20). Within the approximations used here (sub-wavelength regime) this interaction Lagrangian reads

$$\begin{aligned} L_{int} &= \frac{v_a^2 v_b}{4\pi T} \int d\omega \chi_b(\omega) |\chi_a(\omega)|^2 \left| \left[ \mathbf{E}(\omega) \frac{\partial}{\partial \mathbf{R}} \right] \frac{\mathbf{R}}{R^3} \right|^2 \\ &= \frac{v_a^2 v_b}{4\pi T} \int d\omega \chi_b(\omega) |\chi_a(\omega)|^2 \left( |\mathbf{E}(\omega)|^2 + 3 \frac{|\mathbf{E}(\omega)\mathbf{R}|^2}{R^2} \right)^2. \end{aligned} \quad (32)$$

#### IV. RESONANT FORCE

We compute here resonance integrals of the form

$$I_n = \int d\omega \frac{\omega^n (\omega^2 - \omega_b^2)}{[(\omega^2 - \omega_a^2)^2 + \omega^2 \gamma_a^2][(\omega^2 - \omega_b^2)^2 + \omega^2 \gamma_b^2]} \quad (33)$$

for even positive integers  $n$  (the integrals are vanishing for odd integers  $n$ ). We consider first the case of distinct bodies ( $a \neq b$ ). The function to be integrated in Eq. (33) has 4 (simple) poles in the upper half-plane, given by

$$\Omega_{a,b} = \pm \sqrt{\omega_{a,b}^2 - \gamma_{a,b}^2/4} + i\gamma_{a,b}/2 \quad (34)$$

and another set of 4 (simple) poles in the lower half-plane, given by  $\Omega_{a,b}^*$ . The integration must be performed over the upper half-plane. By straightforward calculations we get the contribution

$$I_n^{(1)} = \frac{\pi \omega_a^{n-2}}{\gamma_a} \operatorname{Re} \frac{\omega_a^2 - \omega_b^2 + i\omega_a \gamma_a}{(\omega_a^2 - \omega_b^2)^2 + 2i\omega_a \gamma_a (\omega_a^2 - \omega_b^2) + \omega_a^2 (\gamma_b^2 - \gamma_a^2)} \quad (35)$$

of the  $\Omega_a$ -poles and

$$I_n^{(2)} = -\pi \omega_b^{n-1} \operatorname{Im} \frac{1}{(\omega_b^2 - \omega_a^2)^2 + 2i\omega_b \gamma_b (\omega_b^2 - \omega_a^2) + \omega_b^2 (\gamma_a^2 - \gamma_b^2)} \quad (36)$$

of the  $\Omega_b$ -poles. We can see that in the limit  $\gamma_{a,b} \ll \omega_{a,b}$  the contribution of the  $\Omega_b$ -poles is vanishing and

$$I_n = \frac{\pi \omega_a^{n-2}}{\gamma_a} \cdot \frac{1}{\omega_a^2 - \omega_b^2}. \quad (37)$$

This integral is vanishing for a large difference  $|\omega_a - \omega_b|$  (distinct bodies).

The same result can be obtained by noticing that

$$\frac{1}{(\omega^2 - \omega_a^2)^2 + \omega^2 \gamma_a^2} = \left| \frac{1}{\omega^2 - \omega_a^2 + i\omega \gamma_a} \right|^2 \quad (38)$$

and using the representation

$$\begin{aligned} \frac{1}{\omega^2 - \omega_a^2 + i\omega \gamma_a} &\simeq \frac{1}{2\omega_a} \left( \mathcal{P} \frac{1}{\omega - \omega_a} - \mathcal{P} \frac{1}{\omega + \omega_a} \right) \\ &\quad - \frac{i\pi}{2\omega_a} [\delta(\omega - \omega_a) - \delta(\omega + \omega_a)] \end{aligned} \quad (39)$$

in the limit  $\gamma_a \rightarrow 0$ , where  $\mathcal{P}$  denotes the principal value. The main contribution to the integral  $I_n$  comes from the  $\delta^2$ -terms. According to our procedure, we put

$$\delta(\omega = 0) = \frac{1}{2\pi} \int dt = T/2\pi \quad (40)$$

and get

$$I_n = \frac{\pi T \omega_a^{n-2}}{4} \cdot \frac{\omega_a^2 - \omega_b^2}{(\omega_a^2 - \omega_b^2)^2 + \omega_a^2 \gamma_b^2}, \quad (41)$$

which coincides with Eq. (37) in the limit  $\gamma_b \rightarrow 0$ , providing we identify  $T = 4/\gamma_a$ . Therefore, we may take  $T$  of the order of  $1/\gamma_{a,b}$ , up to minor numerical factors which arise from various limiting procedures (for instance, in the example given above we assumed that the limit  $\gamma_a \rightarrow 0$  is taken first). Indeed, according to Eq. (25), the infrared frequency cutoff is of the order of  $\gamma_{a,b}$ . As a rule, we assume that the damping coefficients  $\gamma_{a,b}$  are small in comparison with the relevant frequencies, but finite.

Let us assume, for instance,  $\omega_a \simeq \omega_b = \omega_c$ , such as  $\omega_a - \omega_b = \delta\omega \ll \omega_c$  and, similarly,  $\gamma_a - \gamma_b = \delta\gamma \ll \gamma$  ( $=\gamma_a \simeq \gamma_b$ ). The careful estimation of the expressions given by Eqs. (35) and (36) shows that

$$I_n^{(1)} = -I_n^{(2)} \simeq \frac{\pi \omega_c^{n-3}}{\gamma} \frac{\delta\omega}{4\delta\omega^2 + \delta\gamma^2}, \quad (42)$$

*i.e.*, the integral  $I_n$  is vanishing (within this approximation).

We consider now two identical substances ( $a = b$ ), with  $\omega_a = \omega_b = \omega_c$  and  $\gamma_a = \gamma_b = \gamma$ . The integrals given by Eq. (33) read

$$I_n = \int d\omega \frac{\omega^n (\omega^2 - \omega_c^2)}{[(\omega^2 - \omega_c^2)^2 + \omega^2 \gamma^2]^2}. \quad (43)$$

The limiting procedure illustrated above (Eqs. (38)–(40)) cannot be applied. The integrand in Eq. (43) has now two double poles

$$\Omega_{1,2} = \pm \sqrt{\omega_c^2 - \gamma^2/4} + i\gamma/2 \quad (44)$$

in the upper half-plane (and two other double poles  $\Omega_{1,2}^*$  in the lower half-plane). The evaluation of the integral is straightforward, the result being

$$I_n = \frac{\pi(n-3)\omega_c^{n-4}}{4\gamma}. \quad (45)$$

We can see that the dipolar force is practically vanishing for distinct substances and has a non-vanishing value for identical substances.

It is worth emphasizing that the results given by Eqs. (37) and (45) have a resonant character, due to the occurrence of the  $\gamma$  parameter in the denominator (as well as the difference  $\omega_a^2 - \omega_b^2$  in Eq. (37)). This remarkable resonance phenomenon originates in the presence of the complex-conjugate poles in the lower half-plane, which may coalesce with the poles in the upper half-plane in the limit of a vanishing  $\gamma$ .

For identical substances, according to Eq. (45),  $I_0 = -3\pi/4\gamma\omega_c^4$ , and the force (Eq. (33)) can be written as

$$\mathbf{F} = -\frac{9v_a^2 v_b \Delta t^2}{8(4\pi)^3 T \gamma \omega_c^4} \omega_p^6 E^2 \left\{ \left[ 1 + 4 \frac{(\mathbf{nR})^2}{R^2} \right] \mathbf{R} - (\mathbf{nR}) \mathbf{n} \right\} \frac{1}{R^8}, \quad (46)$$

where  $\mathbf{n} = \mathbf{E}/E_0$  is the unit vector along the direction of the external field  $\mathbf{E}$  (we keep the distinction between the volumes of the two bodies). According to our procedure, we put  $T\gamma = 1$  in Eq. (46). We can see that the force given by Eq. (46) has an axial symmetry and falls off as the 7-th power of the distance. If we take the  $z$ -axis along the vector  $\mathbf{n}$  and denote by  $\theta$  the angle between  $\mathbf{n}$  and  $\mathbf{R}$ , the radial component of the force is proportional to  $1 + 3 \cos^2 \theta$ ; it is an attractive force. The tangential component of the force is proportional to  $\sin 2\theta$ . In view of its  $R^{-7}$ -dependence, the force given by Eq. (46) is reminiscent of the van der Waals-London dipolar force.

The resonance integral given by Eq. (33) has been estimated here for  $\omega_{a,b} \neq 0$ . If one, at least, of these two frequencies is vanishing (as for conductors) the situation is more complex. First, we note in Eq. (33) an asymmetry with respect to the frequencies  $\omega_{a,b}$  (and  $\gamma_{a,b}$ ), arising from the fact that the external field acts upon only one body (body  $a$ ). For  $\omega_b = 0$  we get  $I_0 = \pi(\gamma_a + \gamma_b)/\gamma_a \gamma_b \omega_a^4$  (compare with

Eq. (37)). For  $\omega_a = 0$  (or  $\omega_a = \omega_b = 0$ ), the integrand of  $I_0$  acquires an additional factor  $1/\omega^2$ . According to our infrared cutoff, we take this factor as the limit of  $1/(\omega^2 + \varepsilon^2)$ , where  $\varepsilon = \min(\gamma_a, \gamma_b)$ . For  $\omega_a = 0$ ,  $\omega_b \neq 0$ , and  $\varepsilon = \gamma_b < \gamma_a$ , we get  $I_0 = -\pi\gamma_b^2/\gamma_a^3(\gamma_a^2 - \gamma_b^2)\omega_b^2$ , which is vanishing in the limit  $\gamma_b \rightarrow 0$ . For  $\omega_a = 0$ ,  $\omega_b \neq 0$ , and  $\varepsilon = \gamma_a \leq \gamma_b$ , the integral is  $I_0 = \pi/2\gamma_a^3\omega_b^2$ , which is more singular than Eq. (37), due to the denominator  $\gamma_a^3$  in the limit  $\gamma_a \rightarrow 0$  (the integral has a double pole, in this case, at  $\omega = i\gamma_a$ ). For  $\omega_a = \omega_b = 0$ , we get  $I_0 = \pi/\max(\gamma_a, \gamma_b)(\gamma_a^2 - \gamma_b^2)^2$  for  $\gamma_a \neq \gamma_b$  and  $I_0 = 3\pi/8\gamma^5$  for  $\gamma_a = \gamma_b = \gamma$ , with an even more pronounced singular character (compare with Eq. (45)). In addition, the radial component of the force is repulsive in this case. However, it is worth noting that the equation of motion (24) and the electric susceptibility given by Eq. (25) are not applicable anymore for  $\omega_{a,b} = 0$  and  $\gamma_{a,b} \rightarrow 0$ . Including these particularities, the general conclusion regarding the resonant character of the force is preserved.

It is difficult to make a numerical estimation of the force  $F$  given by Eq. (46), since it depends on many unknown parameters. For illustrative purposes we take, for instance,  $\nu_p = \omega_p/2\pi = 10^{10}$  Hz,  $\nu_c = \omega_c/2\pi = 1$  MHz ( $\lambda_c = 300$  m),  $v_a = v_b = 1$  cm<sup>3</sup>,  $\Delta t = 1/\omega_p$ ,  $E = 10^3$  V/m ( $\simeq 3 \times 10^{-2}$  statV/cm), and  $R = 10$  m. Using these input data, we get a maximum value  $F = 10^{-11}$  dyn. It is worth noting that Eq. (46) includes the greater-than-unity factor  $(\omega_p/\omega_c)^4$  that can increase appreciably the force. Another numerical example might be  $\omega_p = \omega_c \simeq 10^{15}$  s<sup>-1</sup>, corresponding to a wavelength  $\lambda_c \simeq 1$   $\mu$ m, and  $v_a = v_b = 10^{-18}$  cm<sup>3</sup>, as for nanostructures of a linear dimension 100 Å (10 nm). Assuming that a classical treatment of the electromagnetic interaction is valid for such “quantum” dots, we get a maximum force  $F \simeq 10^{-32}$  dyn for  $\Delta t = 1/\omega_p$ ,  $E = 10^3$  V/m at the distance  $R = \lambda_c = 1$   $\mu$ m, which is a very weak force; it may increase for shorter distances and lower frequencies  $\omega_c$  (the external electric field could also be much higher in this case).

## V. ENERGY LOSS

The energy dissipated per unit time in body  $b$  by the electric field generated by body  $a$  (energy loss) is given by

$$W = \int d\mathbf{r} \mathbf{j}_b(\mathbf{r}, t) \mathbf{E}_a(\mathbf{r}, t) = v_b \frac{\partial \mathbf{P}_b(t)}{\partial t} \mathbf{E}_a(\mathbf{r} = 0, t), \quad (47)$$

where the current density  $\mathbf{j}_b(\mathbf{r}, t)$  is given by Eq. (10) and the electric field  $\mathbf{E}_a(\mathbf{r} = 0, t)$  is given by the Fourier transform of Eq. (21). In order to compute this energy loss we proceed as in the preceding sections, by taking the temporal Fourier transforms, averaging over the time  $T$  and using the electric susceptibilities of the type given by Eq. (25). We limit ourselves to distances  $R$  much smaller than the characteristic wavelengths  $\lambda_{a,b}$ , and get

$$W = \frac{iv_a^2 v_b}{2\pi TR^6} \int d\omega \omega \chi_b^*(\omega) |\chi_a(\omega)|^2 \left( |\mathbf{E}(\omega)|^2 + 3 \frac{|\mathbf{E}(\omega) \mathbf{R}|^2}{R^2} \right) \quad (48)$$

(compare with the interaction Lagrangian given by Eq. (32)) or, introducing the pulsed external field and the electric susceptibilities,

$$W = \frac{2v_a^2 v_b \gamma_b \Delta t^2}{(4\pi)^4 T R^6} \omega_{pa}^4 \omega_{pb}^2 J_0 (1 + 3 \cos^2 \theta) E^2, \quad (49)$$

where  $\theta$  is the angle between the external field  $\mathbf{E}$  and the position vector  $\mathbf{R}$  and

$$J_0 = \int d\omega \frac{\omega^2}{[(\omega^2 - \omega_a^2)^2 + \omega^2 \gamma_a^2][(\omega^2 - \omega_b^2)^2 + \omega^2 \gamma_b^2]} \quad (50)$$

(compare with the integral  $I_0$  for the force, given by Eq. (33)). It is worth noting the occurrence of the damping factor  $\gamma_b$  in Eq. (49), as expected. It is also worth noting the directional character of the energy loss (through the factor  $1 + 3 \cos^2 \theta$  in Eq. (49)).

The calculation of the integral  $J_0$  is straightforward. For  $\omega_a \neq \omega_b$  ( $\omega_{a,b} \neq 0$ ) and  $\gamma_{a,b} \ll \omega_{a,b}$ , we get

$$J_0 = \frac{\pi(\gamma_a + \gamma_b)}{\gamma_a \gamma_b (\omega_a^2 - \omega_b^2)^2}; \quad (51)$$

we can see that the energy loss is practically vanishing for distinct substances, *i.e.*, for large values of the difference  $|\omega_a - \omega_b|$  or for ideal bodies ( $\gamma_{a,b} \rightarrow 0$ ). For  $\omega_a = \omega_b = \omega_c$  ( $\neq 0$ ) and  $\gamma_a = \gamma_b = \gamma$  (identical substances), the integral is  $J_0 = \pi/2\omega_c^2 \gamma^3$  and the energy loss (Eq. (49)) becomes

$$W = \frac{v_a^2 v_b \Delta t^2}{4(4\pi)^3 T R^6 \omega_c^2 \gamma^2} \omega_p^6 (1 + 3 \cos^2 \theta) E^2. \quad (52)$$

The presence of the damping factor  $\gamma$  in the denominator of Eq. (52) ( $T\gamma = 1$ ) can induce an appreciable (resonant) energy loss in the ideal limit  $\gamma \rightarrow 0$ . This is again an indication of the resonant character of this interaction. It is worth emphasizing that the singular character exhibited by the physical quantities at the resonance point is in fact indicative merely of the inadequacy (and the limitations) of the harmonic oscillator-like models, as those employed here. In particular, for instance, the parameters of the polarizability change on decreasing the distance between the two bodies (proximity effect).<sup>29,44</sup>

For  $\omega_a = 0$ ,  $\omega_b \neq 0$  (or  $\omega_b = 0$ ,  $\omega_a \neq 0$ ) we get  $J_0 = \pi(\gamma_a + \gamma_b)/\gamma_a \gamma_b \omega_{b,a}^4$  (identical with  $I_0$  for  $\omega_b = 0$ ), which gives a vanishing energy loss in the ideal limit  $\gamma_{a,b} \rightarrow 0$ . For  $\omega_a = \omega_b = 0$ , the integral  $J_0$  is identical with the integral  $I_0$  ( $J_0 = \pi/\max(\gamma_a, \gamma_b)(\gamma_a^2 - \gamma_b^2)^2$  for  $\gamma_a \neq \gamma_b$  and  $J_0 = 3\pi/8\gamma^5$  for  $\gamma_a = \gamma_b = \gamma$ ), having a pronounced singular character.

The comparison of the energy loss given by Eq. (52) with the energy spent by the external field  $E\Delta t\delta(t)$  provides a criterion of validity for the approximations employed in the above theoretical considerations. The ratio of the two energies should be much lesser than unity. The energy per unit time of the external field in the exciting body  $a$  is  $W_0 = E^2 v_a / 8\pi\Delta t$ , so we should have

$$\frac{W_{max}}{W_0} = \frac{v_a v_b \Delta t^3}{8\pi^2 T R^6 \omega_c^2 \gamma^2} \omega_p^6 \ll 1, \quad (53)$$

where  $W_{max}$  is given by Eq. (52) for  $\theta = 0, \pi$ . This criterion is satisfied for the two numerical examples given before. Indeed, for  $\Delta t = 1/\omega_p$ ,  $T\gamma = 1$ , and  $v_a = v_b = 1 \text{ cm}^3$ ,  $\nu_p = 10^{10} \text{ Hz}$ ,  $\nu_c = 1 \text{ MHz}$ ,  $\gamma = 1 \text{ kHz}$ ,  $R = 10 \text{ m}$  we get  $W_{max}/W_0 \simeq 10^{-5}$ . Similarly, for  $v_a = v_b = 10^{-18} \text{ cm}^3$ ,  $\omega_p = \omega_c \simeq 10^{15} \text{ s}^{-1}$  and  $R = 1 \mu\text{m}$  (all the other data remaining the same) we get  $W_{max}/W_0 = 10^{-2}$ . Such a validity criterion as that given in Eq. (53) imposes important restrictions on the range of parameters used in estimating the resonant force.

## VI. MONOCHROMATIC WAVES

The Fourier transform of a monochromatic external field  $\mathbf{E}(t) = \mathbf{E} \cos \Omega t$  of frequency  $\Omega$  can be written as

$$\mathbf{E}(\omega) = \mathbf{E}^{(+)} \delta(\omega - \Omega) + \mathbf{E}^{(-)} \delta(\omega + \Omega), \quad (54)$$

where  $\mathbf{E}^{(\pm)} = \pi \mathbf{E}$ . This decomposition in  $\pm\Omega$ -components is transmitted to all the relevant quantities, like the charge and current densities, electromagnetic fields, etc. In particular, the polarizations can be written as

$$\mathbf{P}_{a,b}(\omega) = \mathbf{P}_{a,b}^{(+)}(\Omega) \delta(\omega - \Omega) + \mathbf{P}_{a,b}^{(-)}(\Omega) \delta(\omega + \Omega),$$

$$\mathbf{P}_a^{(+)}(\Omega) = \mathbf{P}_a^{(-)*}(\Omega) = \pi \alpha_a(\Omega) \mathbf{E}, \quad (55)$$

$$\mathbf{P}_b^{(+)}(\Omega) = \mathbf{P}_b^{(-)*}(\Omega) = \alpha_b(\Omega) \mathbf{E}_a^{(+)}(\Omega, \mathbf{r} = 0),$$

where

$$\begin{aligned} \mathbf{E}_a^{(+)}(\Omega, \mathbf{r} = 0) &= \mathbf{E}_a^{(-)*}(\Omega, \mathbf{r} = 0) \\ &= \pi v_a \alpha_a(\Omega) \left[ \mathbf{E} \frac{\Omega^2}{c^2} + \left( \mathbf{E} \frac{\partial}{\partial \mathbf{R}} \right) \frac{\partial}{\partial \mathbf{R}} \right] \frac{e^{i\Omega \mathbf{R}}}{R}. \end{aligned} \quad (56)$$

The force calculated from Eq. (1) (averaging over the period  $\sim 1/\Omega$ ) is given by

$$\begin{aligned} \mathbf{F} &= \frac{v_a v_b}{(2\pi)^2} \left[ \frac{\Omega^2}{c^2} \mathbf{P}_a^{(+)*}(\omega) \mathbf{P}_b^{(+)}(\omega) \right. \\ &\quad \left. + \mathbf{P}_{ai}^{(+)*}(\omega) \mathbf{P}_{bj}^{(+)}(\omega) \frac{\partial^2}{\partial R_i \partial R_j} \right] \frac{\partial}{\partial \mathbf{R}} \frac{e^{i\Omega \mathbf{R}}}{R} + c.c.. \end{aligned} \quad (57)$$

The parameter  $\mathbf{R}$  is omitted in these equations and the complex conjugate (*c.c.*) is the contribution of the  $-\Omega$ -components (superscript  $(-)$ ). In the sub-wavelength regime, we get

$$\mathbf{F} = \frac{1}{4} v_a^2 v_b |\alpha_a(\Omega)|^2 \alpha_b(\Omega) \frac{\partial}{\partial \mathbf{R}} \left[ \left( \mathbf{E} \frac{\partial}{\partial \mathbf{R}} \right) \frac{\mathbf{R}}{R^3} \right]^2 + c.c., \quad (58)$$

or, using the electric susceptibility for the polarizability,

$$\begin{aligned} \mathbf{F} &= \frac{3v_a^2 v_b}{(4\pi)^3 R^8} \omega_{pa}^4 \omega_{pb}^2 \frac{\Omega^2 - \omega_b^2}{[(\Omega^2 - \omega_a^2)^2 + \Omega^2 \gamma_a^2][(\Omega^2 - \omega_b^2)^2 + \Omega^2 \gamma_b^2]} \\ &\quad \cdot \left\{ \left[ E^2 + 4 \frac{(\mathbf{E}\mathbf{R})^2}{R^2} \right] \mathbf{R} - (\mathbf{E}\mathbf{R})\mathbf{E} \right\}. \end{aligned} \quad (59)$$



We can see that this force is similar with the force given by Eqs. (30) and (31), except for the  $\omega$ -integration, as expected.

For identical substances ( $\omega_a = \omega_b = \omega_c, \gamma_a = \gamma_b = \gamma$ ) and  $\Omega$  close to  $\omega_c, \Omega = \omega_c + \delta\omega$ , the force given by Eq. (59) is governed by the factor  $\delta\omega/(4\delta\omega^2 + \gamma^2)^2$ , which has two extrema  $\sim \pm 1/\gamma^3$  for  $\delta\omega \sim \gamma$ . In the limit of ideal substances  $\gamma \rightarrow 0$ , this may lead to an appreciable force (and a corresponding energy loss). It is worth noting that a detuning  $\delta\omega \sim \gamma$  is necessary. For distinct substances, the force goes like  $1/\gamma^2$ , which is much smaller than the force corresponding to identical substances, illustrating again the resonance phenomenon.

From Eqs. (55)–(57) we can also estimate the force in the wave-zone, *i.e.*, for  $R \gg \lambda \gg v_{a,b}^{1/3}$ , where  $\lambda = c/\Omega$  is the wavelength. The force is longitudinal in this case ( $\sim \mathbf{R}/R$ ) and generated by the transverse component  $E_t$  of the external field ( $\mathbf{E}_t \mathbf{R} = 0$ ). The resonance character of the force is preserved. For  $\Omega = \omega_a = \omega_b = \omega_c, \gamma_a = \gamma_b = \gamma$ , we have the estimation

$$\mathbf{F} \sim \frac{v_a^2 v_b}{\lambda^5} \frac{\omega_p^6}{\Omega^3 \gamma^3} E_t^2 \cos 2R/\lambda \cdot \frac{\mathbf{R}}{R^3}. \quad (60)$$

This is a resonant optical force resulting from the mutual polarization of the bodies. We can see that this force is practically vanishing, due to the rapid oscillations of the factor  $\cos 2R/\lambda$ , as expected.

## VII. TWO PARTICULAR CASES

We turn now to Eq. (17) and consider two particular cases. First, we take two independent external fields which do not depend on  $\mathbf{R}$ . Then, the resonance integral, denoted by  $K$ , implies the product  $\chi_a^*(\omega)\chi_b(\omega)$ . It can be represented as

$$K = I_2 - \omega_a^2 I_0 + \gamma_a \gamma_b J_0, \quad (61)$$

where  $I_n$  are given by Eq. (33) and  $J_0$  is given by Eq. (50). Making use of the integrals  $I_n$  and  $J_0$  calculated before, we can see that the force is practically vanishing in this case for distinct substances and has a finite value ( $K = \pi/\gamma\omega_c^2$ ) for identical substances. Therefore, the general resonance character of the force is preserved.

An interesting case occurs by assuming that the external field for body  $a$  is the field generated by body  $b$ ,  $\mathbf{E}_a^{ext} = \mathbf{E}_b(\omega, \mathbf{R})$ , and the external field for body  $b$  is the field generated by body  $a$ ,  $\mathbf{E}_b^{ext} = \mathbf{E}_a(\omega, \mathbf{r} = 0)$ . This situation corresponds to two isolated, interacting bodies (closed system). The field  $\mathbf{E}_b^{ext}$  has been computed in Eq. (21). Similarly we get the field  $\mathbf{E}_a^{ext}$ , so we have

$$E_{a,b}^{ext}(\omega, \mathbf{R}) = v_{b,a} \left[ \frac{\omega^2}{c^2} \mathbf{P}_{b,a}(\omega) + \left( \mathbf{P}_{b,a}(\omega) \frac{\partial}{\partial \mathbf{R}} \right) \frac{\partial}{\partial \mathbf{R}} \right] \frac{e^{i\omega R}}{R}, \quad (62)$$

where we omit to write explicitly the variable  $\mathbf{R}$  in the polarizations  $\mathbf{P}_{a,b}(\omega)$ . Making use of  $\mathbf{P}_{a,b}(\omega) = \alpha_{a,b}(\omega) \mathbf{E}_{a,b}^{ext}(\omega, \mathbf{R})$ , we get a homogeneous system of two coupled equations

$$\begin{aligned} \mathbf{P}_a(\omega) &= v_b \alpha_a(\omega) \left[ \frac{\omega^2}{c^2} \mathbf{P}_b(\omega) + \left( \mathbf{P}_b(\omega) \frac{\partial}{\partial \mathbf{R}} \right) \frac{\partial}{\partial \mathbf{R}} \right] \frac{e^{i\omega R}}{R}, \\ \mathbf{P}_b(\omega) &= v_a \alpha_b(\omega) \left[ \frac{\omega^2}{c^2} \mathbf{P}_a(\omega) + \left( \mathbf{P}_a(\omega) \frac{\partial}{\partial \mathbf{R}} \right) \frac{\partial}{\partial \mathbf{R}} \right] \frac{e^{i\omega R}}{R}. \end{aligned} \quad (63)$$

It is convenient to introduce the longitudinal and transverse polarizations,  $P_{a,b}^l = \mathbf{P}_{a,b} \mathbf{R}/R$  and  $P_{a,b}^t = \mathbf{P}_{a,b} \mathbf{R}_\perp/R$ , where  $\mathbf{R}_\perp$  is a vector perpendicular to  $\mathbf{R}$  and of the same magnitude as  $\mathbf{R}$ . In the sub-wavelength regime we can identify two perturbation parameters  $\varepsilon_{a,b}^l = 2v_{a,b}/R^3 \ll 1$  and  $\varepsilon_{a,b}^t = -v_{a,b}/R^3 \ll 1$ . Leaving aside the labels  $l, t$ , the system of equations reads

$$P_a = \alpha_a \varepsilon_b P_b, \quad P_b = \alpha_b \varepsilon_a P_a. \quad (64)$$

The eigenfrequencies are given by the roots of the equation  $1 = \alpha_a \alpha_b \varepsilon_a \varepsilon_b$ . Making use of polarizabilities of the form given by Eq. (25), we get the eigenfrequencies

$$\begin{aligned} \Omega_{1,2} &= \pm \omega_a \pm \frac{\omega_{pa}^2 \omega_{pb}^2 \varepsilon_a \varepsilon_b}{2(4\pi)^2 \omega_a (\omega_a^2 - \omega_b^2)}, \\ \Omega_{3,4} &= \pm \omega_b \mp \frac{\omega_{pa}^2 \omega_{pb}^2 \varepsilon_a \varepsilon_b}{2(4\pi)^2 \omega_b (\omega_a^2 - \omega_b^2)} \end{aligned} \quad (65)$$

for  $\omega_a \neq \omega_b$ . The solutions of the system of Eqs. (64) can be written as

$$\begin{aligned} P_a(\omega) &= 2\pi [C_1 \delta(\omega - \Omega_1) + C_2 \delta(\omega - \Omega_2)] \\ &+ \frac{\omega_{pa}^2 \varepsilon_b}{2(\omega_a^2 - \omega_b^2)} [C_3 \delta(\omega - \Omega_3) + C_4 \delta(\omega - \Omega_4)] \end{aligned} \quad (66)$$

and

$$\begin{aligned} P_b(\omega) &= 2\pi [C_3 \delta(\omega - \Omega_3) + C_4 \delta(\omega - \Omega_4)] \\ &- \frac{\omega_{pb}^2 \varepsilon_a}{2(\omega_a^2 - \omega_b^2)} [C_1 \delta(\omega - \Omega_1) + C_2 \delta(\omega - \Omega_2)], \end{aligned} \quad (67)$$

where  $C_i, i = 1, 2, 3, 4$ , are constants. The contributions to the force (Eq. (16)) come from products of the form  $P_a P_b$ , *i.e.*, from  $\delta^2$ -terms. Using  $\delta(\omega = 0) = T/2\pi$ , we can see that these contributions are proportional to  $\varepsilon_{a,b}/(\omega_a^2 - \omega_b^2)$  (similar to Eq. (37)), so that we may say that the force is practically vanishing in this case.

For identical substances ( $\omega_a = \omega_b = \omega_c, \omega_{pa} = \omega_{pb} = \omega_p$ ), the eigenfrequencies are

$$\Omega_i = \pm \omega_c \pm \frac{\omega_p^2 \sqrt{\varepsilon_a \varepsilon_b}}{8\pi\omega_c}, \quad i = 1, 2, 3, 4 \quad (68)$$

and the polarizations are given by

$$\begin{aligned} P_a(\omega) &= 2\pi \sum_{i=1}^4 C_i \delta(\omega - \Omega_i), \quad P_b(\omega) \\ &= 2\pi \sqrt{\frac{\varepsilon_a}{\varepsilon_b}} \sum_{i=1}^4 (-1)^i C_i \delta(\omega - \Omega_i). \end{aligned} \quad (69)$$

The force acquires a finite value in this case (and is proportional to  $1/R^4$ ), indicating again a resonance character. However, in practice, the preparation of the two bodies in states with polarizations corresponding to the frequencies  $\Omega_i$  is very likely to raise difficulties. Usually, it is reasonable to expect equal weights  $|C_1|^2 = |C_2|^2$  and  $|C_3|^2 = |C_4|^2$ , which, due to the factor  $(-1)^i$  in Eq. (69), leads, in fact, to a vanishing force.

It is worth commenting on Eqs. (63) and (64) derived here. If we add an external field  $\mathbf{E}$  acting only upon one body, as discussed previously, these systems of equations become inhomogeneous and their solution has a denominator of the form  $1 - \alpha_a \alpha_b \epsilon_a \epsilon_b$ . This factor is indicative of the geometric series with the ratio  $\alpha_a \alpha_b \epsilon_a \epsilon_b \sim v_a v_b \alpha_a \alpha_b$  arising in the iterative process of polarization, as discussed before, and discarded for point-like bodies.

### VIII. CONCLUDING REMARKS

A dynamic dipolar force has been described here, which has a resonant character, in the sense that it acquires finite values for identical substances, and is practically vanishing for distinct substances. The derivation is based on the well-known electric susceptibility (dielectric response function), which exhibits a set of characteristic frequencies. The force is free from spatial oscillations (which reduce appreciably its effectiveness) in the sub-wavelength regime, *i.e.*, for distances much smaller than the characteristic wavelengths (but much larger than the dimension of the bodies). The calculations have been limited to point-like bodies (neglecting the spatial dispersion) and only one characteristic frequency for each body, but they can be extended, in principle, to finite-size bodies and several characteristic frequencies. The force computed here appears through the mutual polarization of the bodies, driven by a localized external field acting upon only one body. The resonant force derived here is a generalization to classical time-dependent electromagnetic fields of the static force acting between two classical dipoles. Its  $R^{-7}$ -dependence on the distance  $R$  between the two bodies recalls the well-known van der Waals-London force. Particular cases corresponding to independent external fields or to isolated, interacting bodies (closed system) have also been analyzed, with similar conclusions regarding the resonant force. According to our numerical estimations, the force derived here is very small and is subjected to severe limitations arising from its resonant character.

Finally, it is worth commenting upon the nature and relevance of the resonant force derived here, in the wider context of electromagnetic interaction (which may acquire a great variety of particular forms, as discussed in Sec. I). The electromagnetic forces acting upon polarizable particles (including magnetodielectric particles) and the corresponding energy flow, electromagnetic momentum and torques have been widely investigated in connection with the energy conservation, or, more generally, the conservation of the energy-momentum tensor, for estimating the transfer rate of momentum, angular momentum or energy.<sup>48–52</sup> As it is well known, the conservation of the energy-momentum tensor in polarizable matter involves, beside the mechanical (kinetic)

energy of the charges (currents, energy loss included) and the energy flow corresponding to the Poynting vector, the electromagnetic stress forces, the Lorentz force and the electromagnetic momentum (radiation pressure). Under certain circumstances, special forms of such forces have been derived, like those arising from the curl of the spin angular momentum or the “pulling force,”<sup>48–52</sup> some of them exhibiting a non-conservative character. We focused here on a Lorentz-type force (according to Eq. (1)), corresponding to a particular case of resonance of the induced matter polarization for point-like bodies. Surface stress forces, or forces associated with rotations, may be discarded for point-like bodies in the first approximation, and the electromagnetic momentum (and its rate of change with the time) can also be discarded in the sub-wavelength zone we dealt with here. The (average) mechanical energy of the polarizable charges has been estimated as the energy loss in Sec. V. It has been shown there that it sets important limits on the resonant force, in particular upon the harmonic-oscillator model of polarizable dipoles. The bodies discussed in the present paper are under the action of an external field (one body) and their own polarization fields. Part of the energy pumped into the external field is recovered in the energy flows (Poynting vectors), the remaining is lost as energy loss (kinetic energy) in the polarizable bodies and another part may be associated with the mechanical work done by the force derived here, providing we allow the motion of the bodies. This illustrates the energy conservation (within the approximations used here), as expected for polarizable matter interacting with the electromagnetic field.

### ACKNOWLEDGMENTS

The authors are indebted to the members of the Seminar of the Institute of Atomic Physics and the Laboratory of Theoretical Physics, Magurele, Bucharest, for useful discussions and a careful reading of the manuscript. This work was supported by the Romanian Government Research Agency Grant #306/SMIS 26614.

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