Electromagnetic field interacting with a semi-infinite plasma

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Plasmon and polariton modes are derived for an ideal semi-infinite (half-space) plasma by using a general, unifying procedure based on the equation of motion of the polarization and the electromagnetic potentials. Known results are reproduced in a much more direct manner, and new ones are derived. The approach consists of representing the charge disturbances by a displacement field in the positions of the moving particles (electrons). The propagation of an electromagnetic wave in this plasma is treated by using the retarded electromagnetic potentials. The resulting integral equations are solved, and the reflected and refracted fields are computed, as well as the reflection coefficient. Generalized Fresnel relations are thereby obtained for any incidence angle and polarization. Bulk and surface plasmon–polariton modes are identified. As is well known, the field inside the plasma is either damped (evanescent) or propagating (transparency regime), and the reflection coefficient exhibits an abrupt enhancement on passing from the propagating regime to the damped one (total reflection).

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1. INTRODUCTION

Plasmons, polaritons, and, in general, electromagnetic fields interacting with matter in structures with special, restricted geometries have always enjoyed particular interest. There is a vast literature on this subject regarding structures such as a half-space (semi-infinite) plasma, a plasma slab of finite thickness, a two-plasma interface (two plasmas bounding each other), a two-dimensional sheet with an aperture, a slab with a cylindrical hole, structures with surface gratings or regular hole patterns, layered films, cylindrical rods, and spherical particles, etc. These studies were aimed mainly at identifying new plasmon modes such as surface plasmons [1–8], and experiments accounting for electron energy loss and exploring the interaction of the electron plasma with electromagnetic radiation (polariton excitations) [9–21]. More recently, a possible enhancement of the electromagnetic radiation scattered on electron plasmas with special geometries enjoyed a particular interest [22–24]. In all these studies the plasmon and polariton modes are of fundamental importance [25–29]. The methods used in deriving such results are of great diversity, resorting often to particular assumptions, such that the basic underlying mechanism of plasmons or polaritons is often obscured. The need is therefore felt for having a general, unifying procedure for deriving plasmon and polariton modes in structures with special geometries, as based on the equation of motion of the charge density and Maxwell's equations. Such a procedure is presented in this paper for an ideal semi-infinite plasma.

We represent the charge disturbances as $\delta n=-n\,\mathrm{div}\,\mathbf{u}$, where n is the (constant, uniform) charge concentration and $\mathbf{u}$ is a displacement field of the mobile charges (electrons). This representation is valid for $\mathbf{K}\mathbf{u}(\mathbf{K})\ll 1$, where $\mathbf{K}$ is the wave vector and $\mathbf{u}(\mathbf{K})$ is the Fourier component of the displacement field. We assume a rigid neutralizing background of positive charge, as in the well-known jellium model. In the static limit, i.e., for Coulomb interaction, the Lagrangian of the electrons can be written as

$$L=\int d\mathbf{r}\left[\frac{1}{2}mn\dot{\mathbf{u}}^2-\frac{1}{2}\int d\mathbf{r}'U(|\mathbf{r}-\mathbf{r}'|)\delta n(\mathbf{r})\delta n(\mathbf{r}')\right]+e\int d\mathbf{r}\Phi(\mathbf{r})\delta n(\mathbf{r}), \quad (1)$$

where m is the electron mass, $U(r)=e^2/r$ is the Coulomb energy, $-e$ is the electron charge, and $\Phi(\mathbf{r})$ is an external scalar potential. Equation (1) leads to the equation of motion,

$$m\ddot{\mathbf{u}}=n\,\mathrm{grad}\int d\mathbf{r}'U(|\mathbf{r}-\mathbf{r}'|)\mathrm{div}\,\mathbf{u}(\mathbf{r}')+e\,\mathrm{grad}\,\Phi, \quad (2)$$

which is the starting equation of our approach. We leave aside the dissipation effects [which can easily be included in Eq. (2)].

By using the Fourier transform for an infinite plasma it is easy to see that the eigenmode of the homogeneous Eq. (2) is the well-known bulk plasmon mode given by $\omega_p^2=4\pi ne^2/m$. On the other side, the relation $\delta n=-n\,\mathrm{div}\,\mathbf{u}$ is equivalent to Maxwell's equation $\mathrm{div}\,\mathbf{E}_i=-4\pi e\delta n$, where $\mathbf{E}_i=4\pi ne\mathbf{u}$ is the internal electric field (equal to $-4\pi\mathbf{P}$, where $\mathbf{P}$ is the polarization). Making use of the electric displacement $\mathbf{D}=-\mathrm{grad}\,\Phi=\varepsilon(\mathbf{D}+\mathbf{E}_i)$, we get the well-known dielectric function $\varepsilon=1-\omega_p^2/\omega^2$ in the long-wavelength limit from the solution of the inhomogeneous

Eq. (2). Similarly, since the current density is $\mathbf{j}=-en\dot{\mathbf{u}}$, we get the well-known electrical conductivity $\sigma=i\omega_p^2/4\pi\omega$.

We apply this approach to a semi-infinite plasma. First, we derive the surface and bulk plasmon modes and obtain the dielectric response. Further on, we consider the interaction of the semi-infinite plasma with the electromagnetic field, as described by the usual term $(1/c)\int d\mathbf{r}\mathbf{j}\mathbf{A} - \int d\mathbf{r}\rho\Phi$ in the Lagrangian, where $\mathbf{A}$ is the vector potential, $\rho=en\,\mathrm{div}\,\mathbf{u}$ is the charge density and Φ is the scalar potential. We limit ourselves to the interaction with the electric field, and compute the reflected and refracted fields, as well as the reflection coefficient. Generalized Fresnel relations are obtained for any incidence angle and polarization. The well-known continuity of the tangential components of the electric field and the normal component of the electric displacement at the surface follow from our calculation, as well as the continuity of the normal component of the Poynting vector. We find it more convenient to use the radiation formulas for the retarded potentials, instead of using directly the Maxwell equations, and the resulting integral equations are solved. Bulk and surface plasmon–polariton modes are identified. The field inside the plasma is either damped (evanescent) or propagating (transparency regime), and the reflection coefficient exhibits an abrupt enhancement on passing from the propagating to the damping regime (total reflection). The present approach can be extended to various other plasma structures with special geometries.

2. PLASMA EIGENMODES

We consider an ideal semi-infinite plasma extending over the half-space $z>0$ (and bounded by the vacuum for $z<0$). The displacement field $\mathbf{u}$ is then represented as $(\mathbf{v},u_3)\theta(z)$, where $\mathbf{v}$ is the displacement component in the (x,y)-plane, u_3 is the displacement component along the z-direction and $\theta(z)=1$ for $z>0$ and $\theta(z)=0$ for $z<0$ is the step function. In equation of motion (2) div $\mathbf{u}$ is then replaced by

$$\mathrm{div}\,\mathbf{u} = \left(\mathrm{div}\,\mathbf{v} + \frac{\partial u_3}{\partial z}\right)\theta(z) + u_3(0)\delta(z), \tag{3}$$

where $u_3(0)=u_3(\mathbf{r},z=0)$, $\mathbf{r}$ being the in-plane (x,y) position vector. Equation (2) becomes

$$m\ddot{\mathbf{u}} = ne^2\,\mathrm{grad}\int d\mathbf{r}'dz'\frac{1}{\sqrt{(\mathbf{r}-\mathbf{r}')^2+(z-z')^2}} \times\left[\mathrm{div}\,\mathbf{v}(\mathbf{r}'.z') + \frac{\partial u_3(\mathbf{r}',z')}{\partial z'}\right] + ne^2\,\mathrm{grad}\int d\mathbf{r}'\frac{1}{\sqrt{(\mathbf{r}-\mathbf{r}')^2+z^2}}u_3(\mathbf{r}',0) + e\,\mathrm{grad}\,\Phi \tag{4}$$

for $z>0$. One can see the (de)polarizing field occurring at the free surface $z=0$ [the second integral in Eq. (4)].

We use Fourier transforms of the type

$$\mathbf{u}(r,z;t) = \sum_{\mathbf{k}}\int d\omega\,\mathbf{u}(\mathbf{k},z;\omega)e^{i\mathbf{k}\mathbf{r}}e^{-i\omega t} \tag{5}$$

(for in-plane unit area), as well as the Fourier representation

$$\frac{1}{\sqrt{r^2+z^2}} = \sum_{\mathbf{k}}\frac{2\pi}{k}e^{-k|z|}e^{i\mathbf{k}\mathbf{r}} \tag{6}$$

for the Coulomb potential. Then, it is easy to see that Eq. (4) leads to the integral equation

$$\omega^2 v = \frac{1}{2}k\omega_p^2\int_0^\infty dz' v e^{-k|z-z'|} + \frac{1}{2k}\omega_p^2\int_0^\infty dz'\frac{\partial v}{\partial z'}\frac{\partial}{\partial z'}e^{-k|z-z'|} - \frac{iek}{m}\Phi, \tag{7}$$

and $iku_3=\partial v/\partial z$, where we have dropped out for simplicity the arguments $\mathbf{k}$, z, and ω. The $\mathbf{v}$-component of the displacement field is directed along the wave vector $\mathbf{k}$ (in-plane longitudinal waves). This integral equation can easily be solved. Integrating by parts in its r.h.s we get

$$\omega^2 v = \omega_p^2 v - \frac{1}{2}\omega_p^2 v_0 e^{-kz} - \frac{iek}{m}\Phi, \tag{8}$$

hence

$$v = \frac{iek\omega_p^2}{m}\frac{\Phi_0}{(\omega^2-\omega_p^2)(2\omega^2-\omega_p^2)}e^{-kz} - \frac{iek}{m}\frac{\Phi}{\omega^2-\omega_p^2},$$

$$u_3 = -\frac{ek\omega_p^2}{m}\frac{\Phi_0}{(\omega^2-\omega_p^2)(2\omega^2-\omega_p^2)}e^{-kz} - \frac{e}{m}\frac{\Phi'}{\omega^2-\omega_p^2}, \tag{9}$$

where $v_0=v(z=0)$, $\Phi_0=\Phi(z=0)$ and $\Phi'=\partial\Phi/\partial z$. One can see the surface contributions (terms proportional to $\Phi_0 e^{-kz}$) and bulk contributions (Φ,Φ' terms).

The solutions given by Eqs. (9) exhibit two resonances, the bulk plasmon $\omega_b=\omega_p$ and the surface plasmon $\omega_s=\omega_p/\sqrt{2}$, as is well known. Indeed, the homogeneous Eq. (8) ($\Phi=0$) has two solutions: the surface plasmon $v=v_0e^{-kz}$ for $\omega^2=\omega_p^2/2$ and the bulk plasmon $v_0=0$ for $\omega^2=\omega_p^2$. Making use of this observation we can represent the general solution as an eigenmode series

$$v(\mathbf{k},z) = \sqrt{2k}v_0(\mathbf{k})e^{-kz} + \sum_\kappa\sqrt{\frac{2k^2}{\kappa^2+k^2}}v(\mathbf{k},\kappa)\sin\kappa z, \tag{10}$$

for $z>0$, where $v(\mathbf{k},-\kappa)=-v(\mathbf{k},\kappa)$, and $iku_3(\mathbf{k},z)=\partial v(\mathbf{k},z)/\partial z$. Then, it is easy to see that the Hamiltonian $H=T+U$ corresponding to the Lagrangian $L=T-U$ given by Eq. (1) becomes

$$T = nm\sum_{\mathbf{k}}\dot{v}_0^*(\mathbf{k})\dot{v}_0(\mathbf{k}) + nm\sum_{\mathbf{k}\kappa}\dot{v}^*(\mathbf{k},\kappa)\dot{v}(\mathbf{k},\kappa),$$

$$U = 2\pi n^2 e^2 \sum_{\mathbf{k}} v_0^*(\mathbf{k}) v_0(\mathbf{k}) + 4\pi n^2 e^2 \sum_{\mathbf{k}\kappa} v^*(\mathbf{k},\kappa) v(\mathbf{k},\kappa), \tag{11}$$

where T is the kinetic energy and U is the potential energy. We can see that this Hamiltonian corresponds to harmonic oscillators with frequencies $\omega_s = \omega_p/\sqrt{2}$ and $\omega_b = \omega_p$.

Making use of $\mathbf{E}_i = 4\pi ne\mathbf{u}$ and Eqs. (9) we can write the internal field (polarization) as

$$E_\perp(\mathbf{k},z;\omega) = \frac{ik\omega_p^4 \Phi(\mathbf{k},0;\omega)}{(\omega^2-\omega_p^2)(2\omega^2-\omega_p^2)} e^{-kz} - \frac{ik\omega_p^2 \Phi(\mathbf{k},z;\omega)}{\omega^2-\omega_p^2},$$

$$E_\parallel(\mathbf{k},z;\omega) = -\frac{k\omega_p^4 \Phi(\mathbf{k},0;\omega)}{(\omega^2-\omega_p^2)(2\omega^2-\omega_p^2)} e^{-kz} - \frac{\omega_p^2 \Phi'(\mathbf{k},z;\omega)}{\omega^2-\omega_p^2}, \tag{12}$$

where $E_\perp$ is directed along the in-plane wave vector $\mathbf{k}$ and $E_\parallel$ is parallel to the z-axis (perpendicular to the surface $z=0$). Equations (12) give the dielectric response of the semi-infinite plasma to an external potential.

We take an external potential of the form $\Phi(\mathbf{k},z) = \Phi^0(\mathbf{k})e^{i\kappa z}$ (leaving aside the frequency argument ω) and get the electric displacement $\mathbf{D}_\perp(\mathbf{k},z) = -i\mathbf{k}\Phi^0(\mathbf{k})e^{i\kappa z}$ and $D_\parallel(\mathbf{k},z) = -i\kappa\Phi^0(\mathbf{k})e^{i\kappa z}$ from $\mathbf{D} = -\text{grad}\,\Phi$. We can see that the surface terms do not contribute to this response, as expected, since these terms are localized. Making use of $\mathbf{E}_i = (1/\varepsilon - 1)\mathbf{D}$, we get the well-known dielectric function $\varepsilon(\kappa,\omega) = 1 - \omega_p^2/\omega^2$ in the long-wavelength limit.

3. INTERACTION WITH THE ELECTROMAGNETIC FIELD: POLARITONS

We assume a plane wave incident on the plasma surface under angle α. Its frequency is given by $\omega = cK$, where c is the velocity of light and the wave vector $\mathbf{K} = (\mathbf{k},\kappa)$ has the in-plane component $\mathbf{k}$ and the perpendicular-to-plane component κ, such as $k = K\sin\alpha$ and $\kappa = K\cos\alpha$. In addition, $\mathbf{k} = k(\cos\phi, \sin\phi)$. The electric field is taken as $\mathbf{E}_0 = E_0(\cos\beta, 0, -\sin\beta)e^{i\mathbf{kr}}e^{i\kappa z}e^{-i\omega t}$, and we impose the condition $\cos\beta\sin\alpha\cos\phi - \sin\beta\cos\alpha = 0$ (transversality condition $\mathbf{KE}_0 = 0$). The angle β defines the direction of the polarization of the incident field.

In the presence of an electromagnetic wave we use the equation of motion

$$\omega^2 \mathbf{u} = \frac{e}{m}\mathbf{E} + \frac{e}{m}\mathbf{E}_0 e^{i\kappa z}, \tag{13}$$

for $z>0$, where $\mathbf{E}$ is the polarizing field; in Eq. (13) we have preserved explicitly only the z-dependence (i.e., we leave aside the factors $e^{i\mathbf{kr}}e^{-i\omega t}$). We find it convenient to employ the vector potential

$$\mathbf{A}(\mathbf{r},z;t) = \frac{1}{c}\int d\mathbf{r}' \int dz' \frac{\mathbf{j}(\mathbf{r}',z';t-R/c)}{R} \tag{14}$$

and the scalar potential

$$\Phi(\mathbf{r},z;t) = \int d\mathbf{r}' \int dz' \frac{\rho(\mathbf{r}',z';t-R/c)}{R}, \tag{15}$$

where

$$\mathbf{j} = -ne\dot{\mathbf{u}}\theta(z)e^{i\mathbf{kr}}e^{-i\omega t}$$

is the current density,

$$\rho = ne\,\text{div}\,\mathbf{u} = ne(i\mathbf{kv} + \partial u_3/\partial z)\theta(z)e^{i\mathbf{kr}}e^{-i\omega t} + neu_3(0)\delta(z)e^{i\mathbf{kr}}e^{-i\omega t}$$

is the charge density, and $R = \sqrt{(\mathbf{r}-\mathbf{r}')^2 + (z-z')^2}$. The integrals in Eqs. (14) and (15) imply the known integral [30]

$$\int_{|z|}^{\infty} dx J_0(k\sqrt{x^2-z^2})e^{i\omega x/c} = \frac{i}{\kappa}e^{i\kappa|z|}, \tag{16}$$

where J_0 is the zeroth-order Bessel function of the first kind (and $\omega^2/c^2 = \kappa^2 + k^2$). It is convenient to use the projections of the in-plane displacement field $\mathbf{v}$ on the vectors $\mathbf{k}$ and $\mathbf{k}_\perp = k(-\sin\phi, \cos\phi)$, $\mathbf{k}_\perp\mathbf{k} = 0$. We denote these components by $v_1 = \mathbf{kv}/k$ and $v_2 = \mathbf{k}_\perp\mathbf{v}/k$ and use also the components $E_1 = \mathbf{kE}/k$, $E_2 = \mathbf{k}_\perp\mathbf{E}/k$ and similar ones for the external field $\mathbf{E}_0$. We give here the components of the external field,

$$E_{01} = E_0\cos\beta\cos\phi, \quad E_{02} = -E_0\cos\beta\sin\phi,$$

$$E_{03} = -E_0\sin\beta. \tag{17}$$

One can check immediately the transversality condition $E_{01}k + E_{03}\kappa = 0$. Making use of $\mathbf{E} = -1/c\,\partial\mathbf{A}/\partial t - \text{grad}\,\Phi$, Eqs. (14) and (15) give the electric field

$$E_1 = -2\pi ine\kappa\int_0 dz' v_1(z')e^{i\kappa|z-z'|} - 2\pi ne\frac{k}{\kappa}\int_0 dz' u_3(z')\frac{\partial}{\partial z'}e^{i\kappa|z-z'|},$$

$$E_2 = -2\pi ine\frac{\omega^2}{c^2\kappa}\int_0 dz' v_2(z')e^{i\kappa|z-z'|},$$

$$E_3 = 2\pi ne\frac{k}{\kappa}\int_0 dz' v_1(z')\frac{\partial}{\partial z}e^{i\kappa|z-z'|} - 2\pi ine\frac{k^2}{\kappa}\int_0 dz' u_3(z')e^{i\kappa|z-z'|} + 4\pi neu_3, \tag{18}$$

for $z>0$. It is worth observing in deriving these equations the nonintervertibility of the derivatives and the integrals according to the identity

$$\frac{\partial}{\partial z}\int_0 dz' f(z')\frac{\partial}{\partial z'}e^{i\kappa|z-z'|} = \kappa^2\int_0 dz' f(z')e^{i\kappa|z-z'|} - 2i\kappa f(z) \tag{19}$$

for any function $f(z)$, $z>0$; this is due to the discontinuity in the derivative of the function $e^{i\kappa|z-z'|}$ for $z=z'$. Now, we employ equation of motion (13) in Eqs. (18) and get the coupled integral equations

$$\omega^2 v_1 = -\frac{i\omega_p^2\kappa}{2}\int_0 \mathrm{d}z' v_1(z') e^{i\kappa|z-z'|} - \frac{\omega_p^2 k}{2\kappa}\int_0 \mathrm{d}z' u_3(z')\frac{\partial}{\partial z'}e^{i\kappa|z-z'|} + \frac{e}{m}E_{01}e^{i\kappa z},$$

$$\omega^2 v_2 = -\frac{i\omega_p^2\omega^2}{2c^2\kappa}\int_0 \mathrm{d}z' v_2(z') e^{i\kappa|z-z'|} + \frac{e}{m}E_{02}e^{i\kappa z},$$

$$\omega^2 u_3 = \frac{\omega_p^2 k}{2\kappa}\int_0 \mathrm{d}z' v_1(z')\frac{\partial}{\partial z}e^{i\kappa|z-z'|} - \frac{i\omega_p^2 k^2}{2\kappa}\int_0 \mathrm{d}z' u_3(z') e^{i\kappa|z-z'|} + \omega_p^2 u_3 + \frac{e}{m}E_{03}e^{i\kappa z}, \tag{20}$$

for the coordinates $v_{1,2}$ and u_3 in the region $z>0$.

The second of Eqs. (20) can be solved straightforwardly by noting that

$$\frac{\partial^2}{\partial z^2}\int_0 \mathrm{d}z' v_2(z') e^{i\kappa|z-z'|} = -\kappa^2\int_0 \mathrm{d}z' v_2(z') e^{i\kappa|z-z'|} + 2i\kappa v_2. \tag{21}$$

We get

$$\frac{\partial^2 v_2}{\partial z^2} + (\kappa^2 - \omega_p^2/c^2)v_2 = 0. \tag{22}$$

The solution of this equation is

$$v_2 = \frac{2eE_{02}}{m\omega_p^2}\cdot\frac{\kappa(\kappa-\kappa')}{K^2}e^{i\kappa' z}, \tag{23}$$

where

$$\kappa' = \sqrt{\kappa^2 - \omega_p^2/c^2} = \frac{1}{c}\sqrt{\omega^2\cos^2\alpha - \omega_p^2}. \tag{24}$$

The wave vector κ' can also be written in a more familiar form $\kappa' = (\omega/c)\sqrt{\varepsilon - \sin^2\alpha}$, where $\varepsilon = 1-\omega_p^2/\omega^2$ is the dielectric function. The corresponding component of the (total) electric field (the refracted field), can be obtained from Eq. (13); it is given by $(m\omega^2/e)v_2$. For $\kappa^2<\omega_p^2/c^2$ ($\omega\cos\alpha<\omega_p$) this field does not propagate. For $\kappa^2>\omega_p^2/c^2$ (ω greater than the transparency edge $\omega_p/\cos\alpha$) it represents a refracted wave (transparency regime) with the refraction angle α' given by Snell's law:

$$\frac{\sin\alpha'}{\sin\alpha} = \frac{1}{\sqrt{1-\omega_p^2/\omega^2}} = 1/\sqrt{\varepsilon}. \tag{25}$$

The polariton frequency is given by

$$\omega^2 = c^2K^2 = \omega_p^2 + c^2K'^2, \tag{26}$$

as is well known, where $K'^2 = \kappa'^2 + k^2$.

The first and the third of Eqs. (20) can be solved by using an equation similar to Eq. (21) and by noting that they imply

$$\kappa'^2 u_3 = ik\frac{\partial v_1}{\partial z}. \tag{27}$$

We get

$$v_1 = \frac{2eE_{01}}{m\omega_p^2}\cdot\frac{\kappa'(\kappa-\kappa')}{\kappa\kappa'+k^2}e^{i\kappa' z} \tag{28}$$

and

$$u_3 = \frac{2eE_{03}}{m\omega_p^2}\cdot\frac{\kappa(\kappa-\kappa')}{\kappa\kappa'+k^2}e^{i\kappa' z}. \tag{29}$$

Similarly, the corresponding components of the refracted field are given by Eq. (13). It is easy to check the transversality condition $v_1k+u_3\kappa'=0$ [and the vanishing of the bulk charge $ne(i\mathbf{kv}+\partial u_3/\partial z)=0$].

We can see that the polarization field $\mathbf{E}$ in Eq. (13) cancels out the original incident field $\mathbf{E}_0$ and gives the total, refracted field $m\omega^2\mathbf{u}/e$ inside the plasma. This is an illustration of the so-called Ewald–Oseen extinction theorem [14,31].

It is worth investigating the eigenvalues of the homogeneous system of integral Eqs. (20), for parameter κ given by $\kappa=\sqrt{\omega^2/c^2-k^2}$. Such eigenvalues are given by the roots of the vanishing denominator in Eqs. (28) and (29), i.e., by the relation $\kappa\kappa'+k^2=0$. This equation has real roots for ω only for the damped regime, i.e., for $\kappa=i|\kappa|$ and $\kappa'=i|\kappa'|$. Providing these conditions are satisfied, there is only one acceptable branch of excitations, that given by

$$\omega^2 = \frac{2\omega_p^2c^2k^2}{\omega_p^2+2c^2k^2+\sqrt{\omega_p^4+4c^4k^4}}. \tag{30}$$

We can see that $\omega\sim ck$ in the long-wavelength limit, and it approaches the surface-plasmon frequency $\omega\sim\omega_p/\sqrt{2}$ in the nonretarded limit ($ck\to\infty$). These excitations are surface plasmon–polariton modes. We note that they imply $v_2=0$ and $v_1,u_3\sim e^{-|\kappa'|z}$. In addition, a careful analysis of the homogeneous system of Eqs. (20) reveals another branch of excitations, given by $\omega=\omega_p$, which, occurring in this context, may be termed the bulk plasmon–polariton modes. They are characterized by $v_2=0$ and $v_1(\mathbf{k},0)=0$. For all these modes we have $u_3=[ic^2k/(\omega^2-c^2k^2-\omega_p^2)]\partial v_1/\partial z$.

In order to get the reflected wave (the region $z<0$) we turn to Eqs. (18) and use therein the solutions given above for $v_{1,2}$ and u_3. It is worth noting here that the discontinuity term $\omega_p^2u_3$ no longer appears in these equations (because $z'>0$ and $z<0$ and we cannot have $z=z'$). The integrations in Eqs. (18) are straightforward and we get the fields

$$E_1 = E_{01}\frac{\kappa-\kappa'}{\kappa+\kappa'}\cdot\frac{\kappa\kappa'-k^2}{\kappa\kappa'+k^2}e^{-i\kappa z}, \tag{31}$$

$$E_2 = E_{02}\frac{\kappa-\kappa'}{\kappa+\kappa'}e^{-i\kappa z}, \tag{32}$$

and

$$E_3 = -E_{03}\frac{\kappa-\kappa'}{\kappa+\kappa'}\cdot\frac{\kappa\kappa'-k^2}{\kappa\kappa'+k^2}e^{-i\kappa z}. \tag{33}$$

We can see that this field represents the reflected wave ($\kappa \to -\kappa$), and we can check its transversality to the propagation wave vector. Making use of the reflected field $\mathbf{E}_{refl}$ given by Eqs. (31)–(33) and the refracted field $\mathbf{E}_{refr}$ obtained from Eqs. (13) and (18) ($\mathbf{E}_{refr}=\mathbf{E}+\mathbf{E}_0=m\omega^2\mathbf{u}/e$), one can check the continuity of the tangential components of the electric field and the normal component of the electric displacement at the surface ($z=0$) in the form $E_{1,2refl}+E_{01,2}=E_{1,2refr}$, $E_{3refl}+E_{03}=\varepsilon E_{3refr}$, where $\varepsilon=1-\omega_p^2/\omega^2$. The angle of total polarization (Brewster's angle) is given by $\kappa\kappa'-k^2=0$, or $\tan^2\alpha=1-\omega_p^2/\omega^2=\varepsilon$ (for $\alpha<\pi/4$). The above equations provide generalized Fresnel relations between the amplitudes of the reflected, refracted, and incident waves at the surface for any incidence angle and polarization. They can also be written by using $\omega^2=\omega_p^2/(1-\varepsilon)$, where ε is the dielectric function.

Making use of the reflected field $\mathbf{E}_{refl}$ and the refracted field $\mathbf{E}_{refr}$ we can also check the continuity of the energy flow across the surface. Indeed, the Poynting vector $\mathbf{S}=(c/4\pi)\mathbf{E}\times\mathbf{H}=(c^2/4\pi\omega)\mathbf{K}\mathbf{E}^2$, where $\mathbf{H}=(c/\omega)\mathbf{K}\times\mathbf{E}$ is the magnetic field, has a normal component that is continuous at the surface, i.e., $S_{3refl}+S_{03}=S_{3refr}$, while its in-plane components are discontinuous. These latter components are related by $S_{1,2refl}+(\kappa'/\kappa)S_{1,2refr}=S_{1,20}$. One can see that, along the surface, the energy flows at different rates in the vacuum and in the plasma.

Usually, Fresnel relations are given for two particular cases: $\beta=0$ ($\phi=\pi/2$), $E_{01}=E_{03}=0$, which corresponds to the so-called s-wave (electric field perpendicular to the plane of incidence); and $\beta=\alpha$ ($\phi=0$), $E_{02}=0$, corresponding to the so-called p-wave (electric field in the plane of incidence) [31–33]. For the former case we get

$$E_{2refl}=\frac{\cos\alpha-\sqrt{\varepsilon-\sin^2\alpha}}{\cos\alpha+\sqrt{\varepsilon-\sin^2\alpha}}E_{02}e^{-i\kappa z},$$

$$E_{2refr}=\frac{2\cos\alpha}{\cos\alpha+\sqrt{\varepsilon-\sin^2\alpha}}E_{02}e^{i\kappa' z}. \tag{34}$$

which is one well-known pair of Fresnel relations. Another set of Fresnel relations is obtained from our equations given above for $E_{1,3refl}$ and $E_{1,3refr}$ components (p-wave); usually, this pair of Fresnel relations is given in terms of the magnetic field. Making use of our equations derived above we get the well-known p-wave Fresnel relations

$$H_{2refl}=\frac{\varepsilon\cos\alpha-\sqrt{\varepsilon-\sin^2\alpha}}{\varepsilon\cos\alpha+\sqrt{\varepsilon-\sin^2\alpha}}H_{02}e^{-i\kappa z},$$

$$H_{2refr}=\frac{2\varepsilon\cos\alpha}{\varepsilon\cos\alpha+\sqrt{\varepsilon-\sin^2\alpha}}H_{02}e^{i\kappa' z}. \tag{35}$$

The generalization given here in Eqs. (23), (28), (29), and (31)–(33) consists in extending these relations to any incidence angle and polarization, together with including the dielectric function $\varepsilon=1-\omega_p^2/\omega^2$, which follows from the present treatment.

The reflection coefficient $R=|\mathbf{E}_{refl}|^2/|\mathbf{E}_0|^2$ can be obtained straightforwardly from the reflected fields given by Eqs. (31)–(33). It can be written as

$$R=R_1\cos^2\beta\sin^2\phi+R_2(\cos^2\beta\cos^2\phi+\sin^2\beta), \tag{36}$$

where

$$R_1=\left|\frac{\omega\cos\alpha-\sqrt{\omega^2\cos^2\alpha-\omega_p^2}}{\omega\cos\alpha+\sqrt{\omega^2\cos^2\alpha-\omega_p^2}}\right|^2=\left|\frac{\cos\alpha-\sqrt{\varepsilon-\sin^2\alpha}}{\cos\alpha+\sqrt{\varepsilon-\sin^2\alpha}}\right|^2 \tag{37}$$

and

$$R_2=\left|\frac{(\omega^2-\omega_p^2)\cos\alpha-\omega\sqrt{\omega^2\cos^2\alpha-\omega_p^2}}{(\omega^2-\omega_p^2)\cos\alpha+\omega\sqrt{\omega^2\cos^2\alpha-\omega_p^2}}\right|^2=\left|\frac{\varepsilon\cos\alpha-\sqrt{\varepsilon-\sin^2\alpha}}{\varepsilon\cos\alpha+\sqrt{\varepsilon-\sin^2\alpha}}\right|^2. \tag{38}$$

The first term in the r.h.s. of Eq. (36) corresponds to the s-wave ($\beta=0$, $\phi=\pi/2$), while the second term corresponds to the p-wave ($\beta=\alpha$, $\phi=0$). It is easy to see that there exists a cusp (shoulder) in the behavior of the function $R(\omega)$ occurring at the transparency edge $\omega=\omega_p/\cos\alpha$, where the reflection coefficient exhibits a sudden enhancement on passing from the propagating regime to the damped one, as expected (total reflection). The condition for total reflection can also be written as $\sin\alpha=\sqrt{\varepsilon}$, where $R=1$ ($R_{1,2}=1$), as is well known. For illustration, the reflection coefficient is shown in Fig. 1 for $\beta=\pi/6$ and various incidence angles. The reflection coefficient vanishes at $\omega^2=\omega_p^2/(1-\tan^2\alpha)$ ($\tan\alpha=\sqrt{\varepsilon}$) for $\alpha=\beta<\pi/4$ ($R_2=0$, $\phi=0$).

The results obtained in this section for the interaction of the electromagnetic field with a semi-infinite plasma are the same as those obtained within the so-called theory of "effective medium permittivity," where the dielectric function $\varepsilon=1-\omega_p^2/\omega^2$ is introduced into Fresnel relations, the latter being derived by continuity conditions at the surface. On the other hand, we can see that Eqs. (23), (28), and (29) relate the total field $m\omega^2\mathbf{u}/e$ inside the plasma to the amplitude of the external field $\mathbf{E}_0$. However, while the former runs like $e^{i\kappa' z}$, the latter runs like $e^{i\kappa z}$, so we cannot define properly a dielectric function in usual terms (plane waves) for this semi-infinite plasma (the dielectric function $\varepsilon=1-\omega_p^2/\omega^2$ corresponds to the bulk plasma). The same is true for the nonretarded dielectric response, which contains a surface term $\sim e^{-kz}$. This particular feature is related to the nonlocality of the dielectric response and it holds for any structure with restricted geometry.

Finally, we note that we do not use in our approach boundary (continuity) conditions at the surface; instead, the usual continuity conditions follow from our approach with respect to the transverse components of the electric field and the normal component of the electric induction. There is no need for additional boundary conditions because the problem is completely determined by our equations and the external field.

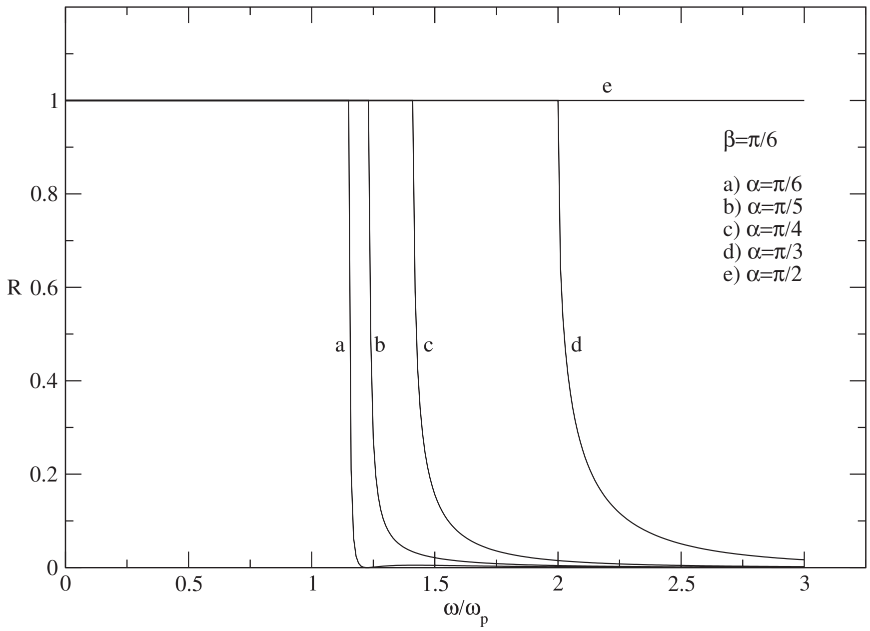

Fig. 1. Reflection coefficient for a semi-infinite plasma for $\beta=\pi/6$ and various incidence angles α. One can see the shoulder occurring at the transparency edge $\omega_p/\cos\alpha$ and the zero occurring at $\omega^2=\omega_p^2/(1-\tan^2\alpha)$ for $\alpha=\beta=\pi/6$ ($R_2=0$, $\phi=0$).

4. CONCLUSIONS

The approach presented here is a quasi-classical one, valid for wavelengths much longer than the amplitude of the Fourier components of the displacement field **u**. This is not a particularly restrictive condition for the classical dynamics of the electromagnetic field interacting with matter. When this condition is violated, as, for instance, for wavelengths much shorter than the mean separation distance between electrons, there appear both higher-order terms in the equations of motion and the coupling to the individual motion of the electrons. These couplings affect in general the dispersion relations and introduce a finite lifetime (damping) for the plasmon and polariton modes.

Making use of the equations of motion for the displacement field **u** and the radiation formulas for the electromagnetic potentials, we have computed herein the plasmon and polariton modes for an ideal semi-infinite electron plasma, as well as the dielectric response, the reflected and refracted fields, and the reflection coefficient. Generalized Fresnel relations have been obtained for any incidence angle and polarization. We have also identified the bulk and surface plasmon–polariton modes. The field inside the plasma is either damped (evanescent) or propagating, as is well known, and the reflection coefficient exhibits a sudden enhancement on passing from the propagating to the damped regime, as expected. The transparency edge is given by $\omega\cos\alpha=\omega_p$, where α is the incidence angle, ω is the frequency of the incident wave, and ω_p is the plasma frequency.

Other effects related to the dynamics of a semi-infinite electron plasma, or, in general, various plasmas with rectangular geometries, can be computed similarly by using the method presented here. The method can also be applied to plasmas with other, more particular, geometries. Dissipation can be introduced (as for metals), and a model can be formulated for dielectrics, amenable to the method presented here. This will allow the treatment of more realistic cases as well as various interfaces, in particular plasmas (or metals) bounded by dielectrics. These investigations are left for forthcoming publications.

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