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ALPHA-LIKE FOUR NUCLEON CORRELATIONS IN SUPERFLUID PHASES OF ATOMIC NUCLEI

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Abstract: α-like four-nucleon correlations are included in the structure of superfluid ground and low-lying excited states of atomic nuclei within a BCS-like approach. New metastable superfluid and normal states are predicted. These states could be associated with some of the recently discovered $I^{\pi}=0^{+}$ states in different regions of atomic nuclei. A new type of elementary excitations may be constructed on these metastable states in the same way as those constructed on the BCS superfluid ground states. The region of superfluid cold nuclei is enlarged due to the fact that the neutron and proton superfluidity can mutually be induced via the α-like four-nucleon ineractions. This type of correlations lead to a further enhancement of the probabilities of the favoured α-clusterization processes (such as α-decay or α-transfer reactions), two-nucleon transfer reactions and other clusterization processes such as e.g. the heavy cluster decay.

1. Introduction

Correlations between the nucleons are generally responsble for the existence of substructures in nuclei. Of these, pairing correlations are by far the best understood correlations as they affect a lot of nuclear processes, which have been extensively studied throughout all regions of nuclei. Among these processes, the two-nucleon-, α-transfer reactions and α-decay display a remarkable degree of selectivity. The pairing correlations separate among these processes the favoured and hindered ones deeper than the shell model without residual interactions. For the favoured transitions coherent sums have to be taken over the shell model configurations in the transition probabilities, in order to include correlations, while for the hindered transitions this coherence is partially or totally absent. Absolute values of the favoured α-decay widths and α-transfer cross-sections are not however sufficiently well explained in the region of superfluid cold nuclei, by pairing correlations alone. Despite the fact that for these nuclei the spectroscopic factors corresponding to the favoured processes are enhanced by factors of thousands when the pairing correlations are included [1]), they are still insufficient in order to reproduce the experimental probabilities.

In the course of time extensive nuclear structure and α-decay studies have brought forth the picture of more or less distinct α-aggregates in nuclei such as α-clusters [2-4]), quartets [5-6]), superfluid four-nucleon substructures [7-13]) (SαS), or shell model

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clusters [14-15]). Thus it is important to distinguish among these two-proton and two-neutron substructures (α-aggregates). For instance an α-particle cluster in the nucleus is not the same as a free α-particle. It is naturally distorted by fields of the surrounding nucleons and may be violently changed by close interactions between them. By α-clusters one should probably understand aggregates formed from two protons and two neutrons with relatively strong spatial correlations, so that the spatial localization of these aggregates be smaller than the spatial localization of the nucleus itself. The SαS may be viewed at least in certain circumstances as formed of two correlated Cooper pairs (one proton pair and one neutron pair). Certainly they have weaker spatial correlations (analogously to the spread of the Cooper pair in a superconductor) as compared to α-clusters, but stronger correlations in the angular momentum space for finite nuclei (or in the momentum space for nuclear matter).

Recently [16]) similar correlations leading to an α-like condensate have been proposed in the framework of the interacting boson model [17]). This model conceals the mechanism of formation of SαS by approximating the pairs by bosons. The underlying fermionic structure of the paired bosons certainly plays an important role in the formation of SαS in nuclei as far as genuine four-nucleon correlations are looked for. Such an investigation raises the question whether a condensed state could directly be obtained by starting with a purely fermionic hamiltonian incorporating two- and four-nucleon interactions.

Early attempts [7-12]) to account for SαS in nuclei use a trial wave function with four-nucleon correlations included, simulating the four-fermion condensate, the interaction remaining of the two-fermion type. In ref. [13]) the four-nucleon correlations are included by four-nucleon interactions within an exact diagonalization procedure. However the two-level model developed in ref. [13]) may be applied probably to light nuclei only, while a generalization to more levels seems to be very difficult.

A different point of view is assumed in the present work and in our preliminary report [18]); it consists essentially in using the BCS-pairing trial wave function and accounts for four-fermion correlations by including a residual coherent two pair (proton and neutron) interaction term in addition to the usual BCS-hamiltonian [1,19,20]). Within this simple approximation the four-fermion interactions lead to condensed superfluid ground and/or metastable states of the Fermi liquid consisting of correlated fermion pairs. Considering a simplified model in which the single-particle part of the hamiltonian for a deformed axially symmetric nucleus [19]), has equidistant energy level spectra for the two types of fermions (protons and neutrons) we reach the following conclusions.

For sufficiently large pairing and α-like coupling constants G_p, G_n, and G_4, the condensed symmetry broken state is energetically favoured with respect to the normal fluid one. Consequently, under these circumstances, a normal fluid–superfluid first-order phase transition is predicted.

For a region of intermediate and large coupling constants normal fluid and superfluid metastable phases are predicted too. The possibility of introducing the concept of normal, superfluid and metastable normal and superfluid bands can also be discussed. In analogy to the well-known rotational bands the elementary excitations built up on normal or superfluid ground or metastable states constitute a normal or superfluid band.

The paper is organized as follows. In sect. 2 we formulate the model. The discussion of the gap equations and the phase diagram is presented in sect. 3 within the schematic model mentioned above.

The procedure for extracting the pairing G_p-, G_n- and α-like G_4-strengths from the experiment, using the general model formulated in sect. 2, is presented in sect. 4. A new binding energy difference involving six nuclei, as a measure for α-like four-nucleon correlations is proposed. In sect. 5 using the experimental strengths we calculated the superfluid enhancement factors for favoured α-clusterization processes, and two-nucleon transfer processes. The results show an enhancement of these factors of about 10–20% for α-clusterization processes and of about 5–10% for two-nucleon transfer processes when the α-like correlations are included. The superfluid enhancement factor for ^{14}C decay, for instance is also estimated to increase up to 100%.

2. Outline of the model

We consider a system of nucleons (protons and neutrons) moving in a certain single-particle self-consistent field as e.g. a deformed Woods–Saxon one [19]).

The hamiltonian for the system of interacting nucleons is

$$H = \sum_{i=p,n} (H_i^{av} + H_i^{pair}) + H_4 , \tag{1}$$

$$H_i^{av} = \sum_{s_i\sigma_i} E_{s_i} a^+_{s_i\sigma_i} a_{s_i\sigma_i} , \tag{2}$$

$$H_i^{pair} = -G_i P_i^+ P_i , \qquad P_i = \sum_{s_i} a_{s_i-} a_{s_i+} ; \qquad i = p, n , \tag{3}$$

$$H_r = -G_4 P_p^+ P_n^+ P_n P_p . \tag{4}$$

Here $a^+_{s_i\sigma_i}$ ($a_{s_i\sigma_i}$) are the Fermi operators which create (destroy) a nucleon in (from) the single-particle state $|s_i\sigma_i\rangle$, where σ_i is the sign of the projection of the angular momentum of the state onto the nuclear symmetry axis, s_i being the rest of the quantum numbers that label the single-particle energy levels.

The last term (4) in eq. (1) is an effective, coherent two-pair (four-nucleon) interaction term, which is expected to induce the α-like four-nucleon correlations in the superfluid phases of the atomic nucleus. The other terms in eq. (1) describe the usual BCS-superfluidity [1,20]). The G_p, G_n and G_4 quantities are the positive valued coupling strength constants nonvanishing in a certain energy range (the cut-off energy range).

As trial wave function we use the BCS wave function

$$|\mathrm{BCS}\rangle = \prod_{s_i, i=\mathrm{p,n}} (U_{s_i} + V_{s_i} a^+_{s_i+} a^+_{s_i-})|0\rangle\,, \tag{5}$$

where $U_s^2 + V_s^2 = 1$ and $|0\rangle$ denotes the absolute vacuum.

Thus the constrained energy functional is:

$$\begin{aligned} W &= \langle \mathrm{BCS}|H - \lambda_\mathrm{p}\hat{N}_\mathrm{p} - \lambda_\mathrm{n}\hat{N}_\mathrm{n}|\mathrm{BCS}\rangle \\ &= \sum_{s_\mathrm{p}} 2(\tilde{E}_{s_\mathrm{p}} - \lambda_\mathrm{p}) V^2_{s_\mathrm{p}} + \sum_{s_\mathrm{n}} 2(\tilde{E}_{s_\mathrm{n}} - \lambda_\mathrm{n}) V^2_{s_\mathrm{n}} - G_\mathrm{p}\chi^2_\mathrm{p} - G_\mathrm{n}\chi^2_\mathrm{n} - G_4\chi^2_\mathrm{p}\chi^2_\mathrm{n}\,. \end{aligned} \tag{6}$$

Here $\lambda_{\mathrm{p(n)}}$ denotes the proton (neutron) Fermi level, $\hat{N}_\mathrm{p}(\hat{N}_\mathrm{n})$ is the proton (neutron) number operator and

$$\chi_i = \sum_{s_i} V_{s_i} U_{s_i}\,, \tag{7}$$

$$\tilde{E}_{s_{\mathrm{p(n)}}} = E_{s_{\mathrm{p(n)}}} - \tfrac{1}{2}(G_{\mathrm{p(n)}} + G_4\chi^2_{\mathrm{n(p)}}) V^2_{s_{\mathrm{p(n)}}} - \tfrac{1}{4} G_4 V^2_{s_{\mathrm{p(n)}}} \sum_{s_{\mathrm{n(p)}}} V^2_{s_{\mathrm{n(p)}}} \tag{8}$$

are the modified single-particle energy levels.

Usually the self-consistent field corrections $\tilde{E}_s - E_s$ are omitted [1]).

The minimization of W given by eq. (6) with respect to the variational parameters leads to the following gap and constraint equations:

$$\begin{aligned} \tfrac{1}{2}(G_{\mathrm{p(n)}} + G_4\chi^2_{\mathrm{n(p)}}) \sum_{s_{\mathrm{p(n)}}} \frac{1}{\varepsilon_{s_{\mathrm{p(n)}}}} &= 1\,, \\ \sum_{s_i} \left(1 - \frac{\tilde{E}_{s_i} - \lambda_i}{\varepsilon_{s_i}}\right) &= N_i\,, \end{aligned} \tag{9}$$

for doubly even mass deformed nuclei. For odd- or odd–odd-mass deformed nuclei the eqs. (9) are modified according to the blocking effect [1]).

The new quantities in eq. (9) are defined as follows:

$$\varepsilon_{s_i} = [(\tilde{E}_{s_i} - \lambda_i)^2 + \Delta_i^2]^{1/2}\,, \tag{10}$$

$$\begin{pmatrix} U^2_{s_i} \\ V^2_{s_i} \end{pmatrix} = \frac{1}{2}\left(1 \pm \frac{\tilde{E}_{s_i} - \lambda_i}{\varepsilon_{s_i}}\right). \tag{11}$$

The eqs. (9) represent a set of coupled nonlinear eqs. for the nontrivial (superfluid) solutions. The original gap and constraint equations including the trivial solutions are:

$$\begin{aligned} \Delta_{\mathrm{p(n)}} &= \chi_{\mathrm{p(n)}}(G_\mathrm{p} + G_4\chi^2_{\mathrm{n(p)}})\,, \\ N_i &= \sum_{s_i} 2V^2_{s_i}\,, \end{aligned} \tag{12}$$

where

$$\chi_i = \sum_{s_{\mathrm{p(n)}}} \frac{\Delta_i}{2\varepsilon_{s_i}}\,. \tag{13}$$

In the eqs. (9) and (12) N_i is the number of protons (neutrons) taken into account in the cut-off energy range.

For a spherical nucleus the labels s_i stand for the single-particle shell-model quantum numbers n, l, j and the eqs. (6) and (9) should be replaced by:

$$W=\sum_{s_p} 2\Omega_{s_p}(\tilde{E}_{s_p}-\lambda_p)V_{s_p}^2+\sum_{s_n} 2\Omega_{s_n}(\tilde{E}_{s_n}-\lambda_n)V_{s_n}^2-G_p\tilde{\chi}_p^2-G_n\tilde{\chi}_n^2-G_4\tilde{\chi}_p\tilde{\chi}_n \quad (14)$$

and

$$\tfrac{1}{2}(G_{p(n)}+G_4\tilde{\chi}^2_{n(p)})\sum_{s_{p(n)}}\frac{\Omega_{s_{p(n)}}}{\varepsilon_{s_{p(n)}}}=1\,,$$
$$N_i=\sum_{s_i}\Omega_{s_i}\left(1-\frac{\tilde{E}_{s_i}-\lambda_i}{\varepsilon_{s_i}}\right), \quad (15)$$

where $\tilde{E}_{s_i}$ has the expression (8) in which χ_i should be replaced by $\tilde{\chi}_i$ and

$$\tilde{\chi}_i=\sum_{s_{p(n)}}\frac{\Omega_{s_i}\lambda_i}{2\varepsilon_{s_i}} \quad (16)$$

with

$$\Omega_s=\tfrac{1}{2}(2j_s+1)\,. \quad (17)$$

Δ_i, ε_{s_i}, U_{s_i} and V_{s_i} have the same expressions as those given by eqs. (10)-(12) in which the correlation functions (16) should be used.

In practical calculations the corrected single-particle energies $\tilde{E}_s$ from eq. (8) are generated within the Woods-Saxon shell model [19]) or the Hartree-Fock approach.

Some more grounds concerning the H_4 term (4) could be the following. Recently a Fermi liquid model for α-clusterization and α-decay has been proposed [21]) and tested [22,23]) on different α-transitions. This model has been born as a result of a comprehensive analysis [24]) of the current α-decay models, where it is shown that the known nuclear structure model wave functions are not sufficient to describe the α-clusterization process entering the α-decay and α-transfer reactions. A scattering amplitude form of the transition operator rather than a potential form is necessary. The Fermi liquid model of α-decay introduces a four-nucleon interaction for the irreducible reaction amplitude of the α-cluster formation in the four-particle channel (fig. 1) based on a Migdal's idea [25]) following the Landau theory of quantum liquids.

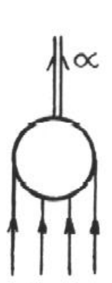

Fig. 1. The α-four-nucleon vertex.

As a matter of fact it may be shown that our H_4 term (4) is equivalent to a field-theoretical one [26]) in which the interaction vertex [21,22,25]) shown in fig. 1 is used.

3. Solutions of the gap equations and the phase diagram

We begin this section by recalling that for $G_4=0$ the gap eqs. (9) and (15) have solutions for coupling constants G_i exceeding the critical values given by the Belyaev conditions [20]):

$$\tfrac{1}{2}G_i^{\mathrm{cr}}\sum_{s_i}\frac{\Omega_{s_i}}{|\tilde{E}_{s_i}-\lambda_i|}=1 \tag{18}$$

with $\Omega_s=1$ for a deformed, axially symmetric nucleus and $\Omega_s=\frac{1}{2}(2j_s+1)$ for a spherical one. The phase structure of this model with respect to the G_i control parameters reduces (independently for protons and neutrons) to normal phases for $G_i \leq G_i^{\mathrm{cr}}$ and superfluid phases in the opposite cases, the phase transitions being of the second order. Such phase transitions have been observed in two-nucleon transfer reactions [27]), α-clusterization processes, etc.

In the case of our model a complete discussion with respect to the three control parameters G_{p}, G_{n} and G_4 and arbitrary single-particle spectra is practically impossible. In order to grasp the character of the phase structure and to identify specific features associated to the new G_4 coupling we consider a simplified model which proves to be rich enough to deserve attention by itself and to suggest the highly nontrivial behaviour of the realistic model.

Let us assume that the single-particle part (2) of our hamiltonian (1) has equidistant level spectra for the two types of fermions and introduce the following notations;

$$\tilde{E}_{s_i}=E_{\mathrm{F}}(i)+\frac{k}{\rho_i}, \tag{19}$$

$$\lambda_i=E_{\mathrm{F}}(i)+\frac{\sigma_i}{\rho_i}, \tag{20}$$

$$g_i=\rho_i G_i, \tag{21}$$

$$x_i=(\rho_i\Delta_i)^2, \tag{22}$$

$$g_4=\rho G_4, \qquad \rho=\tfrac{1}{2}(\rho_{\mathrm{p}}+\rho_{\mathrm{n}}), \tag{23}$$

where $i=\mathrm{p}$, n and k are entire numbers belonging to the Λ shells [28]) (the cut-off energy range) specified by the intervals $[-N_{1i}, N_{2i}]$, while $E_{\mathrm{F}}(i)$ and λ_i are, respectively, the Fermi energies for non-interacting and interacting fermions of type i.

Moreover, let us analyse the "symmetric" situation where the protons and neutrons have the same properties:

$$\begin{aligned}&\rho_{\mathrm{p}}=\rho_{\mathrm{n}}=\rho, \qquad g_{\mathrm{p}}=g_{\mathrm{n}}=g_2, \\ &\lambda_{\mathrm{p}}=\lambda_{\mathrm{n}}=\lambda, \qquad \sigma_{\mathrm{p}}=\sigma_{\mathrm{n}}=\sigma,\end{aligned} \tag{24}$$

for which the gap eqs. have symmetric solutions only

$$x = x_{\mathrm{p}} = x_{\mathrm{n}}\,, \tag{25}$$

and the case of the half-filling of the Λ-shells:

$$N_{1i} = n\,, \qquad N_{2i} = n+1\,, \qquad N_i = 2(n+1)\,. \tag{26}$$

Thus the constraint eqs. from (9) have the solutions

$$\sigma_i = \sigma = \tfrac{1}{2}\,. \tag{27}$$

The correlation energy of our model becomes

$$\begin{aligned} \varepsilon &= \rho E_{\mathrm{corr}} = \rho(W(x) - W(0)) \\ &= 2(n+1)^2 - 4S_1(x) + 4xS_{-1}(x) - 2g_2xS^2_{-1}(x) - g_4x^2S^4_{-1}(x) \end{aligned} \tag{28}$$

with

$$S_j(x) = \tfrac{1}{2}\sum_{k=-n}^{n+1}[x + \tfrac{1}{2}(k-\tfrac{1}{2})^2]^{j/2}\,, \tag{29}$$

which has to be studied on solutions of the gap equation:

$$F(x) = [g_2 + g_4xS^2_{-1}(x)]S_{-1}(x) - 1 = 0\,. \tag{30}$$

In order to find the number and character of the solutions of eq. (30) the following curves in the (g_2, g_4)-plane prove to be useful.

$$F(x) = 0\,, \qquad \frac{\mathrm{d}F}{\mathrm{d}x} = 0\,, \tag{31}$$

$$F(0) = 0\,, \tag{32}$$

$$\varepsilon(x) = 0\,, \qquad F(x) = 0\,. \tag{33}$$

For $n = 20$, curve (31) (point-dash GDECF in fig. 2) separates regions in which the number of solutions of the gap eq. (30) differ by two. Curve (32) (solid and horizontal ADBF in fig. 2), separates regions in which the number of solutions differ by one and in the case $g_4 = 0$ reduces to the critical value given by the Belyaev condition (18). The crossing of the curve(33) (dashed HBEIF in fig. 2) changes the sign of the correlation energy for one solution of the gap equation (30).

One should take into consideration in the following that the global minimum of the correlation energy (28) corresponds to the ground state of the system, while a local mininum may describe a metastable state of the system. Bearing then in mind that the derivatives of the correlation energy (28) with respect to the control parameters g_2 and g_4, for $x = x_0$, with x_0 the solution of the gap equation (30):

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}g_2} = -2x_0S^2_{-1}(x_0) < 0\,, \tag{34}$$

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}g_4} = -x_0^2S^4_{-1}(x_0) < 0\,, \tag{35}$$

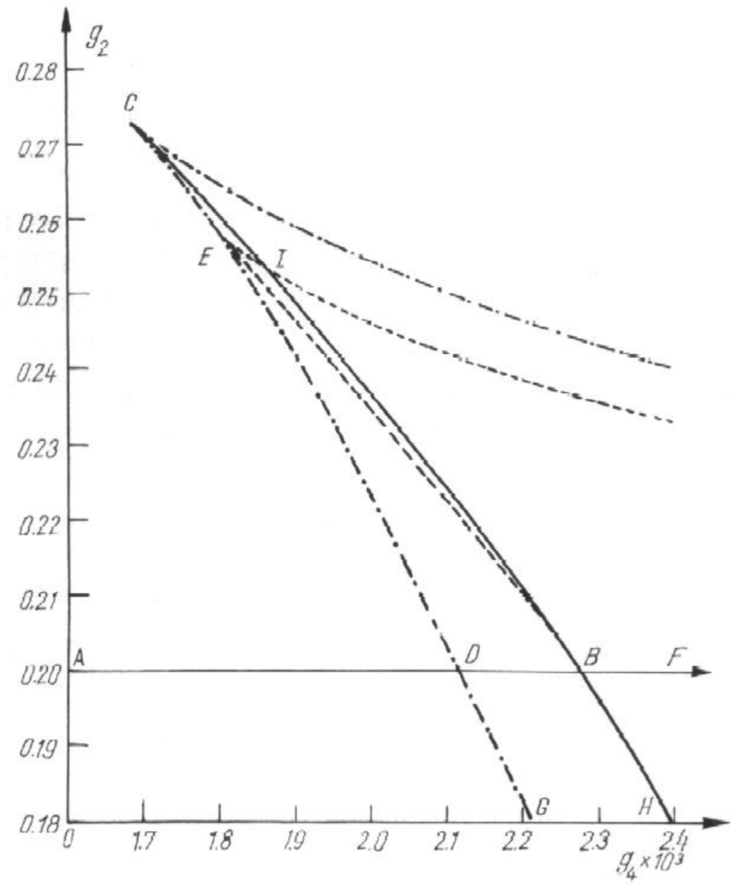

Fig. 2. The phase diagram for the simplified model (see sect. 3).

jump when the gap parameter x_0 has a discontinuity we reach the following conclusions.

In the region OADG (see fig. 2), the gap equation has no solutions and the ground state of the system is in the normal fluid phase.

In the region above ADECF, the gap equation has one solution, which corresponds to a negative minimum of the correlation energy. The ground state of the system is in the superfluid phase of the first kind (see below).

In the region HGDBF, the gap equation has two solutions, one of them being a minimum. Region DBHG has the ground state belonging to the normal fluid phase, while the local minimum, with a positive correlation energy may correspond to a metastable state in a superfluid phase of the second kind (see below).

In the region HBF the ground state of the system belongs to the superfluid phase of the second kind, while the metastable state belongs to the normal fluid phase.

In the region BDECFB the gap equation has three solutions, two of them corresponding to minima. The deepest minimum in the region BDCIB describes the ground state of the system belonging to the superfluid phase of the first kind, the other is associated with a metastable state belonging to the superfluid phase of the second kind (having the gap parameter larger than the gap parameter corresponding to the superfluid phase of the first kind).

The jump from one minimum to the other occurs on curve CIB (in fig. 2), which may be found by determining the self-intersections of curves

$$\varepsilon(x) = \text{const} < 0 , \qquad F(x) = 0 . \tag{36}$$

Crossing the curve CIBH corresponds to a first-order phase transition, while crossing the horizontal line ADB corresponds to a second-order phase transition.

A few more comments may be in order here. First of all our mean field approach is not to be taken too seriously for gaps which are not significantly smaller than the cut-off. In our case this corresponds to x smaller than $(n+1)$, otherwise the situation is one of strong coupling. In particular the transition to the CIB segment that appears for $\sqrt{x} \geqslant 20$ may raise some doubts about its reality. It may be that a renormalization group-type argument improving our correlation energy confirms or eliminates this transition. An alternative approach to clarifying this point may be, to consider the g_2 and g_4 control parameters level dependent, starting for instance from the calculated g_2 for the simple pairing problem to be found in refs. [29,30]), in which case a natural cut-off would occur. In any case a large part of the regions above the ACDF and BDECFB, lies deep inside the domain of validity of our approach, so that at least part of the richness of the behaviour we found is to be taken as granted.

Another interesting aspect is that crossing the CIB segment corresponds to the transition from a region in which the ground state has evolved continuously from that at $g_4 = 0$ which we call the g_2-dominated one to the g_4-dominated one, the segment CIB, being the border. If part of the region CIBF in the neighbourhood of this border is taken as granted, the above mentioned transition may be the analog in our model of a phase transition from a pair superfluid phase (superfluid phase of the first kind) to a quadruplet (α-like) superfluid phase (superfluid phase of the second kind). As to the difficulties in identifying the latter phase we remark that an investigation in the framework of the lattice gauge theory of a nonabelian model [31]) has failed to find a stable phase (at finite scale) in which the symmetry is dynamically broken by a four-fermion condensate despite the fact that the necessity of this phase was strongly motivated from a theoretical point of view. Even if this failure may be traced back to the mean field approach which was used, our treatment (perhaps renormalization group improved, quantum corrections included and restoration of broken symmetries [32-35])) seems to be from a pragmatic point of view a better candidate for describing four-fermion correlations than considering four-fermion condensation.

4. Determination of the coupling constant from the experiment

To fix the coupling constants G_p, G_n and G_4 entering the hamiltonian (1) from the experimental data we use the well-known odd-even mass differences [1]):

$$P_Z = \tfrac{1}{2}\{2\mathscr{E}(Z-1, N) - \mathscr{E}(Z, N) - \mathscr{E}(Z-2, N)\} , \tag{37}$$

$$P_N = \tfrac{1}{2}\{2\mathscr{E}(Z, N-1) - \mathscr{E}(Z, N) - \mathscr{E}(Z, N-2)\} , \tag{38}$$

for G_p and G_n and [18])

$$P_4 = \mathscr{E}(Z, N) - \mathscr{E}(Z-2, N-2) - \mathscr{E}(Z+1, N) + \mathscr{E}(Z-1, N) - \mathscr{E}(Z, N+1) + \mathscr{E}(Z, N-1) \tag{39}$$

for G_4.

Here

$$\mathscr{E}(Z, N) = \sum_{s=s_p,\, s_n} 2E_s V_s^2 - G_p\chi_p^2 - G_n\chi_n^2 - G_4\chi_p^2\chi_n^2 \tag{40}$$

for a doubly even nucleus and

$$\mathscr{E}(Z-1, N) = E_{s_{op}} + \sum_{s_p \neq s_{op}} 2E_{s_p} V_{s_p}^2 + \sum_{s_n} 2E_{s_n} V_{s_n}^2 - G_z\tilde{\tilde{\chi}}_p^2 - G_n\chi_n^2 - G_4\tilde{\tilde{\chi}}_p^2\chi_n^2 \tag{41}$$

for an odd-mass one [1]), where

$$\tilde{\tilde{\chi}}_p = \sum_{s_p \neq s_{op}} U_{s_p} V_{s_p} \,. \tag{42}$$

The experimental P_Z, P_N and P_4 quantities are obtained from the eqs. (37-39) by replacing the energies $\mathscr{E}$ by experimental [36]) binding energies ($-\mathrm{B}$).

The P_4 quantity has been chosen in the same spirit as the analogous quantity for pairing vibrations [27,37]), i.e.:

$$P_4 \simeq \mathscr{E}(Z, N) - \mathscr{E}(Z-2, N-2) - 2\lambda_p - 2\lambda_n \,. \tag{43}$$

The quantities P_Z, P_N and P_4 involve 8 nuclei. For each nucleus of this set we have so solve the gap eqs. (9) or (15). Thus we have to solve in all a nonlinear system of 35 equations with 35 unknowns for each nucleus (Z, N). Taking the $A = Z + N$ dependence of the coupling strength G_p and G_n as usual

$$G_p = \frac{1}{A} C_p \,\mathrm{MeV}, \qquad G_n = \frac{1}{A} C_n \,\mathrm{MeV}, \tag{44}$$

$$G_4 = \frac{1}{A^2} C_4 \,\mathrm{MeV}, \tag{45}$$

we have calculated for some rare-earth and actinide nuclei the C_p, C_n and C_4 parameters. The results are given in table 1.

The expression (45) may be obtained by using the assumption of an approximate factorization of the two-pair vertex interaction into two one-pair vertex interaction with coupling strengths of the form (**44**). In the performed calculations the cut-off energy range contains approximately 40 nucleon energy levels of the deformed Woods-Saxon potential [19]). The Δ_p and Δ_n gap parameters suffer a small increase when G_4 is switched on.

TABLE 1

Results of the numerical determination of the pairing and α-like correlations coupling strength constants G_p, G_n and G_4, the gap parameters Δ_p, Δ_n and the odd-even P_Z, P_N and α-like P_4 mass differences

Nucleus	G_p	G_n	G_4	Δ_p (MeV)	Δ_n (MeV)
$^{152}_{60}Nd_{92}$	19.3	17.1	27.7	1.170	1.160
	26.4	23.6	0	0.920	1.110
$^{156}_{62}Sm_{94}$	18.3	18.0	29.2	0.777	0.889
	24.4	21.8	0	0.685	0.850
$^{160}_{64}Gd_{96}$	21.0	21.6	27.0	0.832	0.991
	26.7	24.9	0	0.752	0.925
$^{164}_{66}Dy_{98}$	24.2	21.8	22.7	0.818	0.833
	27.7	24.6	0	0.739	0.800
$^{168}_{68}Er_{100}$	25.5	22.1	27.6	0.949	0.887
	29.4	26.1	0	0.832	0.839
$^{176}_{72}Hf_{104}$	23.2	19.9	19.1	0.755	0.843
	27.8	22.3	0	0.737	0.840
$^{180}_{74}W_{106}$	26.8	19.8	22.2	0.908	0.803
	31.3	23.0	0	0.878	0.786
$^{184}_{76}Os_{108}$	24.1	25.2	22.1	0.688	1.175
	31.6	26.3	0	0.670	1.080
$^{240}_{94}Pu_{146}$	34.0	21.5	15.2	0.900	0.685
$^{246}_{98}Cf_{148}$	31.7	19.9	14.5	0.609	0.355

Nucleus	P_Z(MeV) exp	P_Z(MeV) th	P_N(MeV) exp	P_N(MeV) th	P_4(MeV) exp	P_4(MeV) th
$^{152}_{60}Nd_{92}$	0.675	0.676	0.971	0.973	0.163	0.104
		0.674		0.971		−0.736
$^{152}_{62}Sm_{94}$	0.474	0.474	0.721	0.721	−0.177	−0.181
		0.473		0.720		−0.617
$^{160}_{64}Gd_{96}$	0.517	0.517	0.754	0.752	−0.216	−0.218
		0.517		0.754		−0.766
$^{164}_{66}Dy_{98}$	0.428	0.429	0.684	0.684	−0.582	−0.576
		0.428		0.684		−0.825
$^{168}_{68}Er_{100}$	0.504	0.504	0.667	0.667	−0.401	−0.401
		0.504		0.667		−0.883
$^{176}_{72}Hf_{104}$	0.594	0.593	0.728	0.727	−0.348	−0.340
		0.593		0.728		−0.576
$^{180}_{74}W_{106}$	0.681	0.681	0.741	0.742	0.076	0.070
		0.680		0.741		−0.508
$^{184}_{76}Os_{108}$	0.439	0.440	0.910	0.912	−0.823	−0.830
		0.439		0.910		−1.222
$^{240}_{94}Pu_{146}$	0.591	0.603	0.443	0.419	−0.313	−0.326
$^{246}_{98}Cf_{148}$	0.538	0.542	0.546	0.543	−0.266	−0.253

5. Superfluid enhancement factor for the α-clusterization probabilities and two-nucleon transfer reaction probabilities

Let us study the processes represented in fig. 3 whose matrix elements may be written as follows:

$$M_{(\tau\ \mathrm{or}\ 2i)} = \langle \mathrm{B}|\Omega_{\tau\ \mathrm{or}\ 2i}|\mathrm{A}\rangle\,, \tag{46}$$

where [1,24])

$$\Omega_\tau = \sum_{\substack{s_p s'_p s_n s'_n\\ \sigma_p \sigma'_p \sigma_n \sigma'_n}} \Omega^{(\tau)}_{\sigma_p\sigma'_p\sigma_n\sigma'_n}(s_p s'_p s_n s'_n)\, a^+_{s_p\sigma_p} a^+_{s'_p\sigma'_p} a^+_{s_n\sigma_n} a^+_{s'_n\sigma'_n}\,, \tag{47}$$

$$\Omega_{2i} = \sum_{s_i s'_i} \sum_{\sigma_i \sigma'_i} \Omega^{(2i)}_{\sigma_i\sigma'_i}(s_i s'_i)\, a^+_{s_i\sigma_i} a^+_{s'_i\sigma'_i} \tag{48}$$

with $i = \mathrm{p}$, n.

Here $\tau = 1$ and $\tau = 2$ corresponds to α-decay and α-transfer reactions respectively and $2i$ corresponds to two-neutron ($i = \mathrm{n}$) and two-proton ($i = \mathrm{p}$) transfer reactions, respectively.

The $|A\rangle$ and $|B\rangle$ wave functions describe the states of nuclei A and B respectively.

The probabilities are $\Gamma_\alpha = |M_{\tau=1}|^2$ for α-decay, $\sigma_\alpha = |M_{\tau=2}|^2$ for α-transfer reactions, and $\sigma_{2i} = |M_{2i}|^2$ for two-nucleon transfer reactions.

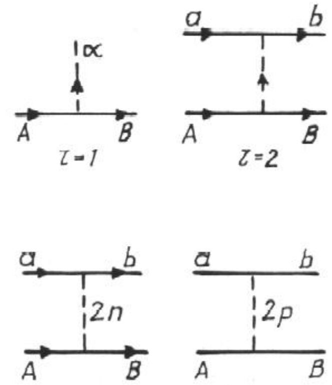

Fig. 3. Diagrams describing the α-decay, α-transfer and two-nucleon transfer reactions.

The matrix elements for favoured processes can be estimated [1]) by the following equations;

$$|M_\tau|^2 \cong |\langle \Omega^{(\tau)}_{+-+-}(s_p s'_p s_n s'_n)\rangle|^2 \chi_p^2 \chi_n^2\,, \tag{49}$$

$$|M_{2i}|^2 \cong |\langle \Omega^{(\sigma)}_{+-}(s_i s'_i)\rangle|^2 \chi_i^2\,, \tag{50}$$

where $\langle\Omega\rangle$ quantities are the averaged shell model without residual interactions matrix elements of the corresponding processes presented in fig. 3 and χ_i are given in the eqs. (7) or (16).

The superfluid enhancement factors $\chi_p^2\chi_n^2$ for favoured α-clusterization processes and χ_i^2 for two-nucleon transfer reactions are given in table 2, calculated with and

TABLE 2

Superfluid enhancement factors for favoured two- and four-nucleon processes

Nucleus	C_4	χ_p^2	χ_n^2
$^{152}_{60}Nd_{92}$	27.7	37.7	53.6
	0	27.9	51.5
$^{156}_{62}Sm_{94}$	29.2	22.4	38.7
	0	19.1	37.5
$^{160}_{64}Gd_{96}$	27.0	23.4	38.3
	0	20.2	35.2
$^{164}_{66}Dy_{98}$	22.7	22.0	29.7
	0	19.0	28.2
$^{168}_{68}Er_{100}$	27.6	26.9	31.2
	0	22.2	29.2
$^{176}_{72}Hf_{104}$	19.1	22.3	44.0
	0	21.9	43.8
$^{180}_{74}W_{106}$	22.2	26.6	38.9
	0	25.4	37.7
$^{184}_{76}Os_{108}$	22.1	15.9	63.1
	0	15.2	56.9
$^{240}_{94}Pu_{146}$	15.2	33.7	47.6

without α-like four-nucleon correlations included. The overall conclusion is that the superfluid enhancement factors increase with G_4 up to 20% in the superfluid region of atomic nuclei. We expect large variations for these superfluid enhancement factors in the regions of phase transitions. Such conclusion may explain the experimental α-reduced widths of $^{184-192}Pb$ isotopes obtained in ref. [38]) lying probably in the region of second-order phase transition for the proton system. The proton superfluidity for these isotopes is probably induced by the neutron superfluidity via the α-like four-nucleon correlations rather than assuming [38]) that $Z = 82$ ceases to be a magic number. Other experimental factors sustaining this point of view may be found in ref. [39]). The difference between the experimental and calculated [39]), within the pairing BCS-theory Te (d, ^{6}Li) Sn cross sections for ground state transitions may be removed by including our α-like four-nucleon correlations. The 0^+ states identified in ref. [39]) as proton pairing vibrational states in Sn isotopes could be metastable 0^+ states in our model in the region of the second-order phase transition. The calculated [39]) cross-sections for transitions to these 0^+ states within the BCS-pairing superfluid model exceed the values corresponding to a normal metastable state in our model and the experimental values [39]).

As a final remark we note that the enhancement superfluid factor for a heavy cluster decay [40]), the cluster containing $2n$ nucleons is estimated to be $(\chi)^{2n}$, where χ is the average value of the correlation function of the type given by eqs. (7) or (16). For ^{14}C decay, for instance, this superfluid enhancement factor may increase up to 100% for $G_4 \neq 0$ (see table 2).

6. Summary and conclusions

The α-like (two-proton and two-neutron) four-fermion correlations are included, in the structure of the ground and low-lying excited states of atomic nuclei in addition to the usual pairing correlations within a BCS-like approach.

Metastable phases whose lowest states are metastable states of atomic nuclei are predicted. With respect to the gauge space associated to particle number conservation these metastable symmetry breaking phases play the same role as rotational symmetry breaking shape isomers [43-46]) do with respect to the ordinary geometrical space. On such metastable states bands of elementary excitations can be constructed in addition to the ground state band, in analogy to the well-known rotational bands [37]). Phase transitions of first and second order are predicted also.

The region of superfluid nuclei could be enlarged firstly by the fact that the proton and neutron BCS pairing superfluidity can mutually be induced by one another via the α-like four-nucleon correlations and secondly by the appearance of the quadruplet (α-like) superfluid phase.

The probabilities for α-clusterization and two-nucleon transfer reactions may change their values as follows. In the region of the pairing superfluid phase these probabilities for favoured processes suffer a small increase, while in the region of the phase transitions they may present large variations.

To improve our approach we should introduce corrections describing quantum and thermal [41,42]) fluctuations which may change our phase diagram. Among the fluctuations we may remember, for instance, those accounted for by models that treat correlations in nuclei without violating the particle number conservation [32-35]). To touch this topic was at the moment beyond the scope of our paper. For comprehensive discussion including references we refer the reader to the monograph by Ring and Schuck [28]).

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