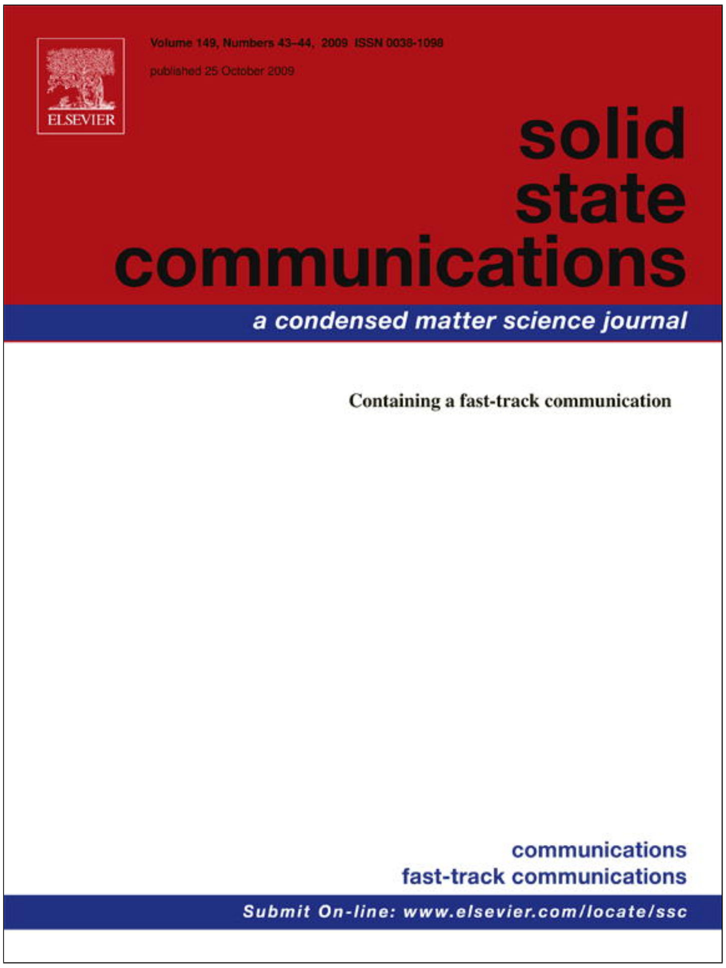
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Reflected and refracted electromagnetic fields in a semi-infinite body

M. Apostol *, G. Vaman

Department of Theoretical Physics, Institute of Atomic Physics, Magurele-Bucharest MG-6, POBox MG-35, Romania

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ABSTRACT

We compute the reflected and refracted electromagnetic fields for an ideal semi-infinite body (either a plasma or a dielectric), as well as the reflection coefficient, by using a general approach based on the polarization equation of motion and electromagnetic potentials. The method consists of representing the charge disturbances by a displacement field in the positions of the moving charges. The propagation of an electromagnetic wave in matter is treated by means of the retarded electromagnetic potentials, and the resulting integral equations are solved. Generalized Fresnel's relations are thereby obtained for any incidence angle and polarization and the angles of total polarization and total reflection are derived (the latter for the plasma). Bulk and surface plasmon–polariton modes are also identified for the plasma. As it is well known, the field inside the plasma is either damped (evanescent) or propagating (transparency regime), and the reflection coefficient exhibits an abrupt enhancement on passing from the propagating regime to the damped one (total reflection).

As is well known, the propagation of the eletromagnetic waves in matter is described usually by Fresnel's theory, which proved to be very successful for describing reflection and refraction [1, 2]. On the other hand, the matter polarization and response is usually represented by the dielectric function (and, in a more general form, by the magnetic permeability and electrical conductivity). This latter point raises some queries in applying the theory to particular cases, especially to structures with restricted geometries. In addition, the dielectric functions are either introduced by various ansatze or are model dependent. It would be desirable, therefore, of describing the reflection and refraction of the electromagnetic field without resorting to particular assumptions on the dielectric function, at least for reasonably realistic models [3–18]. This is particularly relevant for recent investigations of the electromagnetic waves in structures with special geometries, where a possible enhancement of the electromagnetic radiation has been reported [19–21].

We compute herein the reflected and refracted electromagnetic fields for a semi-infinite (half-space) body, first for a plasma and thereafter we describe briefly the extension of the calculations to a dielectric (insulator). The method we use is based on the electromagnetic potentials and the equation of motion for polarization. We represent the charge disturbances as $\delta n = -n div \mathbf{u}$, where n is the (constant, uniform) charge concentration and $\mathbf{u}$ is a displacement field of the mobile charges (electrons). This representation is valid for $\mathbf{Ku}(\mathbf{K}) \ll 1$, where $\mathbf{K}$ is the wavevector and $\mathbf{u}(\mathbf{K})$ is the Fourier component of the displacement field. We assume a rigid neutralizing background of positive charge, as in the well-known jellium model.

We assume a plane wave incident on the body surface under angle α. Its frequency is given by $\omega = cK$, where c is the velocity of light and the wavevector $\mathbf{K} = (\mathbf{k}, \kappa)$ has the in-plane component $\mathbf{k}$ and the perpendicular-to-plane component κ, such as $k = K\sin\alpha$ and $\kappa = K\cos\alpha$. In addition, $\mathbf{k} = k(\cos\varphi, \sin\varphi)$. The electric field is taken as $\mathbf{E}_0 = E_0(\cos\beta, 0, -\sin\beta) \times e^{i\mathbf{kr}}e^{i\kappa z}e^{-i\omega t}$, and we impose the condition $\cos\beta\sin\alpha\cos\varphi - \sin\beta\cos\alpha = 0$ (transversality condition $\mathbf{KE}_0 = 0$). The angle β defines the direction of the polarization of the incident field.

For a plasma, in the presence of an electromagnetic field $\mathbf{E}_0$ we use the equation of motion

$$\ddot{\mathbf{u}} = -\frac{e}{m}\mathbf{E} - \frac{e}{m}\mathbf{E}_0, \tag{1}$$

for the displacement field $\mathbf{u}$, where $-e$ is the electron charge, m is the electron mass and $\mathbf{E}$ is the polarizing field. We leave aside the dissipation effects (which can easily be included in Eq. (1)).

* Corresponding author. Tel.: +40 21 404 62 34.
E-mail address: apoma@theory.nipne.ro (M. Apostol).

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We consider an ideal semi-infinite body extending over the half-space $z > 0$ (and bounded by the vacuum for $z < 0$). The displacement field $\mathbf{u}$ is then represented as $(\mathbf{v}, u_3)\theta(z)$, where $\mathbf{v}$ is the displacement component in the (x, y)-plane, u_3 is the displacement component along the z-direction and $\theta(z) = 1$ for $z > 0$ and $\theta(z) = 0$ for $z < 0$ is the step function. We denote by $\mathbf{u}$ the couple $(\mathbf{v}, u_3)$ and use Fourier transforms of the type

$$\mathbf{u}(r, z; t) = \sum_{\mathbf{k}} \int d\omega \mathbf{u}(\mathbf{k}, z; \omega) e^{i\mathbf{k}\mathbf{r}} e^{-i\omega t} \tag{2}$$

where $\mathbf{r}$ is the (x, y)-in-plane position vector. Eq. (1) becomes

$$\omega^2 \mathbf{u} = \frac{e}{m}\mathbf{E} + \frac{e}{m}\mathbf{E}_0 e^{i\kappa z}, \tag{3}$$

for $z > 0$. In Eq. (3) we have preserved explicitly only the z-dependence (*i.e.* we leave aside the factors $e^{i\mathbf{k}\mathbf{r}}e^{-i\omega t}$). We find it convenient to employ the vector potential

$$\mathbf{A}(\mathbf{r}, z; t) = \frac{1}{c}\int d\mathbf{r}' \int dz' \frac{\mathbf{j}(\mathbf{r}', z'; t - R/c)}{R} \tag{4}$$

and the scalar potential

$$\Phi(\mathbf{r}, z; t) = \int d\mathbf{r}' \int dz' \frac{\rho(\mathbf{r}', z'; t - R/c)}{R}, \tag{5}$$

where $\mathbf{j} = -ne\dot{\mathbf{u}}\theta(z)e^{i\mathbf{k}\mathbf{r}}e^{-i\omega t}$ is the current density, $\rho = ne\,div\mathbf{u} = ne\left(i\mathbf{k}\mathbf{v} + \frac{\partial u_3}{\partial z}\right)\theta(z)e^{i\mathbf{k}\mathbf{r}}e^{-i\omega t} + neu_3(0)\delta(z)e^{i\mathbf{k}\mathbf{r}}e^{-i\omega t}$ is the charge density and $R = \sqrt{(\mathbf{r} - \mathbf{r}')^2 + (z - z')^2}$. The integrals in Eqs. (4) and (5) implies the known integral [22]

$$\int_{|z|}^{\infty} dx J_0\left(k\sqrt{x^2 - z^2}\right) e^{i\omega x/c} = \frac{i}{\kappa} e^{i\kappa|z|}, \tag{6}$$

where J_0 is the zeroth-order Bessel function of the first kind (and $\kappa^2 = \omega^2/c^2 - k^2$). It is convenient to use the projections of the in-plane displacement field $\mathbf{v}$ on the vector $\mathbf{k}$ and on the vector $\mathbf{k}_\perp = k(-\sin\varphi, \cos\varphi)$, $\mathbf{k}_\perp\mathbf{k} = 0$. We denote these components by $v_1 = \mathbf{k}\mathbf{v}/k$ and $v_2 = \mathbf{k}_\perp\mathbf{v}/k$, and use also the components $E_1 = \mathbf{k}\mathbf{E}/k$, $E_2 = \mathbf{k}_\perp\mathbf{E}/k$ and similar ones for the external field $\mathbf{E}_0$. We give here the components of the external field

$$E_{01} = E_0\cos\beta\cos\varphi, \qquad E_{02} = -E_0\cos\beta\sin\varphi, \\ E_{03} = -E_0\sin\beta. \tag{7}$$

One can check immediately the transversality condition $E_{01}k + E_{03}\kappa = 0$. Making use of $\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t} - grad\Phi$, Eqs. (4) and (5) give the electric field

$$\begin{aligned} E_1 &= -2\pi ine\kappa \int_0 dz' v_1(z') e^{i\kappa|z-z'|} \\ &\quad - 2\pi ne\frac{k}{\kappa}\int_0 dz' u_3(z')\frac{\partial}{\partial z'} e^{i\kappa|z-z'|}, \\ E_2 &= -2\pi ine\frac{\omega^2}{c^2\kappa}\int_0 dz' v_2(z') e^{i\kappa|z-z'|}, \\ E_3 &= 2\pi ne\frac{k}{\kappa}\int_0 dz' v_1(z')\frac{\partial}{\partial z} e^{i\kappa|z-z'|} \\ &\quad - 2\pi ine\frac{k^2}{\kappa}\int_0 dz' u_3(z') e^{i\kappa|z-z'|} + 4\pi neu_3 \end{aligned} \tag{8}$$

for $z > 0$. It is worth observing in deriving these equations the non-invertibility of the derivatives and the integrals, according to the identity

$$\frac{\partial}{\partial z}\int_0 dz' f(z')\frac{\partial}{\partial z'} e^{i\kappa|z-z'|} = \kappa^2 \int_0 dz' f(z') e^{i\kappa|z-z'|} - 2i\kappa f(z) \tag{9}$$

for any function $f(z)$, $z > 0$; it is due to the discontinuity in the derivative of the function $e^{i\kappa|z-z'|}$ for $z = z'$. Now, we employ equation of motion (3) in Eqs. (8) and get the integral equations

$$\begin{aligned} \omega^2 v_1 &= -\frac{i\omega_p^2\kappa}{2}\int_0 dz' v_1(z') e^{i\kappa|z-z'|} \\ &\quad - \frac{\omega_p^2 k}{2\kappa}\int_0 dz' u_3(z')\frac{\partial}{\partial z'} e^{i\kappa|z-z'|} + \frac{e}{m}E_{01}e^{i\kappa z}, \\ \omega^2 v_2 &= -\frac{i\omega_p^2\omega^2}{2c^2\kappa}\int_0 dz' v_2(z') e^{i\kappa|z-z'|} + \frac{e}{m}E_{02}e^{i\kappa z}, \\ \omega^2 u_3 &= \frac{\omega_p^2 k}{2\kappa}\int_0 dz' v_1(z')\frac{\partial}{\partial z} e^{i\kappa|z-z'|} - \frac{i\omega_p^2 k^2}{2\kappa} \\ &\quad \times \int_0 dz' u_3(z') e^{i\kappa|z-z'|} + \omega_p^2 u_3 + \frac{e}{m}E_{03}e^{i\kappa z} \end{aligned} \tag{10}$$

for the coordinates $v_{1,2}$ and u_3 in the region $z > 0$, where $\omega_p = \sqrt{4\pi ne^2/m}$ is the plasma frequency.

The second equation (10) can be solved straightforwardly by noticing that

$$\frac{\partial^2}{\partial z^2}\int_0 dz' v_2(z') e^{i\kappa|z-z'|} = -\kappa^2\int_0 dz' v_2(z') e^{i\kappa|z-z'|} + 2i\kappa v_2. \tag{11}$$

We get

$$\frac{\partial^2 v_2}{\partial z^2} + (\kappa^2 - \omega_p^2/c^2)v_2 = 0. \tag{12}$$

The solution of this equation is

$$v_2 = \frac{2eE_{02}}{m\omega_p^2}\cdot\frac{\kappa\left(\kappa - \kappa'\right)}{K^2}e^{i\kappa' z}, \tag{13}$$

where

$$\kappa' = \sqrt{\kappa^2 - \omega_p^2/c^2} = \frac{1}{c}\sqrt{\omega^2\cos^2\alpha - \omega_p^2}. \tag{14}$$

The wavevector κ' can also be written in a more familiar form $\kappa' = (\omega/c)\sqrt{\varepsilon - \sin^2\alpha}$, where $\varepsilon = 1 - \omega_p^2/\omega^2$ is the dielectric function. The corresponding component of the (total) electric field (the refracted field) can be obtained from Eq. (3); it is given by $\left(m\omega^2/e\right)v_2$. For $\kappa^2 < \omega_p^2/c^2$ ($\omega\cos\alpha < \omega_p$) this field does not propagate. For $\kappa^2 > \omega_p^2/c^2$ (ω greater than the transparency edge $\omega_p/\cos\alpha$) it represents a refracted wave (transparency regime) with the refraction angle α' given by Snell's law

$$\frac{\sin\alpha'}{\sin\alpha} = \frac{1}{\sqrt{1 - \omega_p^2/\omega^2}} = 1/\sqrt{\varepsilon}. \tag{15}$$

The polariton frequency is given by $\omega^2 = c^2K^2 = \omega_p^2 + c^2K'^2$, as it is well known, where $K'^2 = \kappa'^2 + k^2$.

The first and the third equations (10) can be solved by using an equation similar with Eq. (11) and by noticing that they imply

$$\kappa'^2 u_3 = ik\frac{\partial v_1}{\partial z}. \tag{16}$$

We get

$$v_1 = \frac{2eE_{01}}{m\omega_p^2}\cdot\frac{\kappa'\left(\kappa - \kappa'\right)}{\kappa\kappa' + k^2}e^{i\kappa' z} \tag{17}$$

and

$$u_3 = \frac{2eE_{03}}{m\omega_p^2}\cdot\frac{\kappa\left(\kappa - \kappa'\right)}{\kappa\kappa' + k^2}e^{i\kappa' z}. \tag{18}$$

Similarly, the corresponding components of the refracted field are given by Eq. (3). It is easy to check the transversality condition $v_1 k + u_3\kappa' = 0$ of the refracted wave.

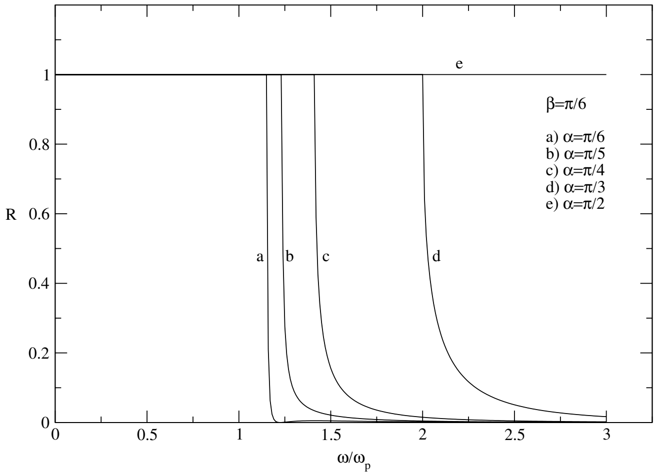

Fig. 1. Reflection coefficient for a semi-infinite plasma for $\beta = \pi/6$ and various incidence angles α. One can see the shoulder occurring at the transparency edge $\omega_p/\cos\alpha$ and the zero occurring at $\omega^2 = \omega_p^2/\left(1-\tan^2\alpha\right)$ for $\alpha = \beta = \pi/6$ ($R_2 = 0, \varphi = 0$).

We can see that the polarization field $\mathbf{E}$ in Eq. (1) cancels out the original, incident field $\mathbf{E}_0$ and gives the total, refracted field $m\omega^2\mathbf{u}/e$ inside the plasma. This is an illustration of the so-called Ewald–Oseen extinction theorem [8,23]. We note that a possible treatment of the propagation of the electromagnetic waves in matter by means of integral equations was suggested previously [23].

In order to get the reflected wave (region $z < 0$) we turn to Eqs. (8) and use therein the solutions given above for $v_{1,2}$ and u_3. It is worth noting here that the discontinuity term $\omega_p^2 u_3$ does not appear anymore in these equations (because $z' > 0$ and $z < 0$ and we cannot have $z = z'$). The integrations in Eqs. (8) are straightforward and we get the field

$$E_1 = E_{01}\frac{\kappa-\kappa'}{\kappa+\kappa'}\cdot\frac{\kappa\kappa'-k^2}{\kappa\kappa'+k^2}\mathrm{e}^{-\mathrm{i}\kappa z}, \tag{19}$$

$$E_2 = E_{02}\frac{\kappa-\kappa'}{\kappa+\kappa'}\mathrm{e}^{-\mathrm{i}\kappa z} \tag{20}$$

and

$$E_3 = -E_{03}\frac{\kappa-\kappa'}{\kappa+\kappa'}\cdot\frac{\kappa\kappa'-k^2}{\kappa\kappa'+k^2}\mathrm{e}^{-\mathrm{i}\kappa z}. \tag{21}$$

We can see that this field represents the reflected wave ($\kappa \to -\kappa$), and we can check its transversality to the propagation wavevector. Making use of the reflected field $\mathbf{E}_{refl}$ given by Eqs. (19)–(21) and the refracted field $\mathbf{E}_{refr}$ obtained from Eqs. (3) and (8) ($\mathbf{E}_{refr} = \mathbf{E} + \mathbf{E}_0 = m\omega^2\mathbf{u}/e$) one can check the continuity of the electric field and electric displacement at the surface ($z = 0$) in the form $E_{1,2refl} + E_{01,2} = E_{1,2refr}$, $E_{3refl} + E_{03} = \varepsilon E_{3refr}$, where $\varepsilon = 1 - \omega_p^2/\omega^2$. The angle of total polarization (Brewster's angle) is given by $\kappa\kappa' - k^2 = 0$, or $\tan^2\alpha = 1 - \omega_p^2/\omega^2 = \varepsilon$ (for $\alpha < \pi/4$). The above equations provide generalized Fresnel's relations between the amplitudes of the reflected, refracted and incident waves at the surface for any incidence angle and polarization. They can also be written by using $\omega^2 = \omega_p^2/(1-\varepsilon)$, where ε is the dielectric function.

The reflection coefficient $R = \left|\mathbf{E}_{refl}\right|^2/\left|\mathbf{E}_0\right|^2$ can be obtained straightforwardly from the reflected fields given by Eqs. (19)–(21). It can be written as

$$R = R_1\left[\cos^2\beta\sin^2\varphi + R_2\left(\cos^2\beta\cos^2\varphi + \sin^2\beta\right)\right], \tag{22}$$

where

$$R_1 = \left|\frac{\sqrt{\omega^2\cos^2\alpha-\omega_p^2}-\omega\cos\alpha}{\sqrt{\omega^2\cos^2\alpha-\omega_p^2}+\omega\cos\alpha}\right|^2 \tag{23}$$

and

$$R_2 = \left|\frac{\cos\alpha\sqrt{\omega^2\cos^2\alpha-\omega_p^2}-\omega\sin^2\alpha}{\cos\alpha\sqrt{\omega^2\cos^2\alpha-\omega_p^2}+\omega\sin^2\alpha}\right|^2. \tag{24}$$

The first term in the *rhs* of Eq. (22) corresponds to $\beta = 0$ ($\varphi = \pi/2$; *s*-wave, electric field perpendicular to the plane of incidence), while the second term corresponds to $\beta = \alpha$ ($\varphi = 0$; *p*-wave, electric field in the plane of incidence). It is easy to see that there exists a cusp (shoulder) in the behaviour of the function $R(\omega)$, occurring at the transparency edge $\omega = \omega_p/\cos\alpha$, where the reflection coefficient exhibits a sudden enhancement on passing from the propagating regime to the damped one, as expected (total reflection). The condition for total reflection can also be written as $\sin\alpha = \sqrt{\varepsilon}$, where $R = 1$ ($R_{1,2} = 1$), as it is well known. For illustration, the reflection coefficient is shown in Fig. 1 for $\beta = \pi/6$ and various incidence angles. The reflection coefficient is vanishing for $\omega^2 = \omega_p^2/\left(1-\tan^2\alpha\right)$ for $\alpha = \beta < \pi/4$ ($R_2 = 0, \varphi = 0$).

Making use of the reflected field given by Eqs. (19)–(21) and the refracted field ($\mathbf{E}_{refr} = m\omega^2\mathbf{u}/e$) given by Eqs. (13), (17) and (18) we can check the continuity of the energy flow across the surface. Indeed, we can compute the Poynting vector $\mathbf{S} = (c/4\pi)\mathbf{E}\times\mathbf{H} = (c^2/4\pi\omega)\mathbf{K}\left|\mathbf{E}\right|^2$, where $\mathbf{H} = (c/\omega)\mathbf{K}\times\mathbf{E}$ is the magnetic field, for the reflected and refracted plane waves. The component normal to the surface is continuous, *i.e.* $S_{3refl} + S_{03} = S_{3refr}$, while the in-plane components are discontinuous, they being related by $S_{1,2refl} + (\kappa'/\kappa)S_{1,2refr} = S_{1,20}$. One can see that, along the surface, the energy flows at different rates in the vacuum and in matter.

The present approach can be extended to a plasma slab of finite thickness d, $0 < z < d$, where the displacement field $\mathbf{u}$ can be represented as $(\mathbf{v}, u_3)\left[\theta(z) - \theta(z-d)\right]$. We have computed the electromagnetic field inside the slab, the reflected and transmitted fields and the reflection and transmission coefficients. The field inside the slab consists of a superposition of two plane waves $\mathrm{e}^{\pm\mathrm{i}\kappa' z}$, where κ' is given by the same equation (14). The transparency edge is given by the same equation $\omega\cos\alpha = \omega_p$ as for a semi-infinite plasma. Generalized Fresnel's relations have thereby been obtained, for both surfaces of the slab, any incidence angle and polarization. Apart from characteristic oscillations, the reflection and transmission coefficients exhibit an appreciable enhancement on passing from the propagating regime to the damped regime. The method can also be applied to other structures with more particular geometries.

The same method can be used for treating the plasmons in structures with special geometries. Indeed, the electric force in equation of motion (1) must then be replaced by the Coulomb (non-retarded) force. By using this procedure we have obtained for a semi-infinite plasma the well-known bulk plasmons with frequency ω_p and surface plasmons with frequency $\omega_p/\sqrt{2}$. Similarly, for a plasma slab we have derived the plasmon frequencies given by $\omega_p^2\left(1\pm\mathrm{e}^{-kd}\right)/2$ [24–31]. We have also computed the energy loss for these plasmas and the dielectric response. It is shown that the surface terms do not change the bulk dielectric function as usually defined (*i.e.* for a plane wave), since the surface contributions to the dielectric response are localized. The surface contribution to the energy loss exhibits characteristic oscillations in the transient regime near the surfaces.

It is worth investigating the eigenvalues of the homogeneous system of integral equations (10), for parameter κ given by $\kappa = \sqrt{\omega^2/c^2 - k^2}$. Such eigenvalues are given by the roots of the vanishing denominator in Eqs. (17) and (18), *i.e.* by equation $\kappa\kappa' + k^2 = 0$. This equation has real roots for ω only for the damped regime, *i.e.* for $\kappa = \mathrm{i}\left|\kappa\right|$ and $\kappa' = \mathrm{i}\left|\kappa'\right|$. Providing these conditions are satisfied, there is only one acceptable branch of excitations, given by

$$\omega^2 = \frac{2\omega_p^2 c^2 k^2}{\omega_p^2 + 2c^2k^2 + \sqrt{\omega_p^4 + 4c^4k^4}}. \quad (25)$$

We can see that $\omega \sim ck$ in the long wavelength limit and it approaches the surface plasmon frequency $\omega \sim \omega_p/\sqrt{2}$ in the non-retarded limit ($ck \to \infty$). These excitations are surface plasmon–polariton modes. They imply $v_2 = 0$ and $v_1, u_3 \sim e^{-|\kappa'|z}$. In addition, a careful analysis of the homogeneous system of Eqs. (10) reveals another branch of excitations, given by $\omega = \omega_p$, which, occurring in this context, may be termed the bulk plasmon–polariton modes. They are characterized by $v_2 = 0$ and $v_1(\mathbf{k}, z = 0) = 0$. For all these modes we have $u_3 = \left[ic^2k/\left(\omega^2 - c^2k^2 - \omega_p^2\right)\right](\partial v_1/\partial z)$.

For dielectrics, instead of Eq. (1), we use the equation of motion

$$\ddot{\mathbf{u}} = -\frac{e}{m}\mathbf{E} - \frac{e}{m}\mathbf{E}_0 - \omega_0^2\mathbf{u}, \quad (26)$$

where the frequency ω_0 is a parameter, usually much greater than any characteristic electromagnetic frequency of the body. This equation is well known in the elementary theory of classical dispersion. In particular, it leads to the well-known (bulk) dielectric function

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2 - \omega_0^2} \simeq 1 + \frac{\omega_p^2}{\omega_0^2}, \quad (27)$$

where both ω_p and ω_0 are adjustable parameters. Making use of the Eq. (26) all the results presented above for a semi-infinite plasma are formally preserved, except for the wavevector κ' which becomes

$$\kappa' \simeq \sqrt{\kappa^2 + \frac{\omega_p^2}{\omega_0^2} \cdot \frac{\omega^2}{c^2}} = \frac{\omega}{c}\sqrt{\varepsilon - 1}. \quad (28)$$

One can see that within this model of dielectrics there is no damping regime, as expected. The reflection coefficient R given by Eq. (22) does not depend on ω; it preserves its form given by Eq. (22) with

$$R_1 = \frac{(\varepsilon - 1)^2}{\left(\cos\alpha + \sqrt{\varepsilon - \sin^2\alpha}\right)^4} \quad (29)$$

and

$$R_2 = \left(\frac{\cos\alpha\sqrt{\varepsilon - \sin^2\alpha} - \sin^2\alpha}{\cos\alpha\sqrt{\varepsilon - \sin^2\alpha} + \sin^2\alpha}\right)^2. \quad (30)$$

Finally, we comment here upon two points. First, we can see that Eqs. (13), (17) and (18) connect the total field $m\omega^2\mathbf{u}/e$ to the amplitude of the external field $\mathbf{E}_0$. However, while the former goes like $e^{i\kappa' z}$, the latter goes like $e^{i\kappa z}$, so we cannot define a dielectric function in usual terms (plane waves) for this semi-infinite plasma (the dielectric function $\varepsilon = 1 - \omega_p^2/\omega^2$ corresponds to the bulk plasma). The same is true for the non-retarded dielectric response, which contains a surface term $\sim e^{-kz}$. This particular feature is related to the non-locality of the dielectric response and it holds for any structure with restricted geometry.

Second, it is worth noting that we do not use in our approach boundary conditions at the surface; instead, the usual continuity conditions follow from our approach, for the transverse components of the electric field and the normal component of the electric induction. There is no need for additional boundary conditions because the problem is completely determined by our equations and the external field.

Other effects related to the dynamics of plasmons and polaritons for a semi-infinite electron plasma, or, in general, various bodies with rectangular geometries, as well as structures with more particular geometries, can be computed similarly by using the method presented here. The dissipation can be included in this treatment (as for metals, or dielectrics with loss). This will allow the treatment of more realistic cases as well as various interfaces, in particular plasmas (or metals) bounded by dielectrics. These investigations are left for forthcoming publications.

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