PLANAR CHANNELING AND TRANSFER MATRIX TECHNIQUE

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Received 24 April 1973

A two-dimensional model for planar channeling is given by means of a transfer matrix technique. Critical energies and angles are calculated and the influence of the temperature is also discussed.

As is well known, when energetic charged particles enter a monocrystalline target within a certain critical angle of either a low-index plane or a low-index direction they show anomalous large penetration and small energy losses. These steering effects are called planar and axial channeling, respectively. The aim of this letter is to give a simple model for the planar channeling.

Lindhard [1] has considered that in the planar channeling the motion of a particle is due to the collective collision of this particle with all the atoms in a plane. This image holds especially when the energy of the particle is very large. But, we shall consider instead, that the motion of a particle of lower energy (but obviously large in comparison with the diffraction energy) rather proceeds by individual collisions with the atoms or with the groups of atoms. With this conjecture we solve the problem of planar channeling in terms of classical collisions of the moving particle with its neighbouring atoms located in both the layers which ensure the channeling. We notice that the planar channeled particle is to be oscillating less parallel to the channeling layers than perpendicular to these layers and one knows indeed that planar channeling is more frequent in lamellar structures. We shall be concerned with these latter oscillations and we shall consider the former ones as negligible. We are mostly interested to know at a given moment of time the total length of the projection of the moving particle path in the plane parallel to the two channel layers at an equal distance with respect to these layers, and to know also the distance of the moving particle to this median plane. By characterizing the motion with these two coordinates it is easy to describe it as a two-dimensional motion, and to imagine a two-dimensional model of planar channeling.

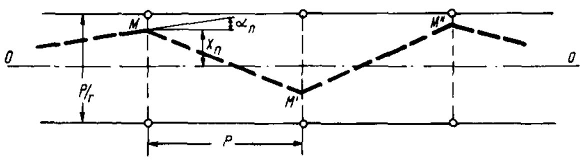

Fig. 1. The two-dimensional channel. M M′ M″ ... denote the particle trajectory.

We suppose that the moving particle interacts with its nearest neighbours only. This interaction varies periodically and we shall approximate it by a periodic succession of average potentials. The sources of these potentials will be considered to be located in the two layers along two parallel lines. We obtain thus a two-dimensional channel shown in fig. 1.

It may happen that for certain planar channeling directions the particle neighbours can be located along lines situated in a planar channel layer. For this case we can use an average line potential which is taken for numerical evaluations as a logarithmic Lindhard potential [1.]

In elastic collisions (within the frame of the momentum approximation the energy loss is a second-order effect which can be neglected) the mechanical state of the moving particle is determined by its position x_n and direction α_n in channel. The basic assumption is that the scattering angles are small, and thus the momentum approximation is valid. This assumption is consistent with our calculations only for repulsive potentials. We obtain the following equations (see fig. 1):

$$\alpha_n = \frac{1}{E}\int_1^\infty \frac{\mathrm{d}u}{(u^2-1)^{1/2}}\frac{\mathrm{d}}{\mathrm{d}u}\left\{V\left[u\left(\frac{P}{2r}-x_n\right)\cos\alpha_{n-1}\right] - V\left[u\left(\frac{P}{2r}+x_n\right)\cos\alpha_{n-1}\right]\right\} + \alpha_{n-1} \qquad (1)$$

$$x_n = x_{n-1} + P \tan \alpha_{n-1}$$

where E is the energy and V the absolute value of the potential. (The collision with the crystal electrons are neglected).

As the scattering angles are assumed to be small, eqs. (1) may be linearized to give:

$$Z_n = A^n Z_0 \qquad \text{where} \qquad Z_n = \begin{pmatrix} P\alpha_n \\ x_n \end{pmatrix} \tag{2}$$

The 2×2 matrix A is

$$A = \begin{pmatrix} 1-4Q & -4Q \\ 1 & 1 \end{pmatrix} \text{ with} \tag{3}$$

$$Q = \frac{r}{E} \int_1^\infty \frac{\mathrm{d}u}{(u^2-1)^{1/2}} \frac{\mathrm{d}}{\mathrm{d}u} \left[u \frac{\mathrm{d}}{\mathrm{d}u} V\left(\frac{P}{2r} u \right) \right]$$

The nth power of the 2×2 matrix A is easily obtained

$$A^n = -\frac{\sin(n-1)\varphi}{\sin \varphi} I + \frac{\sin n\varphi}{\sin \varphi} A \quad \text{with } \tan\varphi = \frac{2(Q-Q^2)^{1/2}}{1-2Q} \tag{4}$$

For $0 < Q < 1$ the trigonometric functions in (4) prove the stability of the trajectory within the channel. For $-\infty < Q \leqslant 0$, $1 \leqslant Q < +\infty$ the trajectory is unstable. Putting $Q = 1$ we obtain the stability condition and, therefore, the critical energy for channeling.

From the condition $\max_n |x_n| = \tau P/2r$, where τ is a factor so that $\tau P/2r$ is the channel halfwidth over which the momentum approximation and the linearization are satisfactory valid, we obtain the critical angle

$$\alpha_c = \frac{\tau}{2r} \sin \varphi = \frac{\tau}{r} (Q-Q^2)^{1/2}. \tag{5}$$

The influence of the temperature is taken into account by averaging the particle motion equations over thermal vibrations of the channel points. It results that instead of Q we must use

$$\langle Q \rangle = \tag{6}$$

$$\left\langle \frac{P}{8E} \int_0^\infty \frac{\mathrm{d}u}{(u^2-1)^{1/2}} \frac{\mathrm{d}}{\mathrm{d}u} \left\{ \frac{u}{P/2r+\xi} \frac{\mathrm{d}}{\mathrm{d}u} V\left[u \left(\frac{P}{2r} + \xi \right) \right] \right\} \right\rangle,$$

where the averaging is taken with respect to the ξ coordinate which is perpendicular to the channel axis. It is easy to see that with the approximation given here, the thermal vibrations which are parallel to the channeling planes do not contribute to the averaging. Since we have taken into account the thermal vibrations for the calculation of the critical angle, the halfwidth of the channel has to be taken as

$$\frac{\tau P}{2r} \left\{ 1 - \left[\langle \xi^2 \rangle / \left(\frac{P}{2r} \right)^2 \right]^{1/2} \right\}.$$

It is important to note that our model is valid for energy larger than the critical energy, the value of which is in the domain of diffraction energies. Another important consequence of the present model is the corrective term Q^2 in eq. (5) to the well known dependence of the critical angle, $\alpha_\tau \sim Q^{1/2} \sim E^{-1/2}$. For large energy this correction is negligible and eq. (5) has the same form as the Lindhard one. It must be pointed out that eq. (5) breaks down for low energies where the linearization of motion equations does not holds.

For numerical evaluations we use the logarithmic Lindhard potential; we take the charge number of crystal ions $Z_2 \approx 10$ and the energies of the light ions $E \approx 1-2$ MeV; then we have $Q \approx 10^{-5}$. At the usual temperatures the stability condition is satisfied and the critical angle obtained from (5) is in good agreement with the experimental values [2]. It decreases as the temperature increases.

Further discussions on this model will be published in a forthcoming paper in Rev. Roum. Phys.

[1] J. Lindhard, Kgl. Danske Videnskab. Selskab. Mat-Fys. Medd. 34 no 14 (1965).
[2] B.R. Appleton, C. Erginsoy and W. Gibson, Phys. Rev. 161 (1967) 330.