Institute for Physics and Nuclear Engineering - Magurele-Bucharest

MOLECULAR FORCES on MACROSCOPIC BODIES

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van der Waals <1900 $1/R^7$

London QM 1930

Casimir conducting plates $1/d^4$ 1948; Bohr: zero-point energy?

Relevance for QED

Macroscopic bodies -polarization, EM field

Lifshitz et co statistical flcs 1954-1964 (vdW-L $1/d^3$)

van Kampen et al zero-point energy, vacuum flcs, EM field

Schwinger propagators 1975

Experimental evidence

Calculations for 2 half-spaces ($1/d^3$, $1/d^4$)

Measurements for sphere-half-space

???

Controversies, Debate, Confusion

What is the cause?

Who is moving in there?

EM field or polarization? How?

Role of Zero-Point Energy - Vacuum Fluctuations

Relation of the vacuum to QM perturbation calculations (?)

Macroscopic bodies: their own polarization modes (“non-additivity” of mol forces)

What are we saying in this Seminar?

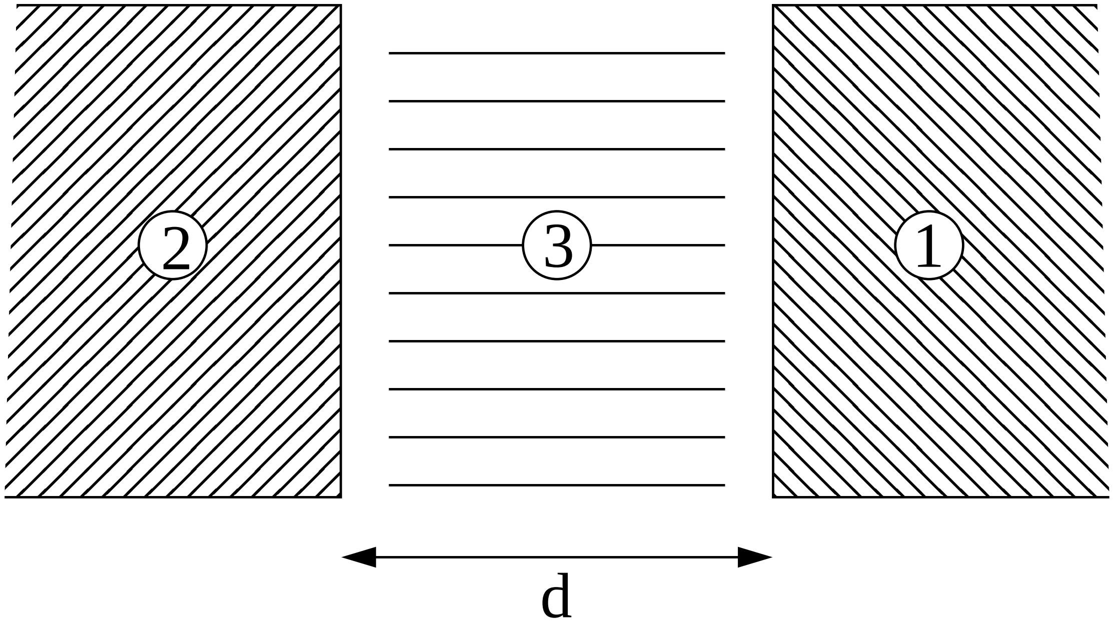
Polarizations, displacement of mobile charges, interacting through EM field

Electromagnetically-coupled bodies

EM field: interaction energy

Normal modes and eigenfrequencies - entangled ensemble

Zero-point energy of vacuum flcs of the polarization of the interacting ensemble


2
3
1
d

Lorentz-Drude equation; dielectric function

$$m\ddot{\mathbf{u}} = q(\mathbf{E} + \mathbf{E}_0) - m\omega_c^2\mathbf{u} - m\gamma\dot{\mathbf{u}}$$

$$\varepsilon(\omega) = \frac{\omega^2 - \omega_c^2 - \omega_p^2}{\omega^2 - \omega_c^2 + i\omega\gamma} = \frac{\omega^2 - \omega_L^2}{\omega^2 - \omega_T^2 + i\omega\gamma}$$

Polarization:$\rho = -div\mathbf{P} = -nqdiv\mathbf{u}$, $\mathbf{j} = \frac{\partial \mathbf{P}}{\partial t} = nq\dot{\mathbf{u}}$ (plasma frequency $\omega_p = \sqrt{4\pi nq^2/m}$)

Kirchoff's potentials

$$\Phi(\mathbf{R},t)=\int d\mathbf{R}'\frac{\rho(\mathbf{R}',t-\left|\mathbf{R}-\mathbf{R}'\right|/c)}{|\mathbf{R}-\mathbf{R}'|}\ ,$$

$$\mathbf{A}(\mathbf{R},t)=\frac{1}{c}\int d\mathbf{R}'\frac{\mathbf{j}(\mathbf{R}',t-\left|\mathbf{R}-\mathbf{R}'\right|/c)}{|\mathbf{R}-\mathbf{R}'|}$$

$div\mathbf{A}+(1/c)\partial\Phi/\partial t=0$; $\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}-grad\Phi$ $(\mathbf{H}=curl\mathbf{A})$

$$\frac{e^{i\lambda\left|\mathbf{R}-\mathbf{R}'\right|}}{|\mathbf{R}-\mathbf{R}'|}=\frac{i}{2\pi}\int d\mathbf{k}\frac{1}{\kappa}e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}')}e^{i\kappa|z-z'|}\ ,\ \kappa=\sqrt{\lambda^2-k^2}\ (\lambda=\omega/c)$$

1

$$E_1 = 2\pi i\kappa \int_d^\infty dz' u_1 e^{i\kappa|z-z'|} - \frac{2\pi k}{\kappa}\frac{\partial}{\partial z}\int_d^\infty dz' u_z e^{i\kappa|z-z'|} \ ,$$

$$E_2 = \frac{2\pi i\lambda^2}{\kappa}\int_d^\infty dz' u_2 e^{i\kappa|z-z'|} \ ,$$

$$E_z = -\frac{2\pi k}{\kappa}\frac{\partial}{\partial z}\int_d^\infty dz' u_1 e^{i\kappa|z-z'|} + \frac{2\pi i k^2}{\kappa}\int_d^\infty dz' u_z e^{i\kappa|z-z'|} - 4\pi u_z \theta(z-d) \ .$$

$$u_{1,2} = A_{1,2} e^{i\kappa' z},\ \kappa' = \sqrt{\varepsilon\lambda^2 - k^2}\ (\omega_p = 4\pi n q^2/m)$$

$$\frac{1}{2} A_1 \omega_p^2 \frac{\kappa\kappa' + k^2}{\kappa'(\kappa' - \kappa)} e^{i(\kappa' - \kappa)d} e^{i\kappa z} = \frac{q}{m} E_{01}\ ,$$

$$\frac{1}{2} A_2 \omega_p^2 \frac{\lambda^2}{\kappa(\kappa' - \kappa)} e^{i(\kappa' - \kappa)d} e^{i\kappa z} = \frac{q}{m} E_{02}\ .$$

$$E_1 = -2\pi n q A_1 \frac{\kappa\kappa' - k^2}{\kappa'(\kappa' + \kappa)} e^{i(\kappa' + \kappa)d} e^{-i\kappa z}\ ,\ z < d\ ,$$

$$E_2 = -2\pi n q A_2 \frac{\lambda^2}{\kappa(\kappa' + \kappa)} e^{i(\kappa' + \kappa)d} e^{-i\kappa z}\ ,\ z < d$$

2

$$v_{1,2} = B_{1,2} e^{-i\kappa' z}$$

$$E_1 = -2\pi n q B_1 \frac{\kappa\kappa' - k^2}{\kappa'(\kappa' + \kappa)} e^{i(\kappa'+\kappa)d} e^{i\kappa z} \ , \ z > -d \ ,$$

$$E_2 = -2\pi n q B_2 \frac{\lambda^2}{\kappa(\kappa' + \kappa)} e^{i(\kappa'+\kappa)d} e^{i\kappa z} \ , \ z > -d$$

$$\frac{1}{2} B_1 \omega_p^2 \frac{\kappa\kappa' + k^2}{\kappa'(\kappa' - \kappa)} e^{i(\kappa'-\kappa)d} e^{-i\kappa z} = \frac{q}{m} E_{01}$$

$$\frac{1}{2} B_2 \omega_p^2 \frac{\lambda^2}{\kappa(\kappa' - \kappa)} e^{i(\kappa'-\kappa)d} e^{-i\kappa z} = \frac{q}{m} E_{02}$$

$$\frac{1}{2}A_1\omega_{p1}^2\frac{\kappa\kappa_1'+k^2}{\kappa_1'(\kappa_1'-\kappa)}e^{i(\kappa_1'-\kappa)d}=-\frac{1}{2}B_1\omega_{p2}^2\frac{\kappa\kappa_2'-k^2}{\kappa_2'(\kappa_2'+\kappa)}e^{i(\kappa_2'+\kappa)d}\ ,$$

$$\frac{1}{2}A_2\omega_{p1}^2\frac{\lambda^2}{\kappa(\kappa_1'-\kappa)}e^{i(\kappa_1'-\kappa)d}=-B_2\omega_{p2}^2\frac{\lambda^2}{\kappa(\kappa_2'+\kappa)}e^{i(\kappa_2'+\kappa)d}$$

$$\frac{1}{2}B_1\omega_{p2}^2\frac{\kappa\kappa_2'+k^2}{\kappa_2'(\kappa_2'-\kappa)}e^{i(\kappa_2'-\kappa)d}=-\frac{1}{2}A_1\omega_{p1}^2\frac{\kappa\kappa_1'-k^2}{\kappa_1'(\kappa_1'+\kappa)}e^{i(\kappa_1'+\kappa)d}$$

$$\frac{1}{2}B_2\omega_{p2}^2\frac{\lambda^2}{\kappa(\kappa_2'-\kappa)}e^{i(\kappa_2'-\kappa)d}=-\frac{1}{2}A_2\omega_{p1}^2\frac{\lambda^2}{\kappa(\kappa_1'+\kappa)}e^{i(\kappa_1'+\kappa)d}$$

Dispersion Equations

Coupled equations $A_{1,2} \sim B_{1,2}$

$$\frac{\kappa_1' - \kappa}{\kappa_1' + \kappa} \cdot \frac{\kappa_2' - \kappa}{\kappa_2' + \kappa} e^{2i\kappa d} = 1 \quad ,$$

$$\frac{\kappa_1' - \kappa}{\kappa_1' + \kappa} \cdot \frac{\kappa_2' - \kappa}{\kappa_2' + \kappa} \cdot \frac{\kappa\kappa_1' - k^2}{\kappa\kappa_1' + k^2} \cdot \frac{\kappa\kappa_2' - k^2}{\kappa\kappa_2' + k^2} e^{2i\kappa d} = 1 \ .$$

$$\frac{\kappa_1' - \varepsilon_1 \kappa}{\kappa_1' + \varepsilon_1 \kappa} \cdot \frac{\kappa_2' - \varepsilon_2 \kappa}{\kappa_2' + \varepsilon_2 \kappa} e^{2i\kappa d} = 1 \ .$$

Solution: No solution!!! (in general)

Particular cases: conductors, good dielectrics ($\left|\kappa'_{1,2}\right| \gg \kappa_{1,2}$, $\left|\varepsilon_{1,2}\right| \kappa_{1,2}$); damped modes (surface plasmon-polaritons):

$$\kappa d = \pi n \ , \ \Omega_n(k) = c\sqrt{k^2 + \frac{\pi^2 n^2}{d^2}}$$

Eigenfrequencies, normal modes, $A_{1,2,n} = 2\pi a_{1,2,n}\delta(\omega - \Omega_n(k))$; labelled by in-between standing waves

$u_{1,2,n} = A_{1,2,n} \Longrightarrow u_{1,2,n} \sim e^{-i\Omega_n t}$, $\ddot{u}_{1,2,n} + \Omega_n^2 u_{1,2,n} = 0$: harmonic oscillators

Casimir force

Retarded regime, propagating in-between (κ real), $d \gg c/\omega_p$

Zero-point energy $E = \sum_{n\mathbf{k}} \frac{1}{2}\hbar\Omega_n(k) = \frac{S\hbar c}{2\pi}\sum_{n=0}\int_0 dk \cdot k\sqrt{k^2 + \frac{\pi^2 n^2}{d^2}}$

Euler-MacLaurin formula $\Delta E = \sum_{m=1} \frac{(-1)^m B_m (\pi/d)^{2m-1}}{(2m)!} f^{(2m-1)}(0)$

$$F = -\frac{\pi^2 \hbar c S}{240} \cdot \frac{1}{d^4}$$

Universal character (quantum limit)

van der Waals-London force

Non-retarded regime, $d \ll c/\omega_p$ ($\kappa = ik$, $\lambda = 0$); another solution

$$(\omega^2 - \omega_{c1}^2 - \frac{1}{2}\omega_{p1}^2)(\omega^2 - \omega_{c2}^2 - \frac{1}{2}\omega_{p2}^2) = \frac{1}{4}\omega_{p1}^2\omega_{p2}^2 e^{-2kd}$$

$$F = -\frac{\hbar\omega_{p1}\omega_{p2}}{16\pi\sqrt{2}C_1C_2(\omega_{p1}C_1 + \omega_{p2}C_2)} \cdot \frac{1}{d^3}\ , \ \ F = -\frac{\hbar\omega_p}{32\pi\sqrt{2}}\left(\frac{\varepsilon_0 - 1}{\varepsilon_0 + 1}\right)^{3/2} \cdot \frac{1}{d^3}$$

$$C_{1,2} = \sqrt{\frac{\varepsilon_{01,2} + 1}{\varepsilon_{01,2} - 1}}$$

A third body in-between

$$v = c\frac{\omega_{c3}}{\sqrt{\omega_{p3}^2 + \omega_{c3}^2}} = \frac{c}{\sqrt{\varepsilon_{30}}}$$

$$F = -\frac{\pi^2 \hbar v S}{240} \cdot \frac{1}{d^4}$$

(Metal: vanishing force)

vdW-L for two identical metals separated by another, distinct metal

$$F = -\frac{\hbar}{32\pi\sqrt{2}} \frac{\omega_p^2 - \omega_{p3}^2}{(\omega_p^2 + \omega_{p3}^2)^{3/2}} \cdot \frac{1}{d^3}$$

Formal dangerous equivalences (Lifshitz et co, Schwinger) (van Kampen)

$$E = \frac{1}{2}\hbar \sum_{n\mathbf{k}} \Omega_n(k) = \frac{\hbar}{4\pi i} \sum_{n\mathbf{k}} \int d\omega \frac{\omega}{\omega - \Omega_n(k)} =$$

$$= \frac{\hbar}{2i} \int dkk \int d\omega\omega \frac{\partial}{\partial\omega} \ln G \ , \ G(\Omega) = 0$$

$$F = -\frac{\hbar}{2\pi^2 c^3} \int_1^\infty dpp^2 \int_0^\infty d\xi\xi^3 \left(\frac{1}{G_1} + \frac{1}{G_2} \right)$$

$$G_1 = \frac{\kappa_1' - \kappa}{\kappa_1' + \kappa} \cdot \frac{\kappa_2' - \kappa}{\kappa_2' + \kappa} e^{2i\kappa d} - 1 (= 0), \ G_2 = \frac{\kappa_1' - \varepsilon_1\kappa}{\kappa_1' + \varepsilon_1\kappa} \cdot \frac{\kappa_2' - \varepsilon_2\kappa}{\kappa_2' + \varepsilon_2\kappa} e^{2i\kappa d} - 1 (= 0)$$

Sphere-1/2

$$F_{1/2} = \frac{CS}{d^n} \;,\; \int dV f = F_{1/2} \;,\; f = \frac{Cn}{d^{n+1}}$$

$$F_s = \int df = Cn \int dV \frac{1}{(R + d - r\cos\theta)^{n+1}} \simeq \frac{2\pi CR}{(n-1)d^{n-1}}$$

Shell:

$$\frac{2\pi CR^2}{d^n}$$

Concluding Remarks, Discussions

Coupled polarizations via EM field: much alike coupled oscillators

Origin of multiplicity n: in-beteeen wavevector κ

Field&Stat Fields authors: P external source for response theory *vs* P included in the EM (via ε) which is a dyn variable!!

$$H_m = q\Phi + \frac{1}{2m}(\mathbf{p} - \frac{q}{c}\mathbf{A})^2$$

for matter interacting with the elctromagnetic field in bodies 1 and 2

$$E_{em} = \frac{1}{8\pi}\int d\mathbf{R}(|\mathbf{E}|^2 + |\mathbf{H}|^2)$$

for $1, 2, 3$

$$E_{em} \to H_{em} = \frac{1}{2}\sum_r (|p_r|^2 + \omega_r^2\,|q_r|^2)\ ,\ \mathbf{A} = \sum_r q_r(t)\mathbf{A}_r(\mathbf{R})$$

No coupling between $1, 2, 3$!!

Continuity conditions: $\equiv$remove the internal interaction (through ε)

Inconsistency: preserving the “interaction” $\mathbf{j}(\dot{\mathbf{u}})\mathbf{A} - q\Phi$ (incorrectly used there as for ext sources)

Relation to QM?

Momentum conservation $f_i + \frac{\partial}{\partial t} G_i = \partial_j \sigma_{ij}$, $\mathbf{f}$ - the Lorentz, $\mathbf{G} = \mathbf{S}/c^2$ - the electromagnetic momentum $\sigma_{ij} = \frac{1}{8\pi} \left[E_i^* E_j + H_i^* H_j - \frac{1}{2} \delta_{ij} (|\mathbf{E}|^2 + |\mathbf{H}|^2) \right]$

Surface stress tensor? - zero-point gives The Force; no Lorentz (plus photons!)