FIELD INDUCED SUPERCONDUCTING TRANSISTOR

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December 2007

FIST Summary

FIST - "Field-Induced Superconducting Transistor”

FCST - "Field-Controlled Superconducting Transistor"

"Field" means the magnetization field of a ferromagnetic sample

-ferromagnet-superconductor junction

-Miniaturization; -High resistance; -Potential barriers, tunneling, point contacts, micro-bridges, etc

-Superconductor as a natural tunneling barrier, Andreev reflection

$$R_s = R_n \sqrt{2\Delta/\pi T} e^{\Delta/T} \ , \ T/\Delta \ll 1$$

(ballistic transport $R_s = R_b \cdot eU/\sqrt{e^2U^2 - \Delta^2}$, typical in classical tunneling just above the gap barrier; Giaever)

What is the Andreev Reflection?

$\mathbf{k}, \alpha$-excitation as a $\mathbf{k}, \alpha$-quasi-particle, moving with velocity $\mathbf{v}$

$\mathbf{k}, \alpha$-excitation as a $-\mathbf{k}, -\alpha$-hole in a superconducting pair, moving backwards in time, therefore with velocity $-\mathbf{v}$

-A reduction factor

$$\mathbf{v}(|\varphi|^2 - |\chi|^2) \sim \mathbf{v}\frac{\sqrt{\hbar^2\omega^2 - \Delta^2}}{\hbar\omega}$$

in the current, origin of high resistance

Question: Can the flow be controlled by magnetization? by a spin-polarization?

The answer is No for a diffusive transport in the ferromagnetic sample, because the conductivity

$$\sim k_{F1}^2 \Lambda_1 + k_{F2}^2 \Lambda_2 \sim (1+m)^{2/3}(1+m)^{1/3} + +(1-m)^{2/3}(1-m)^{1/3} \sim$$

$$\sim 1 + m + 1 - m = 2$$

reduced magnetization $m = M/N\mu_B$

However, in the ballistic regime of transport for the ferromagnetic sample the flow can be controlled by m

Ferromagnetic resistance

$l_f < \Lambda$, $m_t = 1 - (l_f/\Lambda)^3$

$$R_f = R\frac{2}{(1+m)^{2/3}+(1-m)^{2/3}}\ ,\ m < m_t$$

$$R_f = R\frac{2}{(1+m)^{2/3}+\frac{4}{3}\frac{1-m}{(1-m_t)^{1/3}}}\ ,\ m > m_t$$

$\Lambda < l_f < 2^{1/3}\Lambda$, $m_t = (l_f/\Lambda)^3 - 1$

$$R_f = \frac{3}{4}R(1+m_t)^{1/3}\ ,\ m < m_t$$

$$R_f = \frac{3}{4}R\frac{2(1+m_t)^{1/3}}{1-m+\frac{3}{4}(1+m_t)^{1/3}(1+m)^{2/3}}\ ,\ m > m_t$$

-negative jump (negative resistance), positive jump; monotonous increase, etc

-control m by slight change in temperature, just below the magnetic critical temperature, but much below the superconducting critical temperature

-the quasi-particles in the ferromagnetic sample behave like two spin-up, spin-down fluids, with (Fermi) velocities $v_{1,2} = v(1 \pm m)^{1/3}$ and density of states $\sim k_{F1,2}^2 = k_F^2(1 \pm m)^{2/3}$

-crossover from ballistic to diffusive regime, and viceversa, hence the m-dependence and the origin of the jumps

Under what conditions?

For a perfect contact at the junction, as for similar solids, so that the matching conditions be fulfilled (close μ, k_F); problems for $m \to 1$ with the low density-of-states spin-down fluid

Quasi-particle wavefunctions (solutions of Gorkov equations)

$$\sim e^{i\mu t/\hbar} \cdot e^{i\omega t} \cdot e^{-i\mathbf{k}_F \mathbf{r}} \cdot e^{-i\mathbf{k}\mathbf{r}}$$

small ω, small $\mathbf{k}$

-hence, μ- and k_F-values close to each other, respectively (for ferromagnet and superconductor), for continuity (wavefunction and its 1st-order derivative)

-$\hbar\omega = \sqrt{\Delta^2 + \hbar^2 v^2 k^2}$, small vk (superconductor)

-$\omega = v_{1,2}k = v(1 \pm m)k$ (ferromagnet), hence large k for $m \to 1$ for the spin-down fluid, which violates the continuity conditions; however, small contribution to the junction resistance

Conclusion

-the need for a perfect contact for matching conditions at the junction (as for similar solids)

-an extended contact might do in this respect, but it could be difficult to realize a ballistic regime of transport in the ferromagnetic sample in this case

-a possible additional layer at the junction, like an oxide layer, to stabilize the tendency towards an extended contact, would act as a potential barrier; it satisfies the matching conditions and brings its own contribution to the junction resistance through the transmission coefficient

Field Induced SuperconductingTransistor (FIST)

-Miniaturization

-Tunneling barriers, inversion layers, bridges, point-contacts, etc

-Superconducting gap as a potential barrier (Andreev reflection)

-Spin correlations in superconductor→Ferromagnet-Superconductor Junction

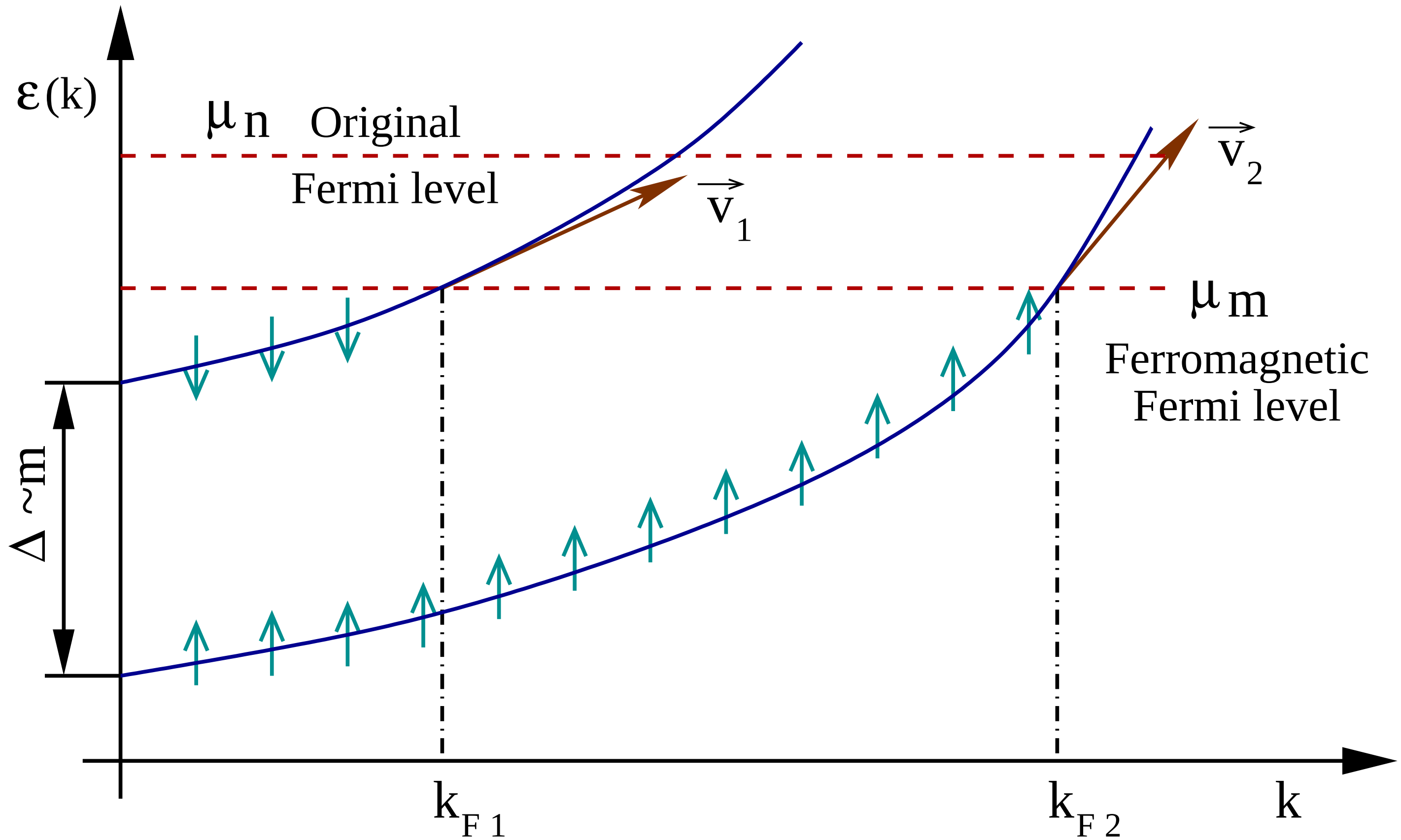


Fig 1. Spectrum of Ferromagnetic Quasi-Particles

Ferromagnet

Two spin fluids of quasi-particles

$$v_{1,2} = v(1 \pm m)^{1/3}$$

Density of states

$$\sim k_{F1,2}^2 = k_F^2 (1 \pm m)^{2/3}$$

Magnetic gap

$$\Delta_m \simeq \frac{2}{3} v k_F m$$

Reduced magnetization $m = M/\mu_B N$

Fermi level

$$\mu_m \simeq -\Delta_m/2 + \mu + v k_F m/3 \simeq \Delta_m/2 + \mu - v k_F m/3 \simeq \mu$$

Superconductor

Spin-singlet, s-wave

Gorkov equations, quasi-particles $\mathbf{k} \sim \mathbf{k}_F$

$$i\hbar \partial \psi_\alpha / \partial t = (-\hbar \mathbf{v} \mathbf{k}_F - i\hbar \mathbf{v} \partial / \partial \mathbf{r}) \psi_\alpha + i \Delta_\alpha \psi^+_{-\alpha}$$

$$-i\hbar \partial \psi^+_{-\alpha} / \partial t = (-\hbar \mathbf{v} \mathbf{k}_F - i\hbar \mathbf{v} \partial / \partial \mathbf{r}) \psi^+_{-\alpha} + i \Delta_\alpha \psi_\alpha$$

Superconducting spectrum ($\Delta_{-\alpha} = -\Delta_\alpha$)

$$\varepsilon = \mu \pm \sqrt{\Delta^2 + \hbar^2 v^2 (k - k_F)^2}$$

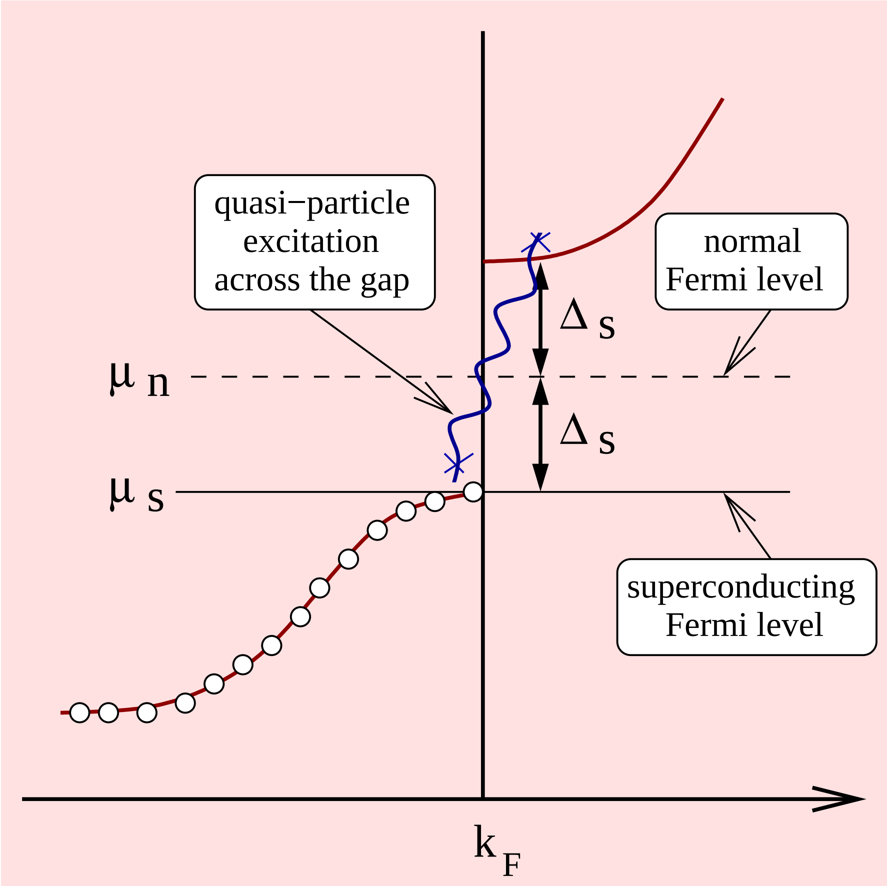


Fig. 2 Superconducting Quasi-Partciles Spectrum

Fermi level

$$\mu_s = \mu - \Delta/2 \simeq \mu$$

Excitation energy $\hbar\omega = \sqrt{\Delta^2 + \hbar^2 v^2 (k - k_F)^2}$

Andreev Reflection

$\mathbf{k}, \alpha$-excitation as a quasi-particle

$$\varphi_\alpha = \langle 0 \left| \psi_\alpha \right| \mathbf{k}\alpha \rangle \quad ,$$

as a $-\mathbf{k}, -\alpha$-quasi-hole

$$\chi_\alpha = \left\langle 0 \left| \psi^+_{-\alpha} \right| \mathbf{k}\alpha \right\rangle$$

in a superconducting pair

φ_α moves with velocity $\mathbf{v}$, χ_α moves with velocity $-\mathbf{v}$ (backwards in time)=Andreev reflection

$$i\hbar\partial\varphi_\alpha/\partial t = (-\hbar\mathbf{v}\mathbf{k}_F - i\hbar\mathbf{v}\partial/\partial\mathbf{r})\varphi_\alpha + i\Delta\chi_\alpha$$

$$-i\hbar\partial\chi_\alpha/\partial t = (-\hbar\mathbf{v}\mathbf{k}_F - i\hbar\mathbf{v}\partial/\partial\mathbf{r})\chi_\alpha + i\Delta\varphi_\alpha$$

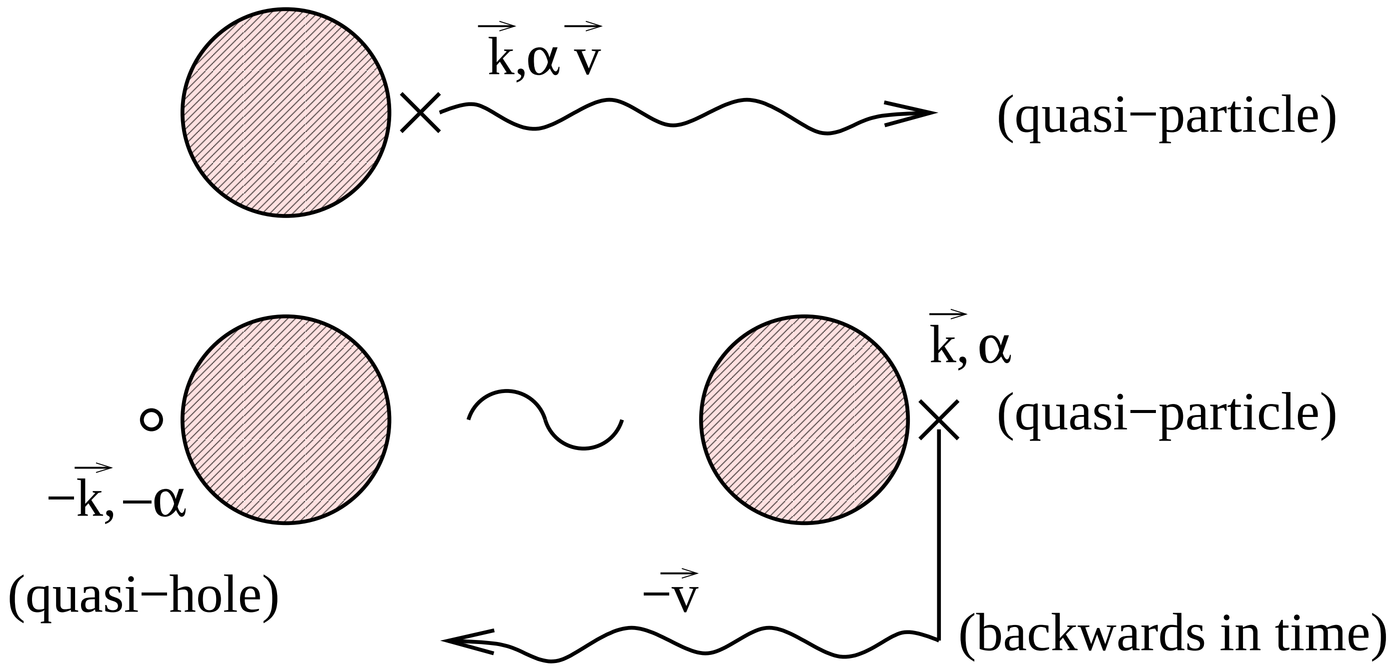
Probability $\sum|\varphi_\alpha|^2 + |\chi_\alpha|^2$

Current $\sum\mathbf{v}(|\varphi_\alpha|^2 - |\chi_\alpha|^2)$

μ-reduced, k_F-reduced (exp$(-i\mu t/\hbar + i\mathbf{k}_F\mathbf{r})$)exp$(-i\omega t)$ exp$(i\mathbf{k}\mathbf{r})$

$$(\omega - \hbar\mathbf{v}\mathbf{k})\varphi_\alpha = i\Delta\chi_\alpha$$

$$(\omega + \hbar\mathbf{v}\mathbf{k})\chi_\alpha = -i\Delta\varphi_\alpha$$



Andreev reflection

Solutions

$$\varphi_\alpha = \frac{C_\alpha}{\sqrt{2}}\sqrt{1 + \mathbf{vk}/\omega}e^{i\mathbf{kr}} \ , \ \chi_\alpha = \frac{-iC_\alpha}{\sqrt{2}}\sqrt{1 - \mathbf{vk}/\omega}e^{i\mathbf{kr}}$$

where

$$\mathbf{vk} = \sqrt{\omega^2 - \Delta^2/\hbar^2} \ , \ \hbar\omega > \Delta$$

and small $\mathbf{k}$; Current

$$\mathbf{j}_\alpha = |C_\alpha|^2 \, \mathbf{v}(\mathbf{vk}/\omega)$$

Reduction factor

$$\mathbf{vk}/\omega \simeq \sqrt{2}\sqrt{\frac{\hbar\omega - \Delta}{\Delta}}$$

(Andreev reduction in transmission, potential barrier)

Asymptotic Boundary, Matching Solutions

$\Delta \to 0$, $\chi_\alpha \to 0$: $\varphi_\alpha \to C_\alpha e^{i\mathbf{kr}}$; quasi-particle wavefunction in the non-superconducting conductor

Warning: Reduction conditions (continuity of the 1st-order derivative of the wavefunctions)

-nearly equal Fermi levels μ; -nearly equal Fermi wavevectors

$$k_{F1} = k_F(1+m)^{1/3} \sim k_F \ , \ k_{F2} = k_F(1-m)^{1/3} \sim k_F$$

$\Rightarrow$problems for high magnetization $m \sim 1$, very short lifetime at the junction, small contribution to resistance for spin-down quasi-particles

-$\hbar\omega = \sqrt{\Delta^2 + \hbar^2 v^2 k^2}$, small k (superconductor)
-$\omega = v_{1,2} k = v(1 \pm m)^{1/3} k$ (ferromagnet), problems for $m \to 1$

-Matching conditions fulfilled, transmission coefficient

$$w = \frac{v(|C_1|^2 + |C_2|^2)}{v(1+m)^{1/3}|C_1|^2 + v(1-m)^{1/3}|C_2|^2} (\mathbf{vk}/\omega) =$$

$$= \frac{2}{(1+m)^{1/3} + (1-m)^{1/3}} \cdot \sqrt{2}\sqrt{\frac{\hbar\omega - \Delta}{\Delta}}$$

Note: m-dependence

Note: spin-balanced population in superconductor ($|C_1|^2 = |C_2|^2$)(not to destroy the superconductivity)

Ferromagnet-Superconductor Junction

-Cohesion of the solids, effective charge z^*

-Bottom of the band (s) $-\varphi$, $\varphi \simeq 4\pi e z^*/q^2 a^3 \sim z^*/a$ (work function)

-Fermi energy μ (Fermi level $-\varphi + \mu$)

Two solids in contact

-potential barrier, width a, height $e^2 z^* \cdot \Delta z^*/a$

-transmission coefficient for atoms

$$T^2 = \frac{4}{4 + (M/m) z^* \Delta z^* (a/a_H)}$$

Free surface

Self-consistent potential

$$\varphi = \sum_i \frac{z_i^*}{|\mathbf{r} - \mathbf{R}_i|} e^{-q|\mathbf{r}-\mathbf{R}_i|}$$

-average $\varphi = 4\pi z^*/a^3q^2$, $aq \sim 2.73$, $q \simeq 0.77{z^*}^{1/3}$

-free surface

$$\varphi = \frac{4\pi z^*}{a^3q^2}(1 - \frac{1}{2}e^{qx}) \ , \ x < 0$$

$$\varphi = \frac{2\pi z^*}{a^3q^2}e^{-qx}) \ , \ x > 0$$

-a change $\delta\varphi$ in the potential, spill-over of the electrons, charge double layer at the surface

$\delta n = q^2 \delta\varphi / 4\pi$, compensating dipole field, work function

$$W = -\varphi(+\infty) + \varphi(-\infty) = -\int dx \cdot \partial\delta\varphi/\partial x = \varphi$$

-surface energy (per unit area)

$$\delta E = -\frac{1}{2}\int dx \cdot \delta\varphi\delta n = -\frac{\pi z^{*2}}{2a^6 q^3}$$

-Additional lifetime:

$$\delta\varepsilon = \pi n^2/2q^3 \cdot A/nAd =$$

$$\pi n/2q^3 d \sim \tfrac{a}{d}\mu$$

per electron,

$$\hbar/\tau = \frac{a}{d}\mu$$

Casimir boundary-scattering (finite-size) time, d linear, finite, dimension of the sample

-Narrow microstructures, dominated by boundary scattering, same m-dependence of the FIST effect

Extended Contact

$$\varphi = \varphi_1 + \frac{1}{2}\Delta\varphi e^{x/\Lambda_c} \ , \ x < 0$$

$$\varphi = \varphi_2 - \frac{1}{2}\Delta\varphi e^{-x/\Lambda_c} \ , \ x > 0$$

Lifetimes

$\hbar/\tau \simeq (\delta\varepsilon)^2/\mu \ , \ T^2/\mu$ (el-el scattering)

$\hbar/\tau \simeq T/F \ , \ F = (M/m)(\hbar\omega_D/\mu)^2$ (el-ph scattering)

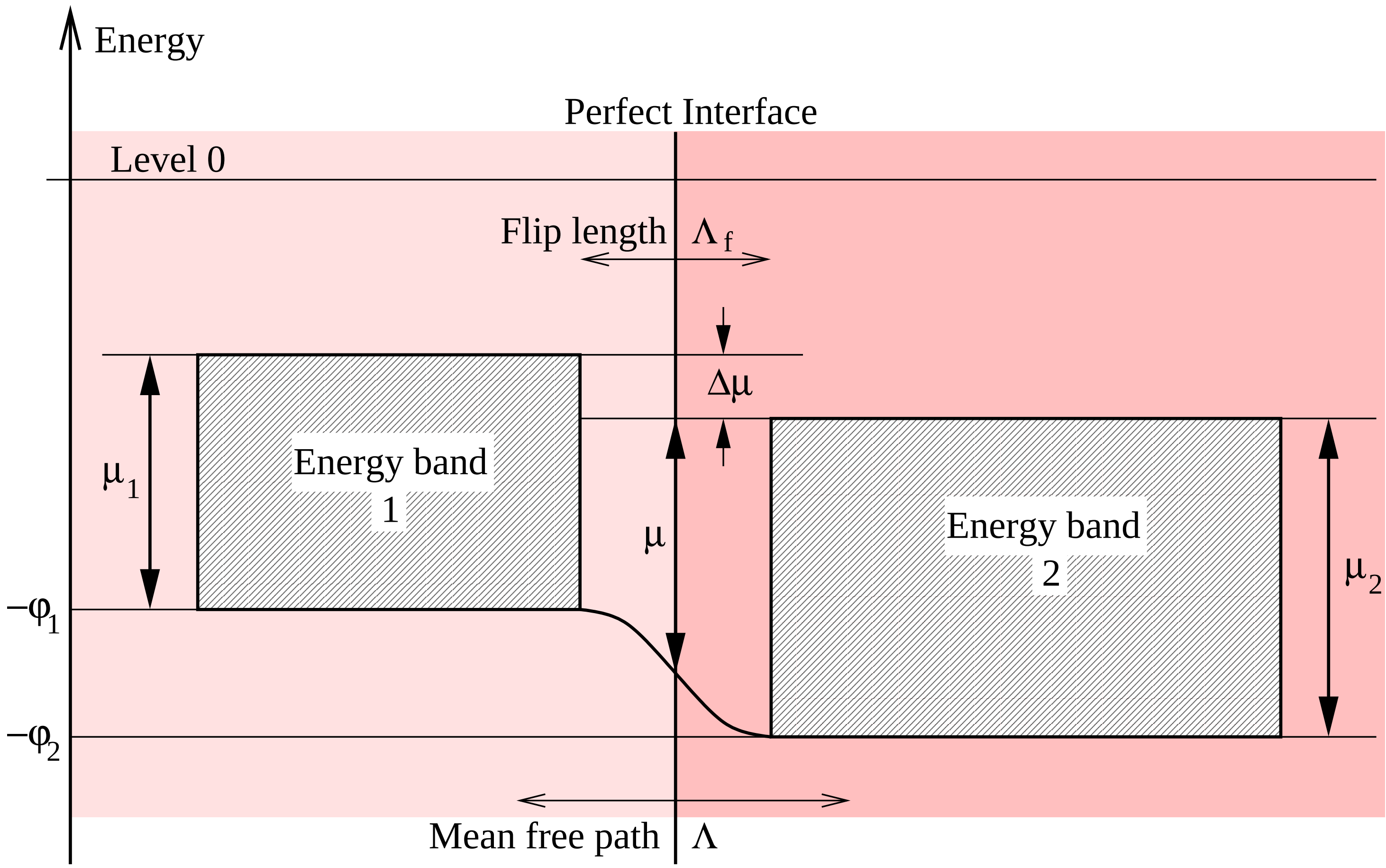


Fig. 4 Two Solids with a Perfect Contact

-distance covered by an atom

$$\Lambda_c \simeq a\frac{M}{m}z^*\Delta z^*(a/a_H)$$

-diffusion (ext fields, temperature for getting the solids into "atomic contact")

Very dissimilar solids, large Δz^*

-extended contacts (if not growth-limited)

-slow spatial variations along such a contact

-matching conditions fulfilled, but

-a "third solid" in-between, with its own contribution to resistance

Similar solids, small Δz^*; typical values $z^* \sim 10^{-1}$, $\Delta z^* \sim 10^{-2}$

-typically, $\Lambda_c \sim 100 - 1000A$

-compared to quasi-particle mean-free path $\Lambda \sim 10^3 - 10^4 A$ at room temperature

-we call this a perfect contact

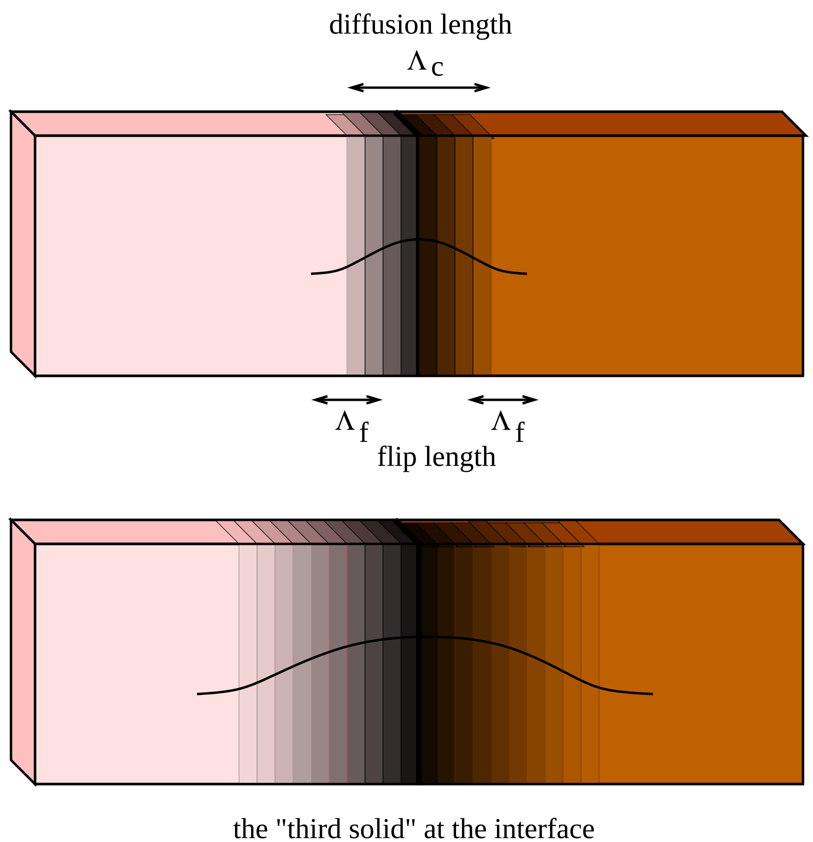


Fig. 5 Two Solids in Contact

Kapitza Resistance

Similar solids, $\Delta\mu$ uncertainty in quasi-particle energy

-lifetime $\hbar/\tau \sim T^2/\mu$, or $\sim T/(M/m)(\hbar\omega_D/\mu)^2$ undergoes a change

$$\Delta(\hbar/\tau) = \frac{\hbar}{\tau}(\Delta\mu/\mu)^2$$

-hence additional (perfect) contact Kapitza resistance $\Delta R/R = (\Delta\mu/\mu)^2$, etc

Spin-Flip, Damping Gap, etc

-Spin flip length $\Lambda_f \simeq (\Delta\mu/\mu)^2\Lambda$ at the junction, fraction of mean-free path, typically 10^{-2}, comparable with Λ_c

-Similarly for vanishing the gap at the junction, etc

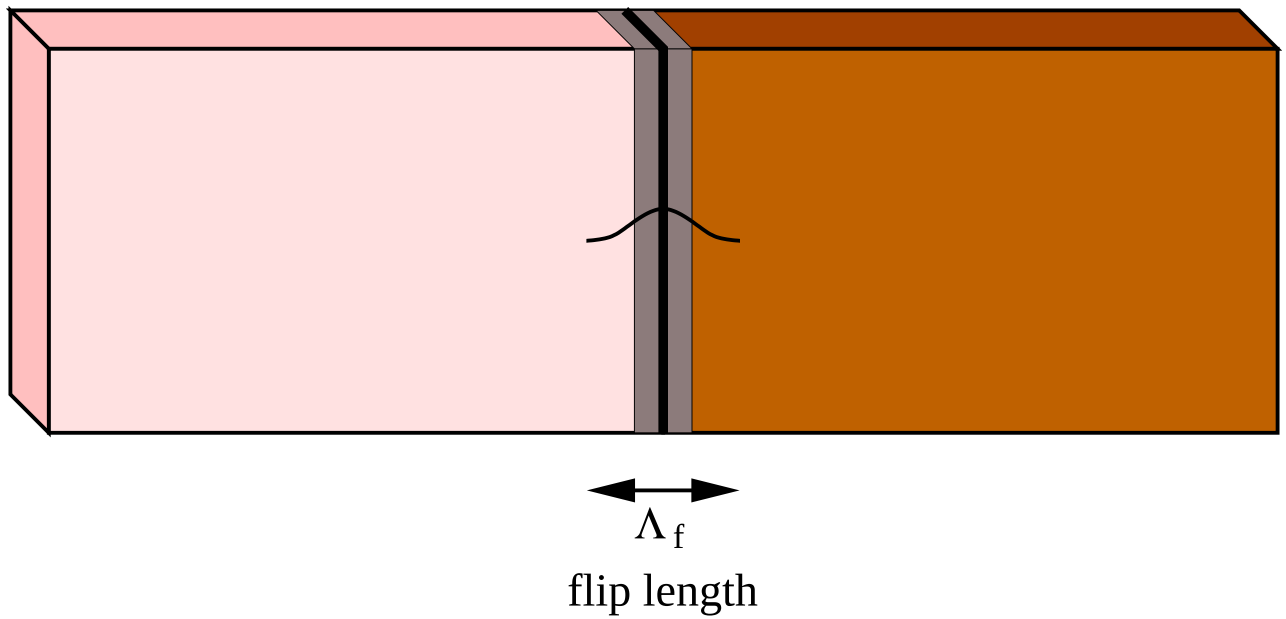


Fig. 6 A Perfect Contact

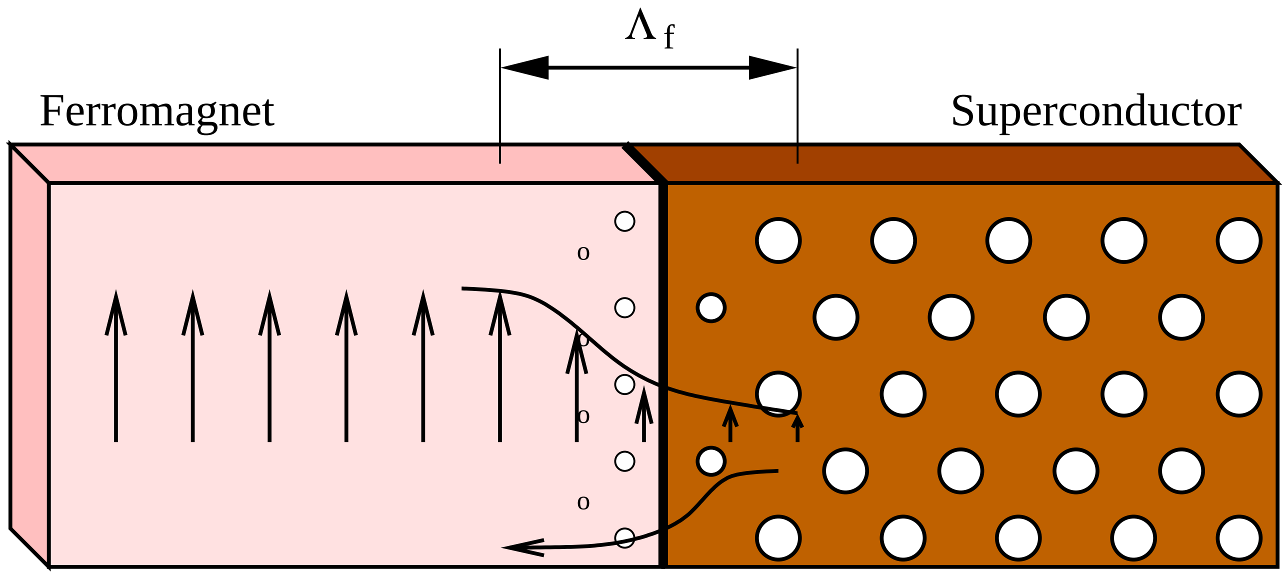


Fig. 7 Spin-Flip and Gap Damping at the Interface

Electric conductivity

Diffusive regime, the flux of charge

$$-e(\partial n/\partial \varepsilon) \cdot (-eU) \cdot v_x \tau$$

-the flow

$$j = \frac{2 \cdot 2\pi \cdot e^2}{(2\pi\hbar)^3} p_F^2 \int du \cdot \frac{d\varepsilon}{v} \cdot \frac{\partial n}{\partial \varepsilon} v_x^2 \tau \cdot (\partial U/\partial x)$$

-conductivity $j = \sigma(-\partial U/\partial x)$

$$\sigma = \frac{e^2 k_F^2}{3\pi^2 \hbar} \Lambda \ , or \ j = \frac{e^2 k_F^2}{3\pi^2 \hbar} \cdot \frac{\Lambda}{l} U$$

hence the resistance (per unit area)

-note the high increase of the resistance due to the ratio l/Λ

-note the independence of magnetization of a ferromagnetic sample resistance

$$k_{F1}^2 \Lambda_1 = k_F^2 \Lambda (1+m)^{2/3} (1+m)^{1/3} == k_F^2 \Lambda (1+m)$$

$$k_{F2}^2 \Lambda_2 = k_F^2 \Lambda (1-m)$$

$$k_{F1}^2 \Lambda_1 + k_{F2}^2 \Lambda_2 = k_F^2 \Lambda$$

Ballistic regime: -$\Lambda \sim l$, $2/3 \to 1/2$ (angle integration)

$$j = \frac{e^2 k_F^2}{4\pi^2 \hbar} \cdot U$$

-low resistance $R = 4\pi^2\hbar/e^2 k_F^2$, quanta e^2/h of conductivity, dependence on m

Resistance of a Superconductor

Diffusive regime, charge flux

$$-e(\partial n/\partial\varepsilon)\cdot(-eU)\cdot(|\varphi|^2-|\chi|^2)\cdot v_x\tau$$

charge flow

$$j=-\frac{2\cdot 2\pi\cdot e^2}{(2\pi\hbar)^3}p_F^2\int du\cdot\int_\Delta\frac{d\varepsilon}{v}\cdot\frac{1}{T}e^{-\varepsilon/T}\cdot\cdot\sqrt{2}\sqrt{\frac{\varepsilon-\Delta}{\Delta}}v_x^2\tau\cdot(\partial U/\partial x)$$

conductivity

$$j=\frac{e^2k_F^2}{3\pi^2\hbar}\cdot\frac{\Lambda}{l}\cdot\sqrt{\pi T/2\Delta}\cdot e^{-\Delta/T}\cdot U$$

or

$$R_s=R_{normal}\cdot\sqrt{2\Delta/\pi T}\cdot e^{\Delta/T}$$

-note high values of R_s for $T/\Delta \ll 1$ due to Andreev reflection

Similarly, in the ballistic regime

$$R_s = R\frac{eU}{\sqrt{e^2U^2 - \Delta^2}}$$

typical for classical tunneling currents

-However, it is preferable the diffusive regime (except for eU just above Δ the tunneling resistance is low; supercond may be destroyed by spin-polarized ballistic currents)

Junction

$$j = \frac{1}{R_f}(U - U_0)$$

$$j = \frac{1}{R_s}U_0$$

-Ballistic regime for the ferromagnetic sample (perfect contact)

-Suficiently low temperature and thin sample $l_f < \Lambda$

-$\Lambda_1 = \Lambda(1+m)^{1/3}$, ballistic regime

-$\Lambda_2 = \Lambda(1-m)^{1/3}$, crossover

-Threshold magnetization $m_t = 1 - (l_f/\Lambda)^3$

$$R_f = R\frac{2}{(1+m)^{2/3}+(1-m)^{2/3}}\ ,\ m < m_t$$

$$R_f = R\frac{2}{(1+m)^{2/3}+\frac{4}{3}\frac{1-m}{(1-m_t)^{1/3}}}\ ,\ m > m_t$$

-Monotonous increase, negative jump (negative resistance)

Another case $\Lambda < l_f < 2^{1/3}\Lambda$, $m_t = (l_f/\Lambda)^3 - 1$

$$R_f = \frac{3}{4}R(1+m_t)^{1/3}\ ,\ m < m_t$$

$$R_f = \frac{3}{4}R\frac{2(1+m_t)^{1/3}}{1-m+\frac{3}{4}(1+m_t)^{1/3}(1+m)^{2/3}}\ ,\ m > m_t$$

-Monotonous increase, positive jump

-Change of m by changing temperature just below the magnetic critical temperature, and well below the superconducting critical temperature

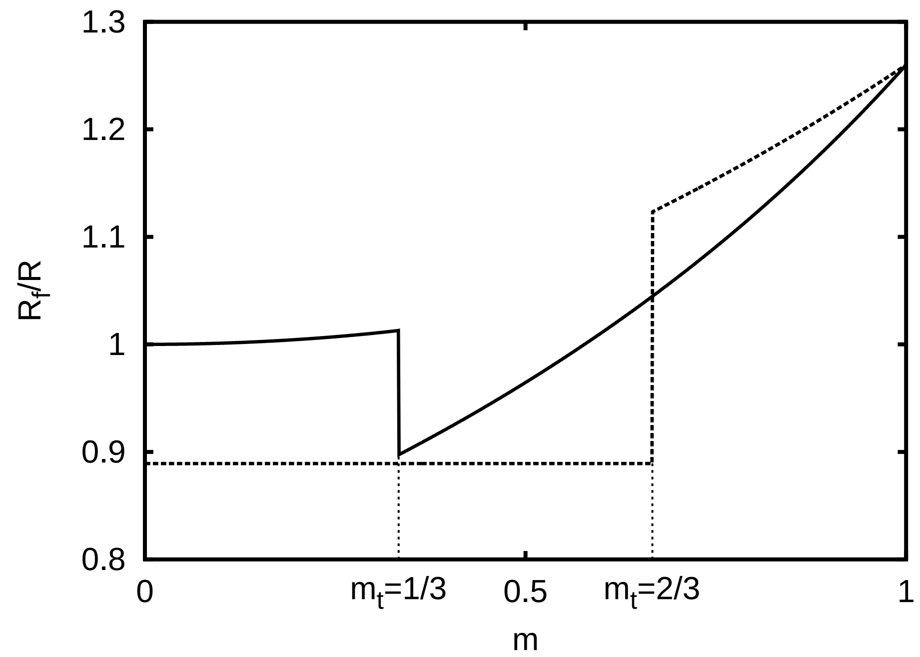


Fig. 9 FIST Resistance vs Magnetization