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Dynamics of Laser Pulses Focalized in Plasma

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What should we know about lasers:

1) Theory of the laser (discovery, 1960, ...)

2) Electron acceleration in plasma: Tajima, Dawson, 1979; desktop accelerators

3) Chirped pulse amplification: Mourou, Strickland, 1985 (see also Cook, 1960)

4) Short pulses, high power; ELI initiative (1-10-100Pw?)

State of the art lasers:

1) Radiation wavelength $1\mu m$ (infrared, frequency $2\times10^{15}s^{-1}$, photon energy $1eV$)

2) Energy per pulse $50J$ ($1kg$, $5m$)

3) Pulse duration $\tau = 50fs = 5 \times 10^{-14}s$ (rep rate $\simeq 1s$)

4) Power $10^{15}w$ ($1Pw$)

5) Pulse length $15\mu m$ (15 wavelengths) (Intensity $\simeq 10^{20}w/cm^2$)

6) Electric fields $\simeq 10^6 statvolt/cm$ ($10^{10}V/m$), Magnetic field $\simeq 10^6 Gs$ ($10^2 Ts$)

7) Comparable with atomic fields (!) (non-linearities)

What else should we know?

Pulsed polariton (polaritonic pulse) - convenient way

(introduced in 2010)

Main limitation: materials - Gw/cm^2 - typical laser described above

A serious difficulty:

Maxwell equations in matter

$$div\mathbf{D} = 4\pi\rho \ , \ div\mathbf{B} = 0 \ ,$$

$$curl\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} \ , \ curl\mathbf{H} = \frac{1}{c}\frac{\partial\mathbf{D}}{\partial t} + \frac{4\pi}{c}\mathbf{j}$$

(plus cont eq)

Two equations and Four unknowns:→**semi-phenomenology, semi-empirical** (ε and μ)

Electromagnetic Theory in Matter

Displacement field: $\mathbf{u}(\mathbf{r},t)$; $\delta n = -ndiv\mathbf{u}$, $\rho = -nqdiv\mathbf{u}$, $\mathbf{j} = nq\dot{\mathbf{u}}$; $\mathbf{P} = nq\mathbf{u}$; Maxwell equations

$$div\mathbf{E} = -4\pi nqdiv\mathbf{u} \ , \ div\mathbf{H} = 0 \ ,$$

$$curl\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} \ , \ curl\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}nq\dot{\mathbf{u}}$$

Two eqs - Three unknowns (non-magnetizable matter)

Newton's law

$$m\ddot{\mathbf{u}} = q(\mathbf{E}_0 + \mathbf{E}) - m\gamma\dot{\mathbf{u}}$$

The third missing eq! (introduced in 2009)

Solution: (infinite matter)

Longitudinal

$$E_1(\mathbf{k},\omega) = -4\pi n q u_1(\mathbf{k},\omega) = \frac{\omega_p^2}{\omega^2 - \omega_p^2 + i\omega\gamma} E_{01}(\mathbf{k},\omega)$$

Transverse

$$u_2(\mathbf{k},\omega) = -\frac{q}{m}\frac{(\omega^2 - c^2k^2)E_{02}(\mathbf{k},\omega)}{\omega^2\left(\omega^2 - \omega_p^2 - c^2k^2\right) + i sgn\omega \cdot 0^+}$$

$$E_2(\mathbf{k},\omega) = \frac{\omega_p^2}{\omega^2 - \omega_p^2 - c^2k^2 + i sgn\omega \cdot 0^+} E_{02}(\mathbf{k},\omega)$$

and magnetic field $H_3(\mathbf{k},\omega) = \frac{ck}{\omega} E_2(\mathbf{k},\omega)$

Eigenmodes

Plasmon $\omega = \omega_p = \sqrt{4\pi n q^2/m}$ (non-propagating)

Polariton $\omega = \Omega(k) = \sqrt{\omega_p^2 + c^2 k^2}$ (dispersive)

Drude-Lorentz model of polarizable matter (1900); dielectric function $\varepsilon(\omega)$

Refraction

$\Omega(k) = \sqrt{\omega_p^2 + c^2k^2} = ck'$ (in vacuum)

Snell law (Huygens, phase velocity)

$$\frac{\sin r}{\sin i} = \frac{k'}{k} = \frac{\Omega}{\sqrt{\Omega^2 - \omega_p^2}} = \frac{1}{\sqrt{\varepsilon(\Omega)}} = \frac{1}{n(\Omega)}$$

($n < 1$; conductors)

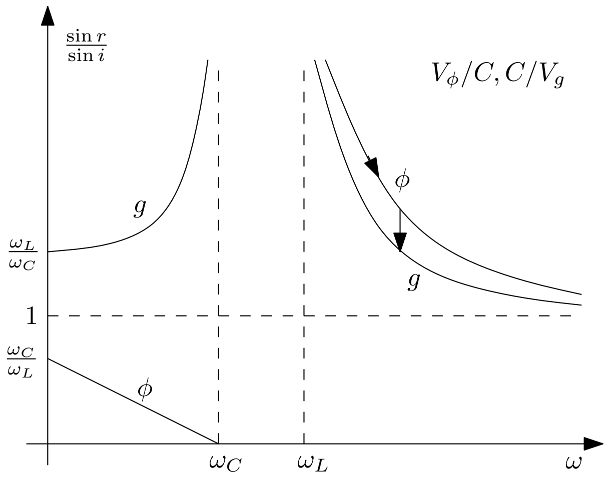
For bound charges (dielectrics) $n < 1$ (X- or gamma rays) and $n > 1$;

Since there is another branch $\Omega(k) = \omega_c ck/(\omega_L + ck)$,$\omega_L = \sqrt{\omega_p^2 + \omega_c^2}$

Snell law

$$\frac{\sin r}{\sin i} = \frac{k'}{k} = \frac{\omega_c - \Omega}{\omega_L} = \frac{1}{n(\Omega)}$$

$(n > 1)$ (no $1/\sqrt{\varepsilon}$-law)



Extinction theorem

(Ewald, Oseen, 1915): $\omega = ck$ (free field)

$$u_2(\mathbf{k},\omega) = -\frac{q}{m}\frac{(\omega^2 - c^2k^2)E_{02}(\mathbf{k},\omega)}{\omega^2\left(\omega^2 - \omega_p^2 - c^2k^2\right) + isgn\omega \cdot 0^+} \to 0$$

$$E_2(\mathbf{k},\omega) = \frac{\omega_p^2}{\omega^2 - \omega_p^2 - c^2k^2 + isgn\omega \cdot 0^+}E_{02}(\mathbf{k},\omega) \to -E_{02}(\mathbf{k},\omega)$$

$$E_{tot} = 0\,!$$

Free fields do not propagate in matter! Presence of surface change things, to some extent! (Refraction)

Wavepackets

(Huygens, 1700)

$$\int d\mathbf{r}_t e^{i\mathbf{k}_t\mathbf{r}_t} = \frac{2\sin k_y d_t/2}{k_y} \cdot \frac{2\sin k_z d_t/2}{k_z} \to 2\pi\delta(\mathbf{k}_t)$$

$$\int d\mathbf{k}_t e^{i\mathbf{k}_t\mathbf{r}_t} = \frac{2\sin y\Delta k_y/2}{y} \cdot \frac{2\sin z\Delta k_z/2}{z} \to 2\pi\delta(\mathbf{r}_t)$$

$$\int dt dx E_0 \cos(\omega_0 t - k_{0x}x) e^{i\omega t} e^{-ik_x x} =$$

$$= \tfrac{1}{2}E_0 \tfrac{2\sin(\omega-\omega_0)\tau/2}{\omega-\omega_0} \cdot \tfrac{2\sin(k_x-k_{0x})d/2}{k_x-k_{0x}} + (\omega_0 \to -\omega_0)$$

Pulsed Polariton

New (approximate and useful) solution of Maxwell eqs in matter (plasma)

External agents (perturbation)

$$[\omega^2 - \Omega^2(k)]u(\mathbf{k},\omega) = 0 \ , \ E(\mathbf{k},\omega) = -\frac{m}{q}\omega^2 u(\mathbf{k},\omega) \ , \ H(\mathbf{k},\omega) = \frac{ck}{\omega}E(\mathbf{k},\omega)$$

Solution:

$$u(\mathbf{k},\omega) = 2\pi u(\mathbf{k})[\delta(\omega - \Omega(k)) + \delta(\omega + \Omega(k)]$$

$$u(\mathbf{r},t) = \frac{1}{(2\pi)^3}\int d\mathbf{k}u(\mathbf{k})e^{-i\Omega(k)t + i\mathbf{k}\mathbf{r}} + c.c.$$

Sommerfeld, Brillouin, 1914: Stationary phase (method of the saddle point or steepest descent)

$$-i\Omega(k)t+i\mathbf{k}\mathbf{r} = -i\Omega(k_0)t+i\mathbf{k}_0\mathbf{r}+\left(-\,i\frac{\partial\Omega}{\partial\mathbf{k}}\Big|_{\mathbf{k}_0} t+i\mathbf{r}\right)\mathbf{q}-\frac{it}{2}\,\frac{\partial^2\Omega}{\partial k_i\partial k_j}\Big|_{\mathbf{k}_0} q_iq_j+..,$$

Group velocity

$$\mathbf{v} = \frac{\partial\Omega}{\partial\mathbf{k}}\Big|_{\mathbf{k}_0}$$

Choose $\mathbf{k}_0 = (k_{0x}, k_{0y} = 0, k_{0z} = 0)$

Transverse wavepacket of extension d_t,

$$\int d\mathbf{q}_t e^{i\mathbf{q}_t\mathbf{r}_t} = \frac{2\sin y/2d_t}{y}\cdot\frac{2\sin z/2d_t}{z} \rightarrow (2\pi)^2\delta(\mathbf{r}_t)$$

We are left, for instance, with

$$E(\mathbf{r},t) \simeq -\frac{m}{q}\Omega_0^2 u_0 \delta(\mathbf{r}_t) \cdot e^{-i\Omega_0 t + ik_{0x}x} \frac{1}{2\pi}\int dq_x e^{-i(vt-x)q_x - \frac{it}{2}\Omega_0'' q_x^2} + c.c. \; ,$$

$(u_0 = u(\mathbf{k}_0))$

$$v = \frac{c\omega_0}{\Omega_0} \; , \; \Omega_0'' = \frac{c^2\omega_p^2}{\Omega_0^3} \; , \; \omega_0 = ck_{x0} \; , \; \Omega_0 = \sqrt{\omega_p^2 + \omega_0^2}$$

The integral

$$\frac{1}{2\pi}\int dq_x e^{-i(vt-x)q_x - \frac{it}{2}\Omega_0'' q_x^2} \simeq \frac{1}{\sqrt{2\pi i t \Omega_0''}} e^{i\frac{(x-vt)^2}{2t\Omega_0''}} \rightarrow_{t\Omega_0'' \rightarrow 0} \delta(x - vt)$$

Lifetime and extension

At $t = 0$ the pulse is $\delta(x)$, $\delta(x) \simeq 1/d$

After time Δt the pulse has a height $\simeq 1/\sqrt{\Delta t \Omega_0''} \longleftrightarrow 1/d$

width $\Delta x \simeq \sqrt{\Delta t \Omega_0''}$

Lifetime:

$$\frac{1}{\sqrt{\Delta t \Omega_0''}} = \frac{1}{df} \; , \; \Delta x = \sqrt{\Delta t \Omega_0''} = fd$$

where f is an arbitrary, small, higher-than-unity number

Limiting case $f = 1$, we get $\Delta t \simeq d^2/\Omega_0''$, or, since $d = v\tau$, we have $\Delta t = v^2\tau^2/\Omega_0''$; *i.e.*

$$\Delta t = \tau^2 \frac{\omega_0^2 \Omega_0}{\omega_p^2}$$

A limiting value for the duration of the pulse

$$\tau = \Delta t = \frac{\omega_p^2}{\Omega_0 \omega_0^2}$$

and a limiting value of the pulse extension

$$d = v\tau = c\frac{\omega_p^2}{\Omega_0^2 \omega_0}$$

During its lifetime Δt the pulse flies the distance $l = v\Delta t = c\tau^2\omega_0^3/\omega_p^2$, which is a pretty long distance for $\omega_p \ll \omega_0$ ($l \simeq d\tau\omega_0^3/\omega_p^2$)

It follows that we can write **the Polaritonic Pulse** as

$$u(\mathbf{r}, t) \simeq -\frac{q}{m\Omega_0^2} E_0 \cdot d\delta(x - vt) \cdot d_t^2 \delta(\mathbf{r}_t) \ ,$$

$$E(\mathbf{r}, t) \simeq H(\mathbf{r}, t) \simeq E_0 \cdot d\delta(x - vt) \cdot d_t^2 \delta(\mathbf{r}_t) \ ;$$

it has an electromagnetic energy $U = E_0^2 d d_t^2/4\pi$, which is transported with velocity $v = c\omega_0/\Omega_0$ during a lifetime $\Delta t \simeq d^2\Omega_0^3/c^2\omega_p^2$ over a distance $l \simeq v\Delta t \simeq d^2\Omega_0^3/c\omega_p^2$.

(There is also a small pulsation with frequency ω_p^2/Ω_0)

Comments on Fresnel and Fraunhofer diffraction: vanishing shadow, fringes in the shadow

“precursors”: high frequencies contribute; propagating with group velocity=c; far away - non-local

Rest frame and equilibrium

$d \rightarrow d' = \gamma d \gg d$ $(\gamma = (1 - v^2/c^2)^{-1/2} = \Omega_0/\omega_p \gg 1)$

$H_z = (\omega_0/\Omega_0)E = \beta E \rightarrow H_z^{'} = 0$: static field

$E_y = E \rightarrow E_y^{'} = E/\gamma = (\omega_p/\Omega_0)E \simeq (\omega_p/\Omega_0)E_0$

Polarization $P = nqu = -(\omega_p^2/4\pi\Omega_0^2)E$, longitudinal field
$E_{ly} = 4\pi P = -(\omega_p^2/\Omega_0^2)E \rightarrow E_{ly}^{'} = -\gamma(\omega_p^2/\Omega_0^2)E = -(\omega_p/\Omega_0)E$: cancellation

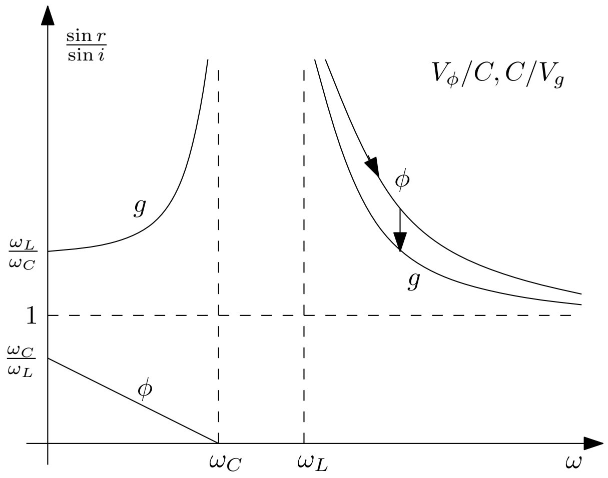
Emergent Physics

Whats new?

1) Pulse refraction (?)

$$\frac{\sin r}{\sin i} = \frac{c}{v_g}$$

Sometimes the same ($v_\Phi v_g = c^2$), sometimes not!



Comment upon γ-ray refraction!

2) Transported (accelerated) charge

$$\delta n = -n div \mathbf{u} = \frac{nq}{m\Omega_0^2 d_t} E_0$$

$$\delta N = \frac{nq}{m\Omega_0^2} d d_t E_0 \ , \ \delta Q = \frac{\omega_p^2}{4\pi\Omega_0^2} d d_t E_0$$

$(\delta N/N = qE_0 d_t / m v_0^2)$

3) **Energy, flows**

$$\mathcal{E} = \frac{mc^2}{\sqrt{1 - v^2/c^2}} = mc^2 \frac{\Omega_0}{\omega_p} \gg mc^2$$

$$\Phi_N = v\delta n = c\frac{nq\omega_0}{m\Omega_0^3 d_t} E_0 \ , \ \Phi = \Phi_N \mathcal{E} = c^3 \frac{nq\omega_0}{\Omega_0^2 \omega_p d_t} E_0$$

compared with the energy flow $\Phi_0 \simeq vE_0^2/4\pi$(intensity)

$$\frac{\Phi}{\Phi_0} = c^2 \frac{nq}{\omega_p \Omega_0 d_t} \sqrt{\frac{4\pi c\omega_0}{\Omega_0 \Phi_0}} = c^2 \frac{\sqrt{mn}}{\Omega_0 d_t} \sqrt{\frac{c\omega_0}{\Omega_0 \Phi_0}}$$

(Rather negative result, $1/\sqrt{\Phi_0}$!; $\mathcal{E}\Phi \simeq const$!)

4) **Coherent** X**- or gamma rays** by Compton (Thomson) backscattering on the pol pulse

Numerical Estimates

Energy

$n = 10^{18} cm^{-3}$ (electrons), $\omega_p = 3 \times 10^{-2} eV$ ($\simeq 5 \times 10^{13} s^{-1}$)

Main frequency $\Omega_0 = 1 eV$ ($2 \times 10^{15} s^{-1}$, wavelength $1 \mu m$)

Ultra-relativistic velocity of the pulse and a particle energy $\mathcal{E} \simeq 20 MeV$

Pulse

Pulse size $d = d_t = 15\mu m$ ($\tau = 50fs = 5 \times 10^{-14}s$), energy $50J$ ($E_0 \simeq 10^6 statvolt/cm$), intensity $\Phi_0 = 4 \times 10^{20} w/cm^2$

$\delta N \simeq 3 \times 10^5$ particles (electrons) (*i.e.* $6TeV$):

Meaning $\simeq 10^{24}$ particles per $cm^2 \cdot s$ and a large amount of energy, $\Phi \simeq 10^{25} MeV/cm^2 \cdot s$ (the only problem is that it is too thin, too short and too brief!)

Displacement: $u \simeq 10$Å; comments with respect to Nuclear Polarization

New Results

1) Polaritonic eigenmodes, their motion

2) Refraction, in particular pulse refraction (?)

3) Lifetime and spread of the Pulsed Polariton

4) Stability, equilibrium (rest frame) of the PP

New Ways

The Way of Thinking Physics Nowadays

"Available optical laser intensities exceeding $10^{22} W/cm^2$"

Wishful thinking - talking the Unreal

"Push the fundamental light-electron interaction to the extreme limit"

Extremist standpoint

“where radiation-reaction effects dominate the electron dynamics”

For single particle, Reaction is compensated by the other particles, cannot construct a useful World from One Particle, or Singular Events

This may explain Everything, which is Bigotry, not Useful Positive Science

“can shed light on the structure of the quantum vacuum”

This is long-, well-known, established knowledge: The Great Fall in Derisory and Triviality

“can trigger the creation of particles such as electrons, muons, and pions and their corresponding antiparticles”

So what?

“novel sources of intense coherent high-energy photons and laser-based particle colliders can pave the way to nuclear quantum optics”

Yes, the Atomic Nucleus may act as lens, prism, diffraction grating, resonant cavity, laser for sharp and intense gamma rays

The only problem is that the effects are extremely weak - since the nuclear polarization is extremely weak

“may even allow for the potential discovery of new particles beyond the standard model”

This is Childish: we need much, much higher energy and luminosity

The Problem lies in Quantity, not in Quality

Future Avenues

1) Charges in Strong Laser Fields: ionization, X-rays, non-linearities, multiphoton scattering, radiation reaction

2) Vacuum Polarization: photon-photon scatt, refractive index, el-pos pairs, $\mu - \overline{\mu}$, $\pi - \overline{\pi}$, non-linear QED

3) Nuclear Physics: Th: $7.6eV = 1.55 \times fifth - harm$; laser+acc els=$X$-, γ-sources (keVs); photoreactions; dipole transitions; $\beta-$ and α-decay

4) Laser accelerators (colliders): pulse; in-phase acceleration; etc

5) Beyond Standard Model: electroweak; axion; etc

Patru situatii experimentale:

1) Disturbante localizate in interiorul plasmei: puls polaritonic

2) Unda plana libera cade pe suprafata; raza, optica geom, beam; propagare polaritonica, cu refractie

3) Focalizare unde in plasma: puls polaritonic, dincolo de opt geom

4) Puls foarte strins, creat in vid si trimis pe suprafata: isi pastreaza destul de bine individ