# ELECTROMAGNETISM in MATTER

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Moon's Problem 2005

Hadronization of the Quark-Gluon Plasma 2006

Curved Space, Covariance, Motion and Quantization 2007

Coherence in Matter Interacting with Radiation 2008

Electromagnetism in Matter 2009

NanoScience and NanoTechnology 2004 (Pitesti)

## PART I

### Electromagnetism

Fresnel, Coulomb, Ampere, Gauss, Weber, Kirchhoff, Hertz, Poincare, Lorentz **Integral Representations**

Faraday, Maxwell **Differential Equations**

Two lines of thought

## Motivation

**Refraction** Fresnel-Huygens theory; **extinction theorem**

"Effective Medium Permittivity" Theory: **the problem of** $\varepsilon$

**Phenomenological** response functions: $\varepsilon$, $\mu$, $\sigma$

The role of matter, its dynamics: **plasmons, polaritons; diffraction**

The **enhancement** in structures with finite geometries

**Metamaterials**

## Maxwell Equations in Vacuum

$$div\mathbf{E} = 4\pi\rho \ \ (Gauss)$$

$$curl\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t} \ \ (Faraday)$$

$$div\mathbf{H} = 0 \ \ (Gauss)$$

$$curl\mathbf{H} = \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j} \ \ (Ampere)$$

Continuity: $\frac{\partial\rho}{\partial t} + div\mathbf{j} = 0$; convection current ($\mathbf{j} = \rho\mathbf{v}$; velocity $\mathbf{v}$)

## Solution

$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - grad\Phi$ , $\mathbf{H} = curl\mathbf{A}$; Lorentz gauge $div\mathbf{A} + \frac{1}{c}\frac{\partial \Phi}{\partial t} = 0$

Wave equations $\frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2} - \Delta\Phi = 4\pi\rho$ , $\frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} - \Delta\mathbf{A} = \frac{4\pi}{c}\mathbf{j}$

Kirchhoff: $R = |\mathbf{r} - \mathbf{r}'|$

$$\Phi(\mathbf{r}, t) = \int d\mathbf{r}' \frac{\rho(\mathbf{r}', t - R/c)}{R}$$

$$\mathbf{A}(\mathbf{r}, t) = \frac{1}{c}\int d\mathbf{r}' \frac{\mathbf{j}(\mathbf{r}', t - R/c)}{R}$$

**Two regimes**: Retarded; Non-retarded (approximate): $\omega/c = 1/\lambda \ll$ (body size)$^{-1}$

**Comment on solvability**

Four Maxwell equations plus two other equations (continuity and Lorentz gauge) = six equations; only four unknown: $\mathbf{E}$, $\mathbf{H}$, $\rho\,,\mathbf{j}$

Actualy **only two independent Maxwell equations**

The only absolute input: $c$

## Maxwell Equations in Matter

$$div\mathbf{D} = 4\pi\rho_0 \ \ (Gauss)$$

$$curl\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} \ \ (Faraday)$$

$$div\mathbf{B} = 0 \ \ (Gauss)$$

$$curl\mathbf{H} = \frac{1}{c}\frac{\partial\mathbf{D}}{\partial t} + \frac{4\pi}{c}\mathbf{j}_0 \ \ (Ampere)$$

$\rho_0$ and $\mathbf{j}_0$ external charge and curent; $\mathbf{D}$ and $\mathbf{H}$ created by external sources

**Comment on solvability**

Two new unknowns: electric displacement $\mathbf{D}$, magnetic induction $\mathbf{B}$ and yet another: the diffusive current $\mathbf{j}$ in matter (no more $\mathbf{j} = \rho\mathbf{v}$).

**Same scheme of solution**, plus phenomenological relations

$$\mathbf{D} = \varepsilon\mathbf{E} \ , \ \mathbf{B} = \mu\mathbf{H} \ , \ \mathbf{j}_0 = \sigma\mathbf{E} \ (Ohm)$$

Dielectric function $\varepsilon$, Magnetic permeability $\mu$, electric conductivity $\sigma$

## Our em equations in matter

$$div\mathbf{E} = 4\pi\rho_0 - 4\pi div\mathbf{P}$$

$$curl\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}\ ,\ div\mathbf{H} = 0$$

$$curl\mathbf{H} = \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} + \frac{4\pi}{c}\frac{\partial\mathbf{P}}{\partial t} + 4\pi curl\mathbf{M} + \frac{4\pi}{c}\mathbf{j}_0$$

Maxwell equations: $\mathbf{H} \rightarrow \mathbf{B}$, $\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P}$, $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$

**Reasoning**: polarization $P$, polarization charge, by cont eq the polarization current $\frac{\partial\mathbf{P}}{\partial t}$

In addition: $div\mathbf{j}$ in the cont eq admits an additional current (magnetization current) $c \cdot curl\mathbf{M}$ ($div \cdot curl = 0$)

## Solvability: Magnetic phenomena

Instead of $\mathbf{D}$ and $\mathbf{H}$ now we have $\mathbf{P}$ and $\mathbf{M}$; what's the gain?

The gain is that we have an additional equation of motion for $\mathbf{M}$:

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{H} \times \mathbf{M}$$

Gyromagnetic factor $\gamma = \mu/\hbar$

**Comment**: Model Ampere molecular currents +spin (quantum relativist) magnetic moments (Bohr magneton $\mu_B = e\hbar/2mc$)

Commonly weak magnetism ($v^2/c^2$): disregard

Except for ferromagnetism and related phenomena: special treatment

Still $\mathbf{j}_0 = \sigma \mathbf{E}_{tot}$; disregard external sources

Have we another equation for $\mathbf{P}$? Had we have→**SOLVABILITY**

## We have another equation for $\mathrm{P}$

Weak displacement field $\mathbf{u}$, **density disturbances** $\delta n = -n div \mathbf{u}$, polarization charge $\rho = -e\delta n = nediv\mathbf{u}$ (as for electrons), polarization $\mathbf{P} = -ne\mathbf{u}$ and polarization current $\mathbf{j} = -ne\frac{d\mathbf{u}}{dt}$

Equation of motion? **Newton's Law:**

$$m\frac{d^2\mathbf{u}}{dt^2} = -e\mathbf{E} - e\mathbf{E}_0 \ \ (plasma,\, ext\, el\, field\, E_0)$$

or

$$m\frac{d^2\mathbf{u}}{dt^2} = -e\mathbf{E} - m\omega_0^2\mathbf{u} - e\mathbf{E}_0 \ \ (dielectrics,\, large\, \omega_0)$$

or

$$m\frac{d^2\mathbf{u}}{dt^2} = -e\mathbf{E} - m\omega_0^2\mathbf{u} - m\gamma\frac{d\mathbf{u}}{dt} - e\mathbf{E}_0 \ \ (loss)$$

or

$$m\frac{d^2\mathbf{u}}{dt^2} = -e\mathbf{E} - m\omega_0^2\mathbf{u} - m\gamma\frac{d\mathbf{u}}{dt} - e\mathbf{E}_0 + \frac{1}{c}\frac{d\mathbf{u}}{dt} \times \mathbf{H}_0 \ (ext\, magn\, field\, H_0)$$

or... whatever else!

**Comment**: discard Lorentz force (relativistic effects); elementary dispersion theory

## Another comment

From $m\frac{d^2\mathbf{u}}{dt^2} = -e\mathbf{E} - e\mathbf{E}_0$ and $\mathbf{E} = -4\pi\mathbf{P} = 4\pi ne\mathbf{u}$ we get the dielectric $\varepsilon$function

$$E_0 = \varepsilon E_{tot} = \varepsilon(\mathbf{E} + \mathbf{E}_0)\ ,\ \varepsilon = 1 - \frac{\omega_p^2}{\omega^2}\ ,\ \omega_p^2 = \frac{4\pi ne^2}{m}$$

And by using $\mathbf{j} = -ne\frac{d\mathbf{u}}{dt}$ we get the conductivity

$$\mathbf{j} = \sigma\mathbf{E}_{tot}\ ,\ \sigma = \frac{i\omega_p^2}{4\pi\omega}$$

All these for jellium-like plasma; similar for dielectrics, loss included, etc

# Our Scheme of Calculation

Eq of motion

$$m\frac{d^2\mathbf{u}}{dt^2} = -e\mathbf{E} - e\mathbf{E}_0$$

Kirchhoff potentials

$$\Phi(\mathbf{r},t) = \int d\mathbf{r}'\frac{\rho(\mathbf{r}',t-R/c)}{R}\ ,\ \mathbf{A}(\mathbf{r},t) = \frac{1}{c}\int d\mathbf{r}'\frac{\mathbf{j}(\mathbf{r}',t-R/c)}{R}$$

Electric field $\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - grad\Phi$

Charge and current

$$\rho = nediv\mathbf{u}\ ,\ \mathbf{j} = -ne\frac{d\mathbf{u}}{dt}$$

Compute $\mathbf{A}$ and $\Phi$ using $\rho$ and $\mathbf{j}$ expressed by $\mathbf{u}$ inside the body, compute $\mathbf{E}$, eliminate $\mathbf{E}$ between the resulting equation and the equation of motion

So we get (coupled) **INTEGRAL EQUATIONS for $\mathbf{u}$**

Once solved, compute $\mathbf{A}$ and $\Phi$ and $\mathbf{E}$ outside, for the reflected, refracted, transmitted, diffracted field

Do the same for the non-retarded regime, where the equation of motion is simple

$$m\frac{d^2\mathbf{u}}{dt^2} = ne^2 grad \int d\mathbf{r}' \frac{div\mathbf{u}}{|\mathbf{r}-\mathbf{r}'|} - e\mathbf{E}_0$$

We can see explicitly its character of integral equation

## Comment

Eigenmodes of the Integral Eq in the non-retarded regime give the **plasmons**

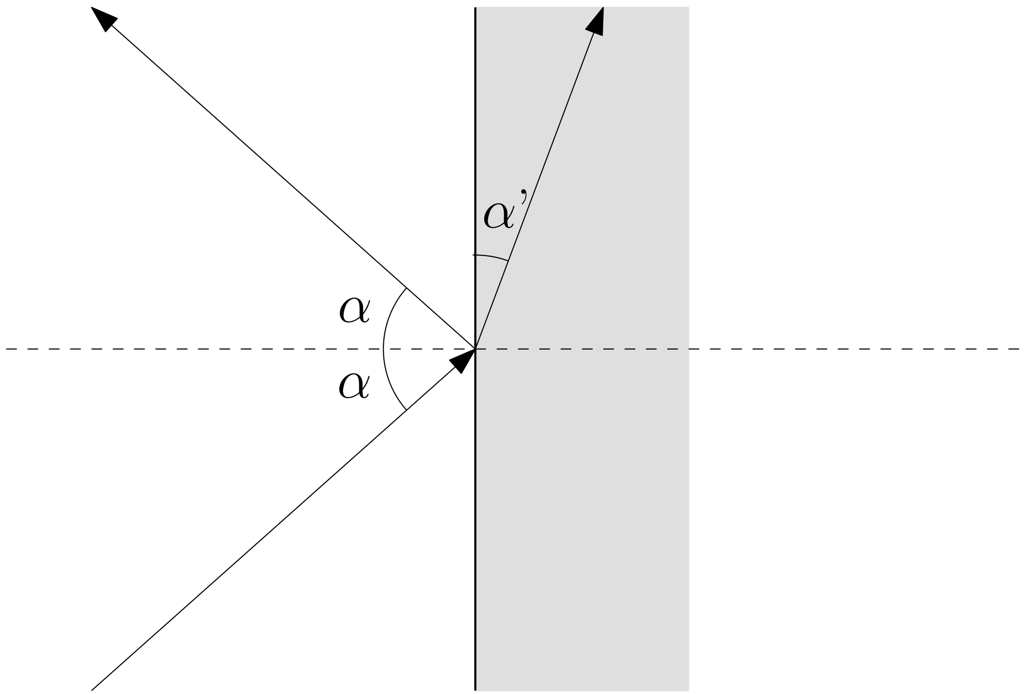
Its full solution gives the **dielectric response**

Eigenmodes of the Integral Eqs in the retarded regime give the **polaritons**

Their full solutions give the **refracted, reflected, transmitted, diffracted fields**

**PART II: APPLICATIONS**

## Semi-infinite body (half-space)



$$\mathbf{u} \rightarrow \mathbf{u}\theta(z) \ , \ div\mathbf{u} \rightarrow div\mathbf{u} \cdot \theta(z) + u_3(z=0)\delta(z)$$

**Non-retarded**:

$$\omega^2 v = \frac{1}{2}k\omega_p^2 \int_0^\infty dz' v e^{-k|z-z'|} + \frac{1}{2k}\omega_p^2 \int_0^\infty dz' \frac{\partial v}{\partial z'} \frac{\partial}{\partial z'} e^{-k|z-z'|} - \frac{iek}{m}\Phi$$

$v = \mathbf{k}\mathbf{u}/k$ , in-plane $\mathbf{k}$

**Dielectric response**

$$\tfrac{e}{m\omega^2} E_{tot\perp} = v = \tfrac{iek\omega_p^2}{m} \tfrac{\Phi_0}{(\omega^2-\omega_p^2)(2\omega^2-\omega_p^2)} e^{-kz} - \tfrac{iek}{m} \tfrac{\Phi}{\omega^2-\omega_p^2} \ ,$$

$$\tfrac{e}{m\omega^2} E_{tot\parallel} = u_3 = -\tfrac{ek\omega_p^2}{m} \tfrac{\Phi_0}{(\omega^2-\omega_p^2)(2\omega^2-\omega_p^2)} e^{-kz} - \tfrac{e}{m} \tfrac{\Phi'}{\omega^2-\omega_p^2}$$

Bulk plasmons $\omega^2 = \omega_p^2$, **surface plasmons** $\omega^2 = \omega_p^2/2$

**No proper dielectric function**: surface contribution

**Energy loss** of an energetic particle

$$P_b = \left(-e^2\omega_p^2/v\right)\ln(vk_0/\omega_p)$$

$$P_s = -\frac{e^2\omega_p}{vt}\left(\sqrt{2}\sin\omega_p t/\sqrt{2} - \sin\omega_p t\right)$$

**Retardation**: $\omega = cK = c\sqrt{k^2+\kappa^2}$, incidence angle $\alpha$, polarization $\beta$, $k = K\cos\varphi$

$$\omega^2 v_1 = -\frac{i\omega_p^2\kappa}{2}\int_0 dz' v_1(z') e^{i\kappa|z-z'|} - \frac{\omega_p^2 k}{2\kappa}\int_0 dz' u_3(z')\frac{\partial}{\partial z'}e^{i\kappa|z-z'|} + \frac{e}{m}E_{01}e^{i\kappa z}$$

$$\omega^2 v_2 = -\frac{i\omega_p^2\omega^2}{2c^2\kappa}\int_0 dz' v_2(z') e^{i\kappa|z-z'|} + \frac{e}{m}E_{02}e^{i\kappa z}$$

$$\left(\omega^2-\omega_p^2\right)u_3 = \frac{\omega_p^2 k}{2\kappa}\int_0 dz' v_1(z')\frac{\partial}{\partial z}e^{i\kappa|z-z'|} - \frac{i\omega_p^2 k^2}{2\kappa}\int_0 dz' u_3(z') e^{i\kappa|z-z'|} +$$

$$+\frac{e}{m}E_{03}e^{i\kappa z}$$

**Refracted field**

$$\frac{e}{m\omega^2}E_{tot1} = v_1 = \frac{2eE_{01}}{m\omega_p^2} \cdot \frac{\kappa'\left(\kappa-\kappa'\right)}{\kappa\kappa'+k^2}e^{i\kappa' z}$$

$$\frac{e}{m\omega^2}E_{tot2} = v_2 = \frac{2eE_{02}}{m\omega_p^2} \cdot \frac{\kappa\left(\kappa-\kappa'\right)}{K^2}e^{i\kappa' z}$$

$$\frac{e}{m\omega^2}E_{tot3} = u_3 = \frac{2eE_{03}}{m\omega_p^2} \cdot \frac{\kappa\left(\kappa-\kappa'\right)}{\kappa\kappa'+k^2}e^{i\kappa' z}$$

$$\kappa' = \sqrt{\kappa^2 - \omega_p^2/c^2} = \frac{1}{c}\sqrt{\omega^2 \cos^2\alpha - \omega_p^2}$$

**Extinction theorem** $\kappa \to \kappa'$; "effective medium theory"?

**Reflected field**

$$E_1 = E_{01}\frac{\kappa-\kappa'}{\kappa+\kappa'} \cdot \frac{\kappa\kappa'-k^2}{\kappa\kappa'+k^2}e^{-i\kappa z}$$

$$E_2 = E_{02}\frac{\kappa-\kappa'}{\kappa+\kappa'}e^{-i\kappa z}$$

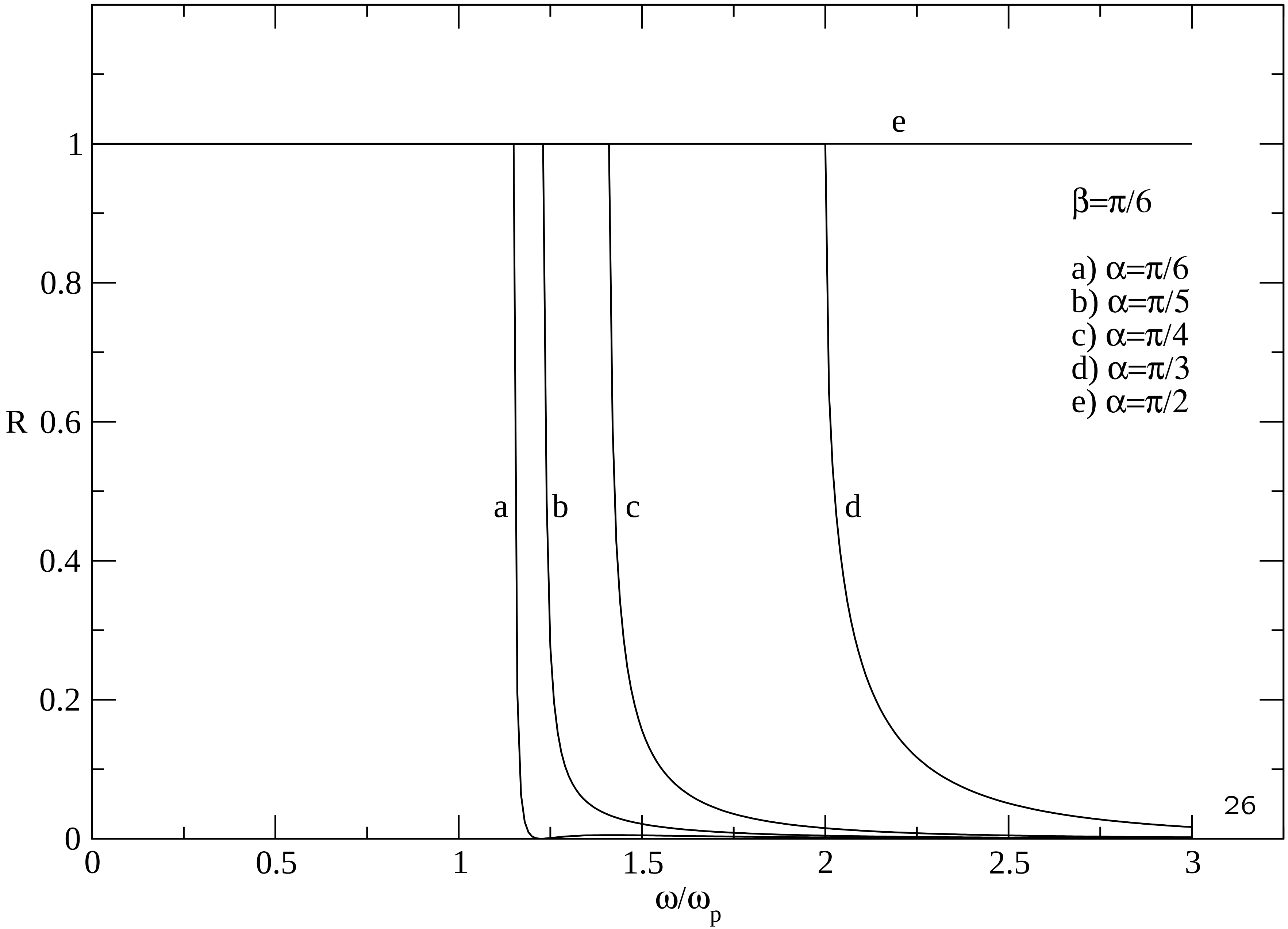
$$E_3 = -E_{03}\frac{\kappa-\kappa'}{\kappa+\kappa'} \cdot \frac{\kappa\kappa'-k^2}{\kappa\kappa'+k^2}e^{-i\kappa z}$$

**Reflection coefficient**

$$R = R_1 \left[\cos^2\beta \sin^2\varphi + R_2 \left(\cos^2\beta \cos^2\varphi + \sin^2\beta\right)\right]$$

$$R_1 = \left|\frac{\sqrt{\omega^2\cos^2\alpha - \omega_p^2} - \omega\cos\alpha}{\sqrt{\omega^2\cos^2\alpha - \omega_p^2} + \omega\cos\alpha}\right|^2$$

$$R_2 = \left|\frac{\cos\alpha\sqrt{\omega^2\cos^2\alpha - \omega_p^2} - \omega\sin^2\alpha}{\cos\alpha\sqrt{\omega^2\cos^2\alpha - \omega_p^2} + \omega\sin^2\alpha}\right|^2$$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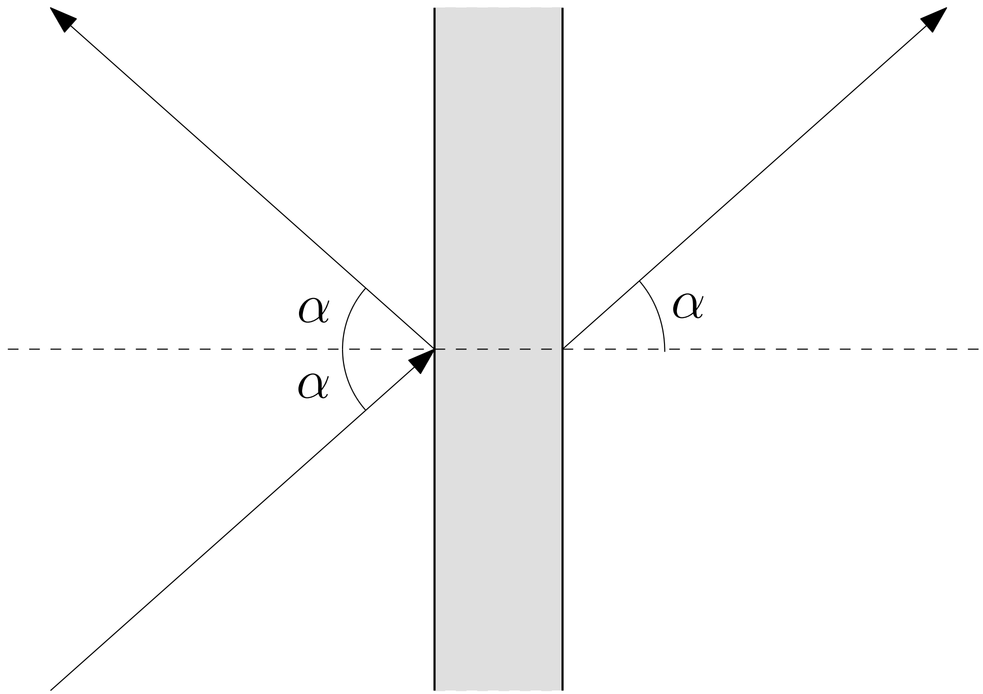
β=π/6
a) α=π/6
b) α=π/5
c) α=π/4
d) α=π/3
e) α=π/2
a
b
c
d
R
0
0.2
0.4
0.6
0.8
1
0.5
1.5
2
2.5
3
ω/ω_p

**Surface plasmon-polariton modes**

$$\omega^2 = \frac{2\omega_p^2 c^2 k^2}{\omega_p^2 + 2c^2k^2 + \sqrt{\omega_p^4 + 4c^4k^4}}$$

Limits $ck$ and $\omega_p/\sqrt{2}$.

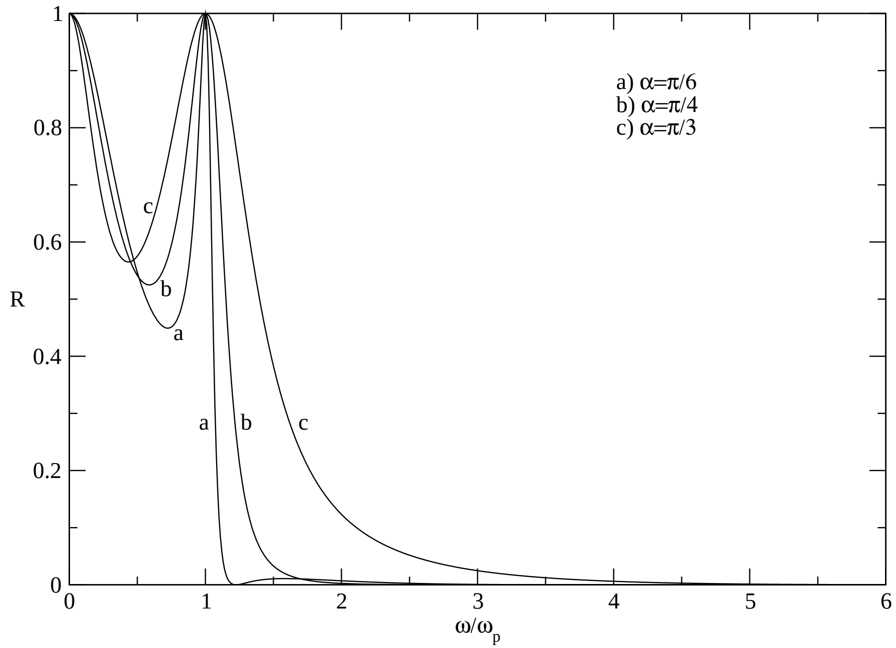
## Slab of thickness $d$



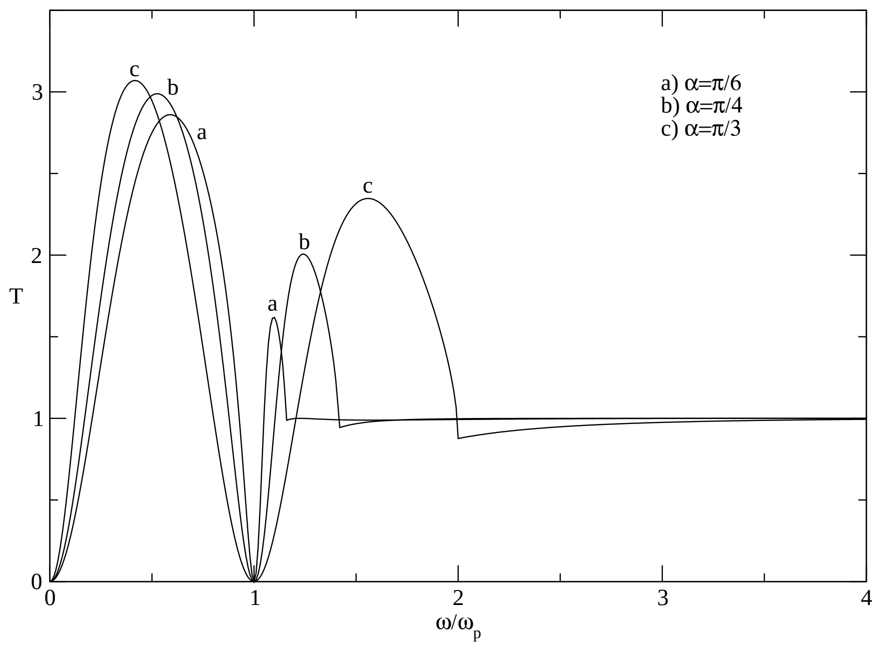
Dielectric response, surface terms, surface plasmons $\omega^2 = \frac{1}{2}\omega_p^2 \left(1 \pm e^{-kd}\right)$

Surface energy loss $\int_0^\infty dt P_s = -\pi \left(\sqrt{2} - 1\right) \frac{e^2 \omega_p}{v}$

Transmitted field $\sim e^{i\kappa z}$, Reflected field $\sim e^{-i\kappa z}$, refracted field $\sim e^{i\kappa' z}$, $e^{-i\kappa' z}$

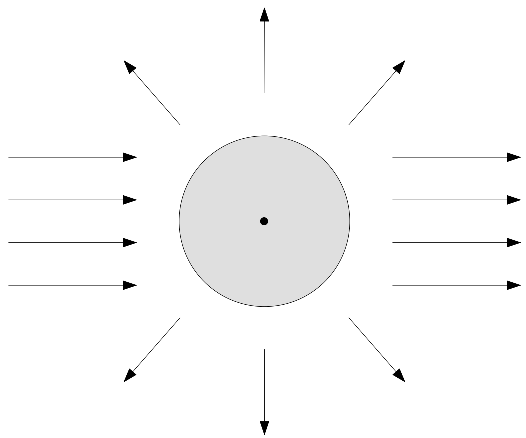



Reflection coefficient (for $p$-wave)




Transmission coefficient (for $p$-wave)

## Sphere



(Mie solution, 1908)

**Spherical plasmons** (sphere) $\omega = \omega_p\sqrt{\frac{l}{2l+1}}$, (void) $\omega = \omega_p\sqrt{\frac{l}{2l+1}}$

**Diffraction** $F^{0,\pm} \sim j_l \mathbf{Y}_{ljm},\ H^{0,\pm} \sim h_l \mathbf{Y}_{ljm},\ \mathbf{Y}_{ljm} \sim C - G,\, \mathbf{e}_1 Y_{lm}$

$$\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} = \frac{ik}{4\pi}\sum_{lm} F^*_{lmk}(\mathbf{r}_<)H_{lmk}(\mathbf{r}_>)$$

## Fields

$$\mathbf{E}_0(\mathbf{r}) = E_0 \sum_{l=1}^{\infty}\sum_{m} \Big( a_{lm}\mathbf{F}^0_{lmk}(\mathbf{r}) + b_{lm}\mathbf{F}^+_{lmk}(\mathbf{r}) \Big)$$

$$\mathbf{E}_i(\mathbf{r}) = \frac{m\omega^2}{e} u(\mathbf{r}) = E_0 \sum_{l=1}^{\infty}\sum_{m} \left[ A_l a_{lm}\mathbf{F}^0_{lmk_1}(\mathbf{r}) + B_l b_{lm}\mathbf{F}^+_{lmk_1}(\mathbf{r}) \right]$$

$$\mathbf{E}_s(\mathbf{r}) = \frac{ka^2}{16\pi^2} E_0 \sum_{l=1}^{\infty}\sum_{m} \left[ A_l a_{lm} f_l^0 \mathbf{H}^0_{lmk}(\mathbf{r}) + B_l b_{lm} f_l^+ \mathbf{H}^+_{lmk}(\mathbf{r}) \right]$$

$$k_1 = \frac{1}{c}\sqrt{\omega^2 - \omega_p^2}$$

**Coefficients**

$$A_l = \frac{16\pi^2(-1)^{l+1}c}{\omega a^2} \cdot \frac{1}{k_1 h_l(ka) j_{l+1}(k_1 a) - k h_{l+1}(ka) j_l(k_1 a)}$$

$$B_l = \frac{16\pi^2(-1)^{l+1}c^3 k_1}{\omega_p^2 \omega a^2} \cdot$$

$$\cdot \frac{1}{\left[\left(-\frac{\omega^2}{\omega_p^2} + \frac{l}{2l+1}\right) h_{l+1}(ka) + \frac{l+1}{2l+1} h_{l-1}(ka)\right] j_l(k_1 a) + \frac{\omega^2 k_1}{\omega_p^2 k} h_l(ka) j_{l+1}(k_1 a)}$$

**Plasmonic resonances** - optical spectroscopy of small spheres

**Cross-section**

$$\sigma = Re\left[r^2 \frac{Q_s}{|\mathbf{S}_0|}|_{r\to\infty}\right] = \frac{a^4}{16\pi^2}\sum_{l=1m}^{\infty}\left(\left|A_l a_{lm} f_l^0\right|^2 + \left|B_l b_{lm} f_l^+\right|^2\right)$$

## Cylindrical geometries

### Two-dimensional screen

Plasmons $\omega^2 = \omega_p^2 kd/2$; Transmitted, reflected fields, discontinuous

### Green Functions

$$\frac{e^{i\frac{\omega}{c}|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} = i \sum_{m=-\infty}^{+\infty} e^{im(\varphi-\varphi')} \int_0^\infty kdk J_m(k\rho) J_m(k\rho') \frac{e^{i\kappa|z|}}{\kappa}$$

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|} = \frac{2}{\pi} \sum_m \int_0^\infty dk e^{im(\varphi-\varphi')} \cos k(z-z') I_m(k\rho_<) K_m(k\rho_>)$$

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|}=\sum_m e^{im(\varphi-\varphi')}\int_0^\infty dk e^{-k(z_>-z_<)}J_m(k\rho)J_m(k\rho')$$

$$\frac{e^{i\frac{\omega}{c}\sqrt{r^2+z^2}}}{\sqrt{r^2+z^2}}=\sum_{\mathbf{k}}\frac{2\pi i}{\kappa}e^{i\mathbf{kr}}e^{i\kappa|z|}$$

**Infinite cylindrical hole** plasmons

$$\omega^2=\omega_p^2\left[1-\kappa a K_m(\kappa a)I_m^{'}(\kappa a)\right]$$

**Infinite cylindrical rod**

$$\omega^2=\omega_p^2\left[1+\kappa a I_m(\kappa a)K_m^{'}(\kappa a)\right]$$

Diffraction: **No**

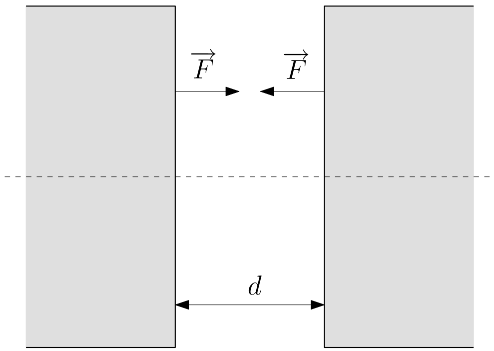
Circular aperture and circular disk: **No**

Finite cylindrical hole and rod: **No**

**PART III**

## Electromagnetic Eigenmodes

## Van der Waals - London - Casimir Forces



Coupled integral equations for the dispalcement $\mathbf{u}$

Their eigenmodes $\Omega_\alpha$

Non-retarded regime

$$\left(\omega^2 - \frac{1}{2}\omega_1^2\right)\left(\omega^2 - \frac{1}{2}\omega_2^2\right) - \frac{1}{4}\omega_1^2\omega_2^2 e^{-2kd} = 0$$

Force

$$F = \frac{\partial}{\partial d}\sum_\alpha \frac{1}{2}\hbar\Omega_\alpha$$

**van der Waals - London** (two identical metals, per unit area)

$$F \simeq \hbar\omega_p/2\pi\sqrt{2}d^3$$

Similar $d^{-3}$ for any pair of bodies (two molecules $\sim 1/R^7$).

**Retarded regime**: dispersion equations $\kappa_{1,2} = \sqrt{\kappa^2 - \omega_{1,2}^2/c^2}$

$$e^{2i\kappa d} = \frac{(\kappa_1 + \kappa)(\kappa_2 - \kappa)}{(\kappa_1 - \kappa)(\kappa_2 + \kappa)}$$

$$e^{2i\kappa d} = \frac{(\kappa_1 + \kappa)(\kappa_2 - \kappa)(\kappa\kappa_1 + k^2)(\kappa\kappa_2 - k^2)}{(\kappa_1 - \kappa)(\kappa_2 + \kappa)(\kappa\kappa_1 - k^2)(\kappa\kappa_2 + k^2)}$$

Solutions

$$\Omega_\alpha = c\sqrt{k^2 + \frac{\pi^2 x_n^2}{d^2}}$$

For surface plasmon-polariton modes (in metals) or for identical bodies; bound cds

Standard renormalization procedure leads to **Casimir force** (per unit area)

$$F = \frac{\pi^2 \hbar c}{240 d^4}$$

Similar $d^4$ for other pairs of bodies, except for distinct dieletrics

**Comment**: original Casimir argument; fluctuation theory

**Point-like particle** and a semi-infinite body $F = \frac{3\hbar\omega_p}{8\sqrt{2}} \cdot \frac{\alpha a^3}{d^4}$ (non-retarded); no force when retardation included

## Part IV

# Conclusion

Some issues in Classical Electromagnetism:

1 Fresnel-Huygens interference in refraction (extinction theorem Ewald-Oseen 1915)?

2 Effective medium theory, $\varepsilon$ problem?

3 Origin of $\varepsilon$?

Answered all, understood none!

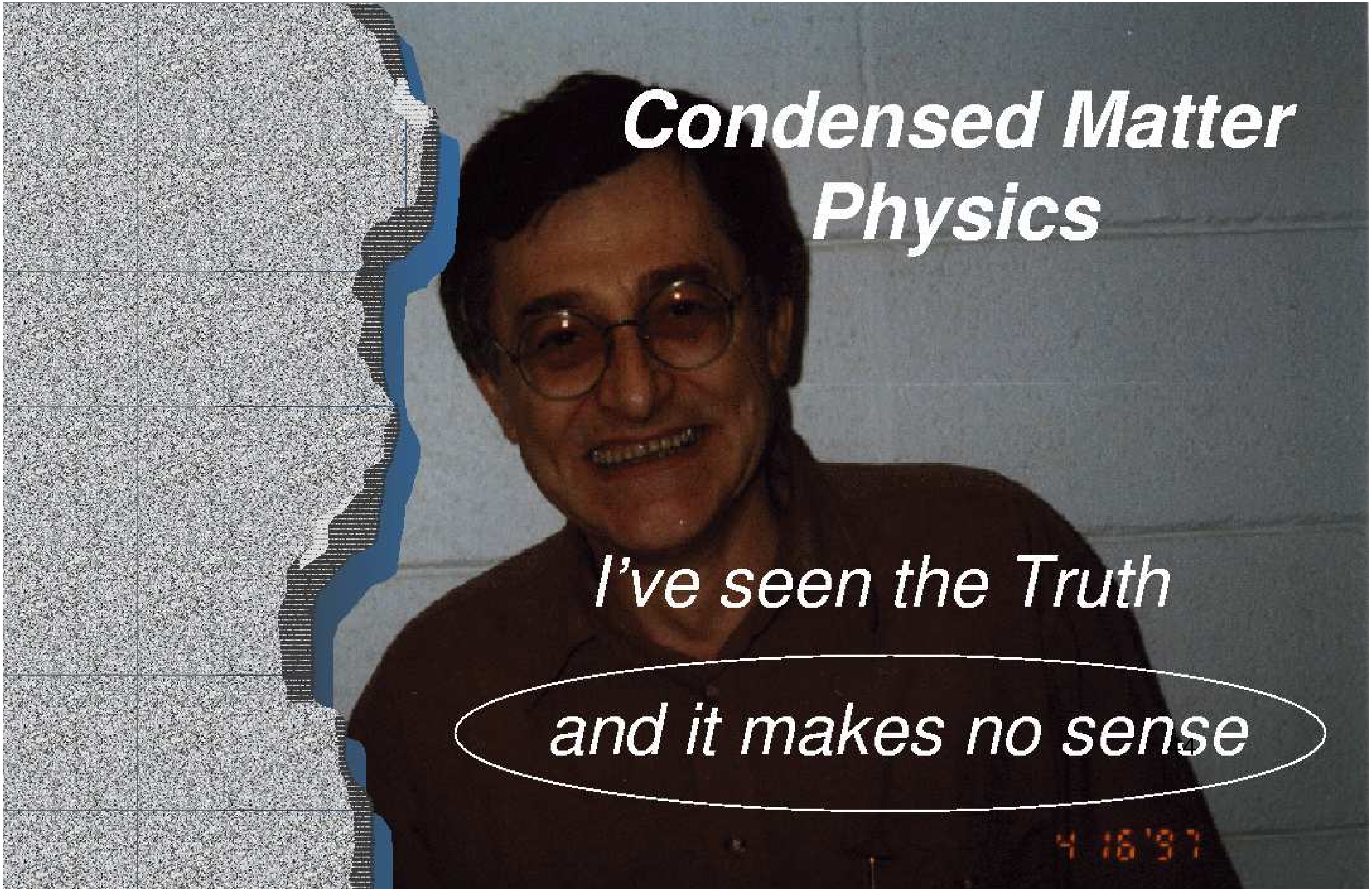
_ Introduced $\mathbf{u}$ and its equation of motion (Newton's) (equation of motion for polarization $\mathbf{P}$)

_similar for magnetization $\mathbf{M}$

_rewriting Maxwell equations in matter with $\mathbf{u}(\mathbf{P})$ and $\mathbf{M}$

_Fully determined system of eqs, no need for phenomenological $\varepsilon$ and $\mu$

_**INTEGRAL EQUATIONS**


Condensed Matter
Physics
I've seen the Truth
and it makes no sense
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