Magnetic moments

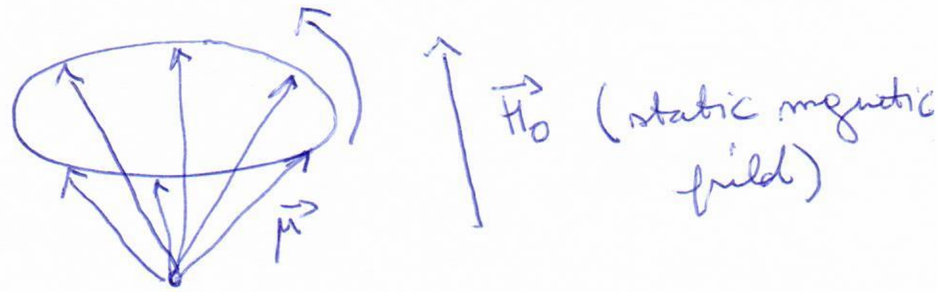
Particles: electron, atom, molecule, nucleus, proton, neutron, ...

Particles may have a magnetic moment $\vec{\mu}$

small magnet

Motion of a magnetic moment

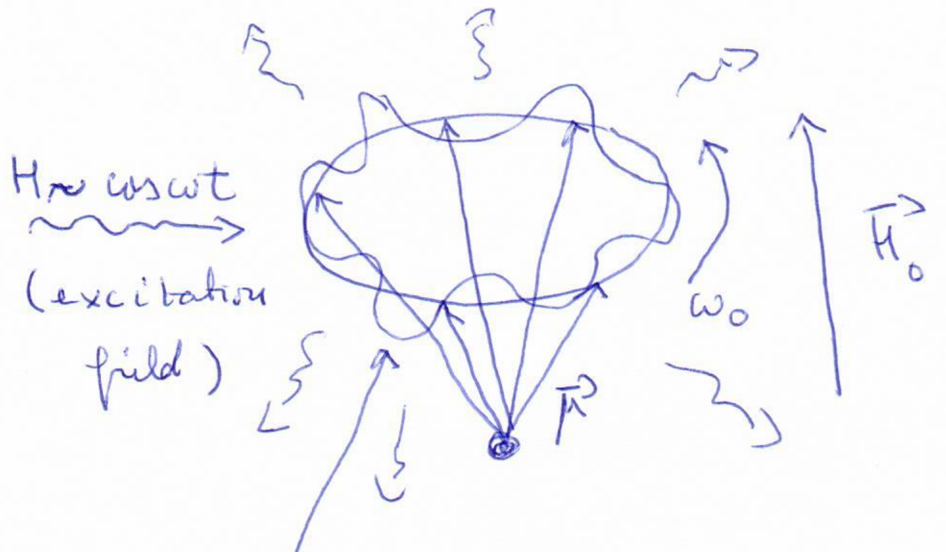
Like a top



Rotation frequency $\omega_0 \sim H_0$

e.g. proton, $H_0 = 1\,Ts \rightarrow \omega_0(\nu) \sim 40\,MHz$

A time-dependent magnetic field



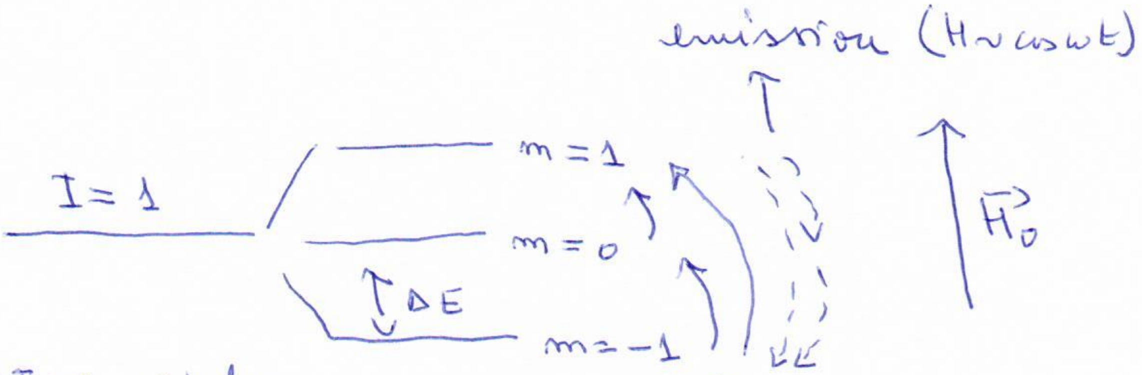
precession with frequency ω_0!

The moment behaves as a harmonic oscillator, with eigenfrequency ω_0 and oscillating frequency ω.

- Absorption of energy from H
- A magnetic moment in motion absorbs and emits energy
- Emission of energy (oscillating magnetic field, induction in a coil, receiving antenna)
- Resonance $\omega \cong \omega_0$?

Quantum Mechanics

- Motion cannot be visualized
- Spin (angular momentum) I (number)
- $\mu \sim I$ ($N = 1$, $Cl = 3/2$, $O = 5/2$)
- Constraints : quantum states



$2I+1$ - States

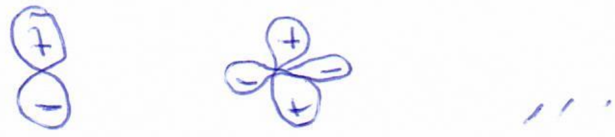
(splitting of energy levels) absorption ($H \sim \cos \omega t$)

- $\Delta E \sim \omega_0$ (quantum jumps, transitions)

This is the principle of the NMR.

Electric moments

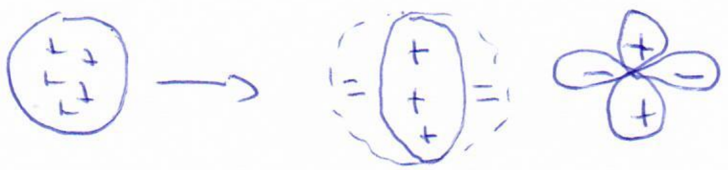
- Electric charge distribution



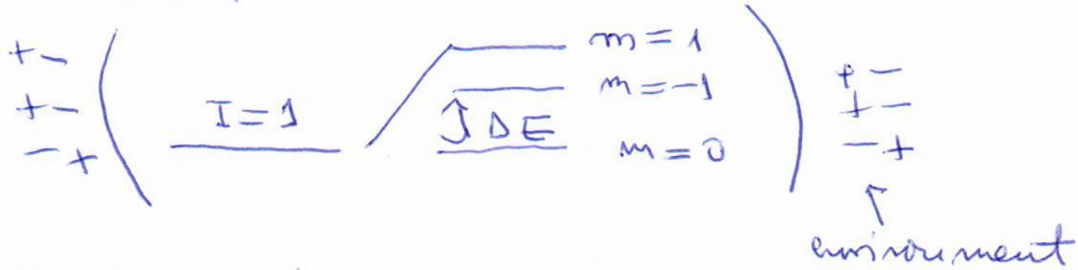
Dipole   Quadrupole

- Electron has no dipole, some atoms have, molecules have, the nuclei do not have

- Some nuclei do have a quadrupole (No $I=0$, $I=1/2$)

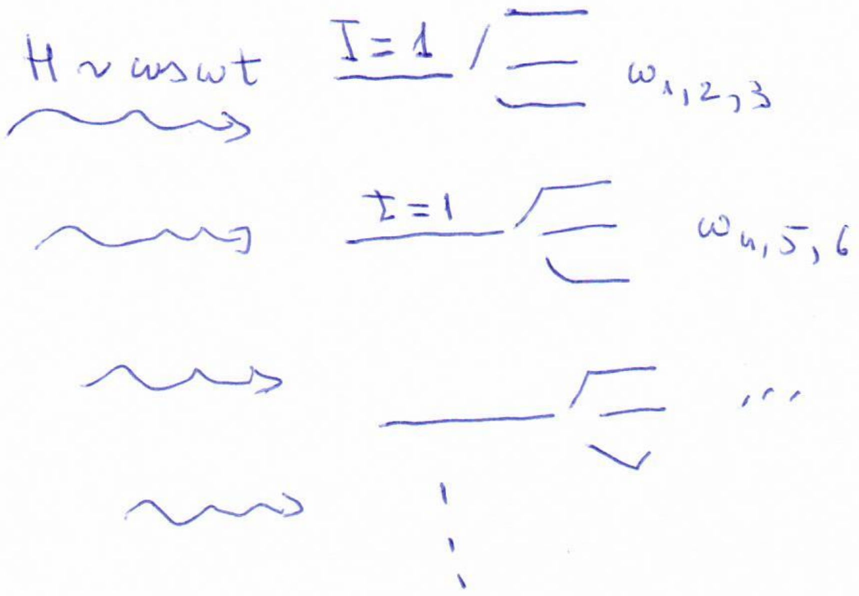


- Microscopic (atomic, chemical) environment acts upon the nucleus quadrupole and splits the magnetic levels!



- The role of H_0 in NMR is played by the chemical environment in NQR
- $\Delta E \sim \omega_0$ depends on the environment
- Several ω_0, inequivalent structural sites (the problem with the TNT!)

NQR



– Several resonances $\omega \sim \omega_1, \omega_2, \omega_3, \ldots = \omega_0$
(RF, $\omega_0 \sim 1-5\,MHz$)

Absorption, emission, relaxation

T_e, t_e

↑

excitation

- $t_e > T_e$ – excitation
- Stop excitation ⟹ free induction decay
- Loss of energy – relaxation time τ

  $T_e < t_e < \tau$
- τ not too long → no emission.

- Another problem
  - thermal population $\frac{N_1}{N_0} \sim e^{-\beta \Delta E}$ $(\beta = 1/T)$.
  - since ΔE small ⟹ almost equal populations ⟹

    ⟹ low efficiency (sensitivity)

Emission process: stimulated, spontaneous

After a long exc – depopulation of E_0 (increase T)

- Consequently: a method of improving efficiency

  Excite long, stop, wait; regular excitation
  in pulses → free decay

Spin echo

$\overset{\omega}{\rightsquigarrow}$ $\cos\omega t$ (signal)

After time τ another pulse

$$\cos\omega\tau \cos\omega(t-\tau) \sim \cos(t-\underline{2\tau})$$

After time 2τ another pulse

$$\cos 2\omega\tau \cos\omega(t-2\tau) \sim \cos(t-\underline{4\tau})$$

- - - -

Optimized with respect to pulse duration and time relaxation

(Another problem with the TNT

- NQR does not appear in gases, nor liquids
- NQR frequencies shift with the temperature
- Low signal (NQR does not appear in amorphous materials)

$\Uparrow$

problems

Double resonance (polarization)

- Put a static magnetic field H_0

  $\Downarrow$

  Spin polarization, splitting of the states $H_0 \sim \omega_0$ (10–20 MHz)

- Reduce the field up to $\omega_0 \sim \omega_{NQR}$!

  $\Downarrow$

  Increase of population of the NQR-excited states

  $\Downarrow$

  increase of the signal

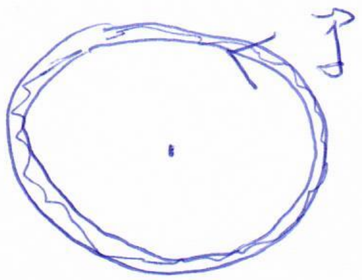
- Switch off H_0!

- Do NQR.

- Disadvantage: if T_1 long, weak signal.
  if T_1 short, no signal

Superconductivity. SQUID

- Permanent current



Superconducting ring (low temperature)

$$\vec{p} \to \vec{p} - \frac{q}{c}\vec{A}$$

$$\psi \sim e^{i\theta} \sim e^{-\frac{i}{\hbar}\frac{q}{c}\int \vec{A}\,d\vec{r}}$$

$$\theta = \frac{q}{c\hbar}\oint \vec{A}\,d\vec{r} = \frac{q}{c\hbar}\iint \operatorname{curl}\vec{A}\cdot d\vec{s} =$$

$$= \frac{q}{c\hbar}\iint \vec{H}\,d\vec{s} = \frac{q\phi}{c\hbar} = 2\pi n$$
(n integer)

$\phi = SH$ - magnetic flux

$q \to 2e$ (electron pairs)

$$\boxed{\begin{aligned} \theta &= \frac{2e\phi}{c\hbar} \\ \phi_n &= \frac{ch}{2e}n \end{aligned}}$$

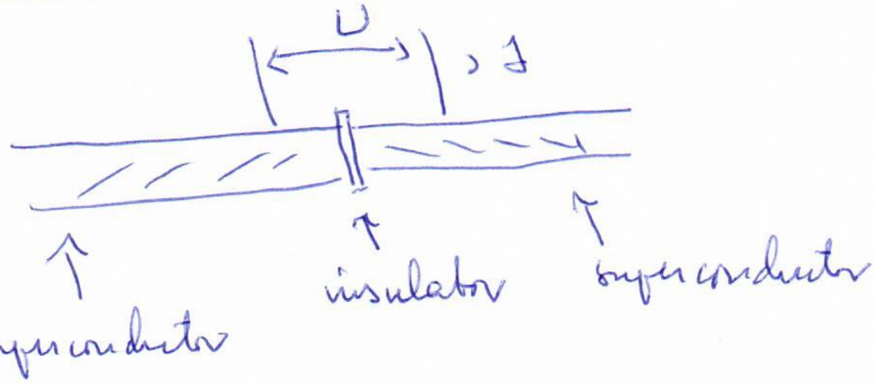
<u>Maxwell</u> $\mathrm{curl}\,\vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t}$

$$\iint \mathrm{curl}\,\vec{E}\, d\vec{S} = \oint \vec{E}\, d\vec{l} = U =$$

$$= \frac{1}{c}\iint \frac{\partial \vec{H}}{\partial t}\, d\vec{S} = \frac{1}{c}\frac{\partial \Phi}{\partial t}$$

$$\boxed{\frac{\partial \theta}{\partial t} = \frac{2e}{c\hbar}\frac{\partial \Phi}{\partial t} = \frac{2e}{\hbar} U}$$

<u>Josephson junction</u>



$$J \sim -\mathrm{Re}\, i e^{i\Delta\theta} = \sin \Delta\theta$$

$$\frac{\partial \theta}{\partial t} = \frac{2e}{\hbar} U, \quad \theta = \frac{2e}{\hbar} U t$$

$$J \sim \sin \frac{2e}{\hbar} U t \sim \sin \frac{2e}{\hbar c}\Phi$$

- Increase the flux, increases current, continue increase the flux, current decreases, continue, current restored $\to$ good measure of small H!
- Etalon for U

- dc - SQUID

$$J \sim \sin \frac{2e}{\hbar c} \varphi = \sin \frac{2eS}{\hbar c} H.$$

- ac - SQUID

$$H = H_0 \sin \omega t$$

$$\varphi = S H_0 \sin \omega t$$

$$\theta = \frac{2e}{c\hbar} S H_0 \sin \omega t$$

$$J \sim \sin \frac{2eS}{c\hbar} H_0 \sin \omega t$$

Detection of H, very sensitive,

since $\frac{2eS}{c\hbar} \cong 3 \cdot 10^{7} / Gs \ (S = 1\,cm^2) \Rightarrow$

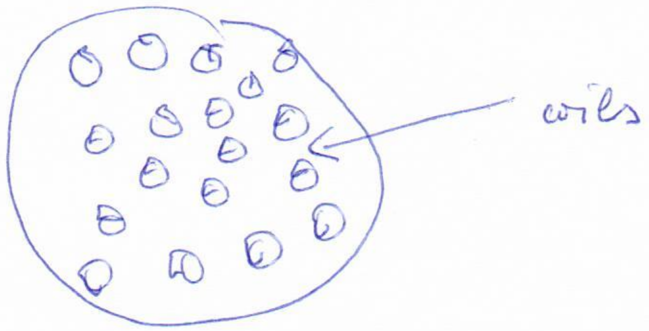
$\Rightarrow$ very large $\Rightarrow$ small H_0 !

SQUID - Superconducting Quantum Interference Device

- Low noise ($\sim 1/\omega$), low frequency, high noise
  SQUID - especially for low frequency

Receiving antenna

- High volume (high noise)
- Decorated antenna



- Keep the excitation phase, use the long axis, apply the regular pulse excs, possibly echo-spin technique

Atomic magnetometers

- Vapours of alkali metals
- Send a laser beam to excite the atoms
  $\Rightarrow$ create induced magnetic moments $\vec{\mu}$
- Put an $\vec{H}$; splits the level
- Another laser pulse detect the rotation of the polarization
- (Or do the NMR)

The problem with the TNT

① – Long T_1 (τ_1) – spin-lattice relaxation
weak spin-lattice coupling

② – Broadening of lines, short T_2 (transverse time)
Strong inter-molecular coupling (heat)

③ – Many lines, low frequency

① gives weak signal
② gives weak & broad signal
③ weak detection

① do not dis-excite, do not give energy, weak signal
② excite molecules

$$\frac{dN_1}{dt} = BN_0\rho - BN_1\rho - AN_1$$

$$\frac{dN_0}{dt} = BN_1\rho + AN_1 - BN_0\rho$$

$$N_0 + N_1 = N = \text{cnt}$$

Eq $$B(N_0 - N_1)\rho = AN_1$$

$$B\left(e^{\beta\Delta E} - 1\right)\rho = A$$

$$\rho = \frac{A}{B}\frac{1}{e^{\beta\Delta E}-1} = \frac{C}{e^{\beta\Delta E}-1} \; ; \; C = \frac{8\pi h\nu^3}{c^3}$$

$$\frac{d\delta N_1}{dt} = (BN_0 - BN_1)\delta\rho + (\underbrace{B\delta N_0 - B\delta N_1}) \rho - A\delta N_1 =$$

$$= \frac{AN_1}{\rho}\delta\rho + B\delta N_0 \rho - (B\rho + A)\delta N_1 =$$

$$= \frac{AN_1}{\rho}\delta\rho - 2B\delta N_1\rho - A\delta N_1 =$$

$$= \frac{AN_1}{\rho}\delta\rho - (2B\rho + A)\delta N_1 =$$

$$= C_1 - C_2\,\delta N_1$$

$$\delta N_1 = \alpha e^{-C_2 t} + \frac{C_1}{C_2} = \frac{C_1}{C_2}\left(1 - e^{-C_2 t}\right)$$

$$\delta N_1 = \frac{AN_1\delta\rho}{\rho}\frac{1}{2B\rho + A}\left(1 - e^{-(2B\rho + A)t}\right)$$

$$2B\rho + A = \frac{2A}{e^{\beta\Delta E}-1} + A = A \cdot \frac{1 + e^{\beta\Delta E}}{e^{\beta\Delta E}-1}$$

$$\delta N_1 = \frac{A N_1 \delta\rho}{c}\left(e^{\beta\Delta E}-1\right)\frac{e^{\beta\Delta E}-1}{A\left(e^{\beta\Delta E}+1\right)}\cdot\left(1-e^{-A\frac{1+e^{\beta\Delta E}}{e^{\beta\Delta E}-1}t}\right)$$

$$= N_1\delta\rho\,\frac{\left(e^{\beta\Delta E}-1\right)^2}{c\left(e^{\beta\Delta E}+1\right)}\left(1-e^{-A\frac{e^{\beta\Delta E}+1}{e^{\beta\Delta E}-1}\cdot t}\right)$$

$$N_1 = N_0\, e^{-\beta\Delta E} = (N-N_1)\,e^{-\beta\Delta E}$$

$$\left(1+e^{-\beta\Delta E}\right)N_1 = N\,e^{-\beta\Delta E}$$

$$N_1 = \frac{N}{e^{\beta\Delta E}+1}$$

$$\delta N_1 = \frac{N}{c}\delta\rho\left(\frac{e^{\beta\Delta E}-1}{e^{\beta\Delta E}+1}\right)^2\left(1-e^{-A\frac{e^{\beta\Delta E}+1}{e^{\beta\Delta E}-1}\cdot t}\right)\cong$$

$$\cong \frac{N}{4c}\delta\rho\cdot(\beta\Delta E)^2\,\frac{2A}{\beta\Delta E}\,t = \frac{NB}{2\cancel{B}}\delta\rho\,(\beta\Delta E)\,t =$$

$$= \frac{NBc}{2\cancel{B}}\frac{\delta\rho}{\rho}\,t = \frac{NA}{2\cancel{B}}\frac{\delta\rho}{\rho}\,t$$

$$t\to\infty \quad \delta N_1 \sim \frac{N}{4c}\delta\rho\,(\beta\Delta E)^2 \cong \frac{N}{4}$$

$$\frac{N}{e^{\beta\Delta E}+1} = N_1+\frac{N}{4} = \frac{5N}{4},\quad \frac{4}{5} = e^{\beta\Delta E}+1$$

$$\delta N_1 = -\frac{N\,\delta\beta\Delta E\, e^{\beta\Delta E}}{\left(e^{\beta\Delta E}+1\right)^2} \cong -\frac{N\,\delta\beta\Delta E}{4} = \frac{N}{4}$$

$$\delta\beta \cong -1/\Delta E,\quad \delta T = T^2/\Delta E$$