INSTITUTE of ATOMIC PHYSICS
Magurele-Bucharest

PULSE and IMPULSE of ELI (VI)

("Extreme Light Infrastructure")

Electron-Positron Pairs Created from Vacuum by External Fields

M Apostol

Institute of Physics and Nuclear Engineering,
Magurele-Bucharest

December 2010

General, Common Belief:

High external em fields generate el-pos pairs from vacuum (in certain circumstances)

The vacuum gets polarized (like a plasma?)

(and has a refractive index)

Focused laser beams may give such high em fields (see ELI) (?)

Questions: How high, what spatial extension, how many pairs, what happens after generation, what we possibly can measure, the meaning?

The Generation of the "Pair Generating" Idea

1 Dirac 1930, Spend $2mc^2$ to create a pair (free, at rest) ($\hbar\omega = 2mc^2$) (not quite!)

2 Bound electrons (in atoms), lower energy, less spending $< 2mc^2$

3 Strongly bound els (large Z), crossover to negative energies ($\varepsilon < -mc^2$), spontaneous creation of positrons (an empty level is a hole in the negative sea, *i.e.* a pos!); unstability in "higher than $1/r$ potentials" ("fall on the centre"); $Z > 137$

QED cannot accept such a spontaneous creation ("open sysems", undefined, unstable), avoid them (eg, $Z < 137$)

Such phenomena are not, in fact, in the field of study of QED

QED does not deal with them

They signify the limits of the QED

We have not tools for dealing with them

If ELI experiments aim at this, then they aim at something which does not exist within the QED

Another Theory? A New Physics?

4 Constant H; pairs created by γ (high energy) (magnetic synchrotron radiation) (**spatial non-uniformities**)

5 H high enough: spontaneous creation of positron (empty negative state), again!

6 Estimation (wrong): $\frac{e\hbar}{mc}H = mc^2$, $H = 10^{13} Gs$!!

7 Why wrong: because $H \simeq H_{internal}$!

8 Bethe&Heitler 1934, Pair production by a photon in the field of nuclei (inverse bremsstrahlung; collisions); el-pos created from γ! (**spatial non-uniformities!**)

9 Uehling&Serber 1935, Vacuum polarization, screened Coulomb law (**weak fields**) (el-pos "created" from the ext field!); Sauter 1931

10 Heisenberg&Euler 1935, Corrections to Maxwell equations, non-linear effects (uniform, constant, **weak fields**); el-pos "created" from the ext field!

11 Schwinger 1951, Vacuum polarization (**weak fields**); similarly; **Schwinger limit**

Estimation of “high” fields: (wrong !!) $E^2\lambda_c^3 \sim mc^2$ ($\lambda_c = 0.3 \times 10^{-10} cm$ Compton wavelength), $E \simeq 10^{16} V/m$

Another wrong estimation $eE\lambda_c \sim mc^2$, $E \simeq 10^{18} V/m$!! (common)

Why wrong: $E_{intern} = \frac{e}{\lambda_c^2} = 10^{16} V/m$: electron would disintegrate !

ELI Lasers

$W = 10^3 J$, $\tau = 1 pico\,(10^{-12} s)$; pulse $d = 3 \times 10^{-2} cm\,(300\mu)$, $P = 1 petaW$

$E \simeq 10^{12} V/m$ - insufficient for those **high and wrong** estimations ($10^{16} V/m$)

Increase power to $100 pW$!!

12 Another idea (personal): Send a γ into high em field (not so high!) to get el-pos pairs

Summary

$\gamma = \hbar\omega$ cannot create a free el-pos pair $2mc^2$ (or higher); conservation laws

External em fields may help, they modify the el-pos states; vacuum gets polarized; this is valid for **weak fields**

Caution: high external fields lead to instabilities, spontaneous pair creation, (crossover of the levels) (such things do not exist for QED)

If so, worlds would be created from nothing; and ours would forever perish!

ELI wouldnt push into this directon. Good that $100 petaW$ is unreacheable!

A Few Simple Observations

1 All the above considerations are for quantum transitions (which we measure)

2 External fields are always classical (when pure, coherent!)

3 **A classical em field can pump any energy levels**

4 We may have pairs!!

5 Irrespective of field intensity

6 High fields are not "dangerous", due to recombination

7 Stationary el-pos plasma

8 Energy conservation is enough: $W = N \times 2mc^2$

9 E depends on the volume, N does not! (what about focalization?)

10 ELI $W = 10^3 J$, Number of pairs $N \simeq W/2mc^2 = 10^{16}$ (separation $10^5 \lambda_c$)

Therefore

Focus some laser energy W over some volume d^3; create el-pos pairs; they recombine; a stationary regime? (el-pos plasma?); an internal polarizaton field; vacuum polarization (refractive index)

Another important Note: do not stick with the rate of production ($1st$ order perturbation theory), because the pairs annihilate (recombine)

So, indeed we may have a **New Theory**, different from the QED

It is the **Coherent Pair Production Theory** in the next slides

Radiation

$$\mathbf{A}(\mathbf{r}) = \sum_{\mu\mathbf{k}} \sqrt{\frac{2\pi\hbar c^2}{V\omega_k}} \left[\mathbf{e}_\mu(\mathbf{k}) a_{\mu\mathbf{k}} e^{i\mathbf{kr}} + \mathbf{e}^*_\mu(\mathbf{k}) a^*_{\mu\mathbf{k}} e^{-i\mathbf{kr}}\right]$$

$\mathbf{E} = -(1/c)\partial\mathbf{A}/\partial t$, $\mathbf{H} = curl\,\mathbf{A}$, three Maxwell's equations are satisfied: $curl\mathbf{E} = -\frac{1}{c}\partial\mathbf{H}/\partial t$, $div\mathbf{H} = 0$, $div\mathbf{E} = 0$.

$$L_f = \tfrac{1}{8\pi} \int d\mathbf{r} \left(E^2 - H^2\right) =$$

$$= \sum_{\mu\mathbf{k}} \tfrac{\hbar}{4\omega_k} \left(\dot{a}_{\mu\mathbf{k}} + \dot{a}^*_{-\mu-\mathbf{k}}\right) \left(\dot{a}_{-\mu-\mathbf{k}} + \dot{a}^*_{\mu\mathbf{k}}\right) -$$

$$- \sum_{\mu\mathbf{k}} \tfrac{\hbar\omega_k}{4} \left(a_{\mu\mathbf{k}} + a^*_{-\mu-\mathbf{k}}\right) \left(a_{-\mu-\mathbf{k}} + a^*_{\mu\mathbf{k}}\right)$$

gives the eq of motion

$$\ddot{a}_{\mu\mathbf{k}} + \ddot{a}^*_{-\mu-\mathbf{k}} + \omega_k^2 \left(a_{\mu\mathbf{k}} + a^*_{-\mu-\mathbf{k}} \right) = 0 \ ,$$

which is the fourth Maxwell's equation $curl\mathbf{H} = (1/c)\partial\mathbf{E}/\partial t$.

Standard Dirac Field

$$\psi(\mathbf{r}) = \sum_{\sigma\mathbf{p}} \frac{1}{\sqrt{2\varepsilon V}} \left(u_{\mathbf{p}\sigma} b_{\mathbf{p}\sigma} e^{\frac{i}{\hbar}\mathbf{pr}} + v_{\mathbf{p}\sigma} c^*_{\mathbf{p}\sigma} e^{-\frac{i}{\hbar}\mathbf{pr}} \right) \quad ,$$

$\varepsilon = \sqrt{c^2p^2 + m^2c^4}$.

$$u_{\mathbf{p}\sigma} = \begin{pmatrix} \sqrt{\varepsilon + mc^2} w_\sigma \\ \sqrt{\varepsilon - mc^2} (\mathbf{n}\overrightarrow{\sigma}) w_\sigma \end{pmatrix} \ , \ v_{\mathbf{p}\sigma} = \begin{pmatrix} \sqrt{\varepsilon - mc^2} (\mathbf{n}\overrightarrow{\sigma}) w'_\sigma \\ \sqrt{\varepsilon + mc^2} w'_\sigma \end{pmatrix} \ ;$$

$\mathbf{n} = \mathbf{p}/p$, $\overrightarrow{\sigma}$ Pauli matrices, $w'_\sigma = -\sigma_y w_{-\sigma}$, $w^*_\sigma w_{\sigma'} = \delta_{\sigma\sigma'}$, $w'^*_\sigma w'^*_\sigma = \delta_{\sigma\sigma'}$
.

$$H_0 = \sum_{\sigma\mathbf{p}} \varepsilon \left(b^*_{\mathbf{p}\sigma} b_{\mathbf{p}\sigma} - c_{\mathbf{p}\sigma} c^*_{\mathbf{p}\sigma} \right) \quad ,$$

eq. of motion $i\hbar\dot{b}_{\mathbf{p}\sigma} = \varepsilon b_{\mathbf{p}\sigma}$, $i\hbar\dot{c}_{\mathbf{p}\sigma} = \varepsilon c_{\mathbf{p}\sigma}$ (Schrodinger).

Interaction

$$H_{int} = -\frac{e}{c}\int d\mathbf{r}\psi^*(\mathbf{r})\mathbf{j}\psi(\mathbf{r})\mathbf{A}(\mathbf{r}) \ ,$$

current $\mathbf{j} = c\overrightarrow{\alpha}$,

$$\overrightarrow{\alpha} = \begin{pmatrix} 0 & \overrightarrow{\sigma} \\ \overrightarrow{\sigma} & 0 \end{pmatrix}$$

The current differs from the group velocity $c^2\mathbf{p}/\varepsilon$ of the free electrons (positrons) - interaction!

Matrix elements $M^{\mu}_{\sigma\sigma'}(\mathbf{p},\mathbf{p}')$

They contain the matrix elements $(\overrightarrow{\overline{\sigma}})_{\sigma\sigma'} = w^*_\sigma\overrightarrow{\sigma}w_{\sigma'}$, $(\overrightarrow{\overline{\sigma}})_{\sigma\sigma'}(\mathbf{p},\mathbf{p}') = w^*_\sigma(\mathbf{p})\overrightarrow{\sigma}w_{\sigma'}(\mathbf{p}')$

First difficulty: arbitrariness in these matrix elements, due to the arbitrariness in the spinors $w_\sigma(\mathbf{p})$

There is no reason to have a "spin" dependence in the interaction

$M(\mathbf{p},\mathbf{p}') = \sum_{\mu\sigma\sigma'} M^{\mu}_{\sigma\sigma'}(\mathbf{p},\mathbf{p}')$

$$H_{int} = -ec\sum_{\mathbf{pk}}\sqrt{\frac{2\pi\hbar}{V\omega_k}}[A(\mathbf{p},\mathbf{p}-\mathbf{k})b^*_{\mathbf{p}}b_{\mathbf{p}-\mathbf{k}} + B(\mathbf{p},-\mathbf{p}+\mathbf{k})b^*_{\mathbf{p}}c^*_{-\mathbf{p}+\mathbf{k}}+$$

$$+B^*(-\mathbf{p}-\mathbf{k},\mathbf{p})c_{\mathbf{p}}b_{-\mathbf{p}-\mathbf{k}} + C(\mathbf{p},\mathbf{p}+\mathbf{k})c_{\mathbf{p}}c^*_{\mathbf{p}+\mathbf{k}}]\left(a_{\mathbf{k}} + a^*_{-\mathbf{k}}\right) \quad ,$$

Solution: take the mean value of $(\overrightarrow{\overline{\sigma}})_{\sigma\sigma'}$ over all possible polarizations, and get $(\overrightarrow{\overline{\sigma}})_{av} = 0$

This amounts to a statistical (uniform) average of the interaction hamiltonian over "spin" states

$$H_{int} = -ec\sum_{\mathbf{pk}}\sqrt{\frac{2\pi\hbar}{V\omega_k}}[A(\mathbf{p},\mathbf{p}-\mathbf{k})b^*_{\mathbf{p}}b_{\mathbf{p}-\mathbf{k}} + B(\mathbf{p},-\mathbf{p}+\mathbf{k})b^*_{\mathbf{p}}c^*_{-\mathbf{p}+\mathbf{k}}+$$

$$+B(\mathbf{p},-\mathbf{p}-\mathbf{k})c_{\mathbf{p}}b_{-\mathbf{p}-\mathbf{k}} + A(\mathbf{p},\mathbf{p}+\mathbf{k})c_{\mathbf{p}}c^*_{\mathbf{p}+\mathbf{k}}]\left(a_{\mathbf{k}} + a^*_{-\mathbf{k}}\right) \ ,$$

where

$$A(\mathbf{p},\mathbf{p}') = \frac{1}{\sqrt{\varepsilon\varepsilon'}}\left[\mathbf{en}\sqrt{(\varepsilon - mc^2)(\varepsilon' + mc^2)} + \mathbf{en}'\sqrt{(\varepsilon + mc^2)(\varepsilon' - mc^2)}\right] \ ,$$

$$B(\mathbf{p},\mathbf{p}') = -\frac{1}{\sqrt{\varepsilon\varepsilon'}}\{e_y\sqrt{(\varepsilon + mc^2)(\varepsilon' + mc^2)}+$$

$$+[(\mathbf{en})n'_y - (\mathbf{nn}')e_y + (\mathbf{en}')n_y]\sqrt{(\varepsilon - mc^2)(\varepsilon' - mc^2)}\}$$

and $\varepsilon' = \sqrt{c^2 p'^2 + m^2 c^4}$, $\mathbf{e} = \sum_\mu \mathbf{e}_\mu(\mathbf{k})$

Note the electron-electron, positron-positron interactions (the terms with the coefficients $A(\mathbf{p}, \mathbf{p} \mp \mathbf{k})$) and the creation and annihilation of pairs (the terms with the coefficients $B(\mathbf{p}, -\mathbf{p} \pm \mathbf{k})$)

Eqs. of motion

$$\ddot{a}_{\mathbf{k}} + \ddot{a}^*_{-\mathbf{k}} + \omega_k^2(\ddot{a}_{\mathbf{k}} + \ddot{a}^*_{-\mathbf{k}}) = 2ec \sum_{\mathbf{p}} \sqrt{\frac{2\pi\omega_k}{\hbar V}} [A(\mathbf{p}, \mathbf{p} + \mathbf{k}) b^*_{\mathbf{p}} b_{\mathbf{p}+\mathbf{k}} +$$

$$+ B(\mathbf{p}, -\mathbf{p} - \mathbf{k}) b^*_{\mathbf{p}} c^*_{-\mathbf{p}-\mathbf{k}} + B(\mathbf{p}, -\mathbf{p} + \mathbf{k}) c_{\mathbf{p}} b_{-\mathbf{p}+\mathbf{k}} + A(\mathbf{p}, \mathbf{p} - \mathbf{k}) c_{\mathbf{p}} c^*_{\mathbf{p}-\mathbf{k}}]$$

and

$$i\hbar\dot{b}_{\mathbf{p}} = \varepsilon_p b_{\mathbf{p}} -$$

$$-ec\sum_{\mathbf{k}}\sqrt{\frac{2\pi\hbar}{V\omega_k}}\left[A(\mathbf{p},\mathbf{p}-\mathbf{k})b_{\mathbf{p}-\mathbf{k}} + B(\mathbf{p},-\mathbf{p}+\mathbf{k})c^*_{-\mathbf{p}+\mathbf{k}}\right](a_{\mathbf{k}} + a^*_{-\mathbf{k}}) \ ,$$

$$i\hbar\dot{c}_{\mathbf{p}} = \varepsilon_p c_{\mathbf{p}} +$$

$$+ec\sum_{\mathbf{k}}\sqrt{\frac{2\pi\hbar}{V\omega_k}}\left[A(\mathbf{p}-\mathbf{k},\mathbf{p})c_{\mathbf{p}-\mathbf{k}} + B(-\mathbf{p}+\mathbf{k},\mathbf{p})b^*_{-\mathbf{p}+\mathbf{k}}\right](a_{\mathbf{k}} + a^*_{-\mathbf{k}}) \ .$$

Charge conservation $Q = \sum_{\mathbf{p}}\left(b^*_{\mathbf{p}}b_{\mathbf{p}} - c^*_{\mathbf{p}}c_{\mathbf{p}}\right)$

This is the **standard framework** (not manifestly covariant) provided by the quantum electrodynamics for an ensemble of interacting electrons, positrons and photons

Second difficulty: solving these equations

For the creation and annihilation of electron-positron pairs in the process of polarization of a piece of macroscopic vacuum we adopt here a special route.

Interacting fermions with spin one-half are admixtures of empty ($|0\rangle$) and occupied ($|1\rangle$) states

Creation and destruction operators can then be expressed by one scalar

For instance, let $|s\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

The destruction operator b has only one non-vanishing matrix element, $\langle 0|\, b\,|s\rangle = \beta$, or $\langle s|\, b\,|1\rangle = \alpha$

We choose $\langle 0|\, b\,|s\rangle = \beta$

The occupation number is given by $\langle s|\, b^* b\,|s\rangle = |\beta|^2$

Since the states $|s\rangle$ for an ensemble of interacting fermions are not, in general, well-defined single-particle states, $|\beta|^2$ is not subjected to the restriction $|\beta|^2 \leq 1$

Consequently, we can take such matrix elements

Which amounts to work with fermionic amplitudes

Which are c-numbers, instead of operators

These amplitudes can be viewed as classical fields

The charge conservation $Q = 0$ for pairs suggests the replacement

$$b_{\mathbf{p}} \to \beta_{\mathbf{p}} \ , \ c^*_{-\mathbf{p}} \to \beta_{\mathbf{p}} \ ,$$

in accordance with the particle-hole symmetry

Similarly, we replace the photon operators by c-numbers,

$$a_{\mathbf{k}} + a^*_{-\mathbf{k}} \to A_{\mathbf{k}} \ ,$$

where $A_{\mathbf{k}}$ are viewed as classical fields

This is the coherent states framework

The interaction hamiltonian becomes

$$H_{int} = -2ec \sum_{\mathbf{pk}} \sqrt{\frac{2\pi\hbar}{V\omega_k}} B(\mathbf{p}, -\mathbf{p}+\mathbf{k}) \beta^*_{\mathbf{p}} \beta_{\mathbf{p}-\mathbf{k}} A_{\mathbf{k}} \ ,$$

and the equations of motion read

$$\ddot{A}_{\mathbf{k}} + \omega_k^2 A_{\mathbf{k}} = 4ec \textstyle\sum_{\mathbf{p}} \sqrt{\frac{2\pi\omega_k}{\hbar V}} B(\mathbf{p}, -\mathbf{p}-\mathbf{k}) \beta^*_{\mathbf{p}} \beta_{\mathbf{p}+\mathbf{k}} \ ,$$

$$i\hbar\dot{\beta}_{\mathbf{p}} = \varepsilon_{\mathbf{p}} \beta_{\mathbf{p}} - 2ec \textstyle\sum_{\mathbf{k}} \sqrt{\frac{2\pi\hbar}{V\omega_k}} B(\mathbf{p}, -\mathbf{p}+\mathbf{k}) \beta_{\mathbf{p}-\mathbf{k}} A_{\mathbf{k}} \ .$$

Classical

The scattering of individual electrons (and positrons) disappears from the interaction hamiltonian (the terms with the A-coefficients)

The interaction is determined by the vacuum polarization (creation and annihilation of pairs), as expected

The product $\beta^*_{\mathbf{p}}\beta_{\mathbf{p}-\mathbf{k}}$ in the interaction hamiltonian can also be viewed as corresponding to the excitation (and dis-excitation) of an esemble of particles, each with two energy levels (labelled by $\mathbf{p}$ and $-\mathbf{p}+\mathbf{k}$), the levels corresponding to positive and, respectively, negative energy states

This latter feature is incorporated in the structure of the B-coefficients

Such an ensemble of particles can be excited (polarized) in a stationary regime by an external classical field of radiation, which pumps energy in the ensemble, resembling to some extent the laser effect

The product $\beta^*_{\mathbf{p}}\beta_{\mathbf{p}+\mathbf{k}}$ appearing in the *rhs* of the first eq. of motion (for the electromagnetic field) is related to the medium polarization (polarization current)

Limit to p, $\hbar k < p_0 \ll mc$, p_0 is a momentum cutoff

Expand $B(\mathbf{p}, \mathbf{p} \pm \mathbf{k})$ in powers of $\mathbf{p}$ and $\mathbf{k}$

Approximate $\varepsilon_\mathbf{p}$ by $\varepsilon_0 = mc^2$

Angular dependence of $B(\mathbf{p}, -\mathbf{p} + \mathbf{k})$ is irrelevant

Effective coupling coefficient

$$b = \left[\overline{B(\mathbf{p}, -\mathbf{p} - \mathbf{k}) B(\mathbf{p}, -\mathbf{p} + \mathbf{k})}\right]^{1/2} = \frac{p_0^2}{\sqrt{35} m \varepsilon_0}$$

Introduce the coupling constant $g_k = 2ecb\sqrt{2\pi/V\hbar\omega_k}$

$$\ddot{A}_{\mathbf{k}} + \omega_k^2 A_{\mathbf{k}} = 2\omega_k g_k \textstyle\sum_{\mathbf{p}} \beta^*_{\mathbf{p}} \beta_{\mathbf{p}+\mathbf{k}} \ ,$$

$$i\dot{\beta}_{\mathbf{p}} = \Omega\beta_{\mathbf{p}} - \textstyle\sum_{\mathbf{k}} g_k \beta_{\mathbf{p}-\mathbf{k}} A_{\mathbf{k}} \ ,$$

where $\Omega = \varepsilon_0/\hbar$

Solution

Fourier transforms

$$\beta_{\mathbf{p}} = \frac{1}{V}\int d\mathbf{r}\beta(\mathbf{r})e^{-\frac{i}{\hbar}\mathbf{pr}} \ , \ \beta(\mathbf{r}) = \sum_{\mathbf{p}} \beta_{\mathbf{p}} e^{\frac{i}{\hbar}\mathbf{pr}}$$

Number of pairs

$$N = 4\sum_{\mathbf{p}} |\beta_{\mathbf{p}}|^2 = \frac{4}{V}\int d\mathbf{r}\, |\beta(\mathbf{r})|^2$$

We have also

$$\sum_{\mathbf{p}} \beta^*_{\mathbf{p}} \beta_{\mathbf{p}+\mathbf{k}} = \frac{1}{V} \int d\mathbf{r}\, |\beta(\mathbf{r})|^2\, e^{-i\mathbf{k}\mathbf{r}}$$

Solution

$$\beta(\mathbf{r}) = B(\mathbf{r}) e^{-i\Omega t + i \int^t dt' \lambda(\mathbf{r}, t')}$$

$$\lambda(\mathbf{r}, t) = \sum_{\mathbf{k}} g_{\mathbf{k}} A_{\mathbf{k}} e^{i\mathbf{k}\mathbf{r}}$$

Number of pairs

$$N = \frac{4}{V} \int d\mathbf{r}\, |B(\mathbf{r})|^2$$

The pair dynamics is quasi-stationary, it conserves the "occupation" number $|\beta(\mathbf{r})|^2 = |B(\mathbf{r})|^2$

The polarization field **does not depend on time**,

$$A_{\mathbf{k}} = \frac{2g_k}{\omega_k}\frac{1}{V}\int d\mathbf{r}\, |B(\mathbf{r})|^2\, e^{-i\mathbf{k}\mathbf{r}}$$

It follows

$$\lambda(\mathbf{r},t) = \lambda(\mathbf{r}) = \sum_{\mathbf{k}} g_{\mathbf{k}} A_{\mathbf{k}} e^{i\mathbf{k}\mathbf{r}} = \sum_{\mathbf{k}} \frac{2g_k^2}{\omega_k}\frac{1}{V}\int d\mathbf{r}' \left|B(\mathbf{r}')\right|^2 e^{-i\mathbf{k}(\mathbf{r}'-\mathbf{r})}$$

and

$$\beta(\mathbf{r}) = B(\mathbf{r}) e^{-i\Omega t + i\lambda(\mathbf{r})t}$$

The single-particle energy $\hbar\Omega - \hbar\lambda(\mathbf{r})$ has a spatial dependence

Reflecting the local force exerted on the pairs by the polarization field

Pairs confined to a spatial region of finite extent, this force tends to localize the pairs in that region; stability (recall of spatial non-uniformities in QED!)

Assume the pairs distributed uniformly in space, *i.e.* $B(\mathbf{r}) = B = \sqrt{N}/2$

This amounts to a condensation of the fermions on the $\mathbf{p} = 0$ state

In fact, the pairs are distributed ("condensed") over the low-momenta fermionic states

The field $A_{\mathbf{k}}$ and the single-particle energy λ exhibit a singularity for $k \rightarrow 0$, as expected for such an infinite uniform distribution

In practice, the pairs are distributed quasi-uniformly in space over a region of finite linear size d, so we may take $A_{\mathbf{k}} \simeq g_k N/2\omega_k$ for $k < k_0 = 1/d$

The single-particle energy becomes

$$-\hbar\lambda \simeq -\frac{2e^2b^2}{\pi d}N \ ,$$

and $\beta_{\mathbf{p}} = Be^{-i\Omega t + i\lambda t}$ for $\mathbf{p} \to 0$

From the conservation of the number of particles

$$N = 4\sum_{\mathbf{p}} |\beta_{\mathbf{p}}|^2 = 4B^2 \frac{V}{(2\pi)^3\hbar^3}\frac{4\pi p_0^3}{3} = 4B^2$$

we get the momentum cutoff $p_0/\hbar = (6\pi^2)^{1/3}/d$

See now the coupling coefficient b,

$$b = \frac{p_0^2}{\sqrt{35} m \varepsilon_0} = \frac{(6\pi^2)^{2/3}}{\sqrt{35}} \left(\frac{\lambda_c}{d}\right)^2 \quad ,$$

where $\lambda_c = \hbar / mc$ is the electron Compton wavelength

Since $\lambda_c \simeq 0.3 \times 10^{-10} cm$, we can see that the coupling coefficient b acquires an extremely small value

The single-particle energy

$$-\hbar\lambda \simeq -\frac{2e^2 b^2}{\pi d} N = -\frac{12}{35} (6\pi^2)^{1/3} \frac{e^2}{d} \left(\frac{\lambda_c}{d}\right)^4 N \quad ,$$

is extremely small

Note the occurrence of the Coulomb energy e^2/d of an electron localized in a spatial region of linear size d

The magnetic field $\mathbf{H} = curl\mathbf{A}$ and the electromagnetic energy E_{em} stored by the polarization field $A_{\mathbf{k}} \simeq g_k N/2\omega_k$ for $k < k_0 = 1/d$

$$E_{em} = \frac{2e^2b^2}{\pi d}N^2 = \frac{12}{35}(6\pi^2)^{1/3}\frac{e^2}{d}\left(\frac{\lambda_c}{d}\right)^4 N^2$$

The number of pairs from the conservation of energy

$$E_{em} + 2mc^2N = \frac{12}{35}(6\pi^2)^{1/3}\frac{e^2}{d}\left(\frac{\lambda_c}{d}\right)^4 N^2 + 2mc^2N = W$$

where W (neglect the single-particle energy $-\hbar\lambda$)

The N^2-term brings an extremely small contribution (due to the fourth power of the ratio $\lambda_c/d \ll 1$)

So, the number of pairs $N \simeq W/2mc^2$

Numerical reference, $W = 1J$ and get $N \simeq 10^{13}$ pairs

The pairs number does not depend practically on the size of the spot where the energy is concentrated

Commenting further upon **solutions**

$$\beta(\mathbf{r}) = Be^{-i\Omega t + i\lambda t}$$

(for $r < d$)

They represent single-particle eigenstates

They correspond to electrons (positrons) localized in space

Divide the space in small, identical cells of volume $v \ll V$, and write the number of particles

$$N = \frac{4}{V}\int d\mathbf{r}\, |B(\mathbf{r})|^2 = \frac{4v}{V}\sum_{\mathbf{r}} B^2 = \frac{vN}{V}\sum_{\mathbf{r}} 1$$

One can see that the "occupation" number in each cell of volume v is unity, as for fermions

Therefore, the vacuum can be polarized with electron-positron pairs, which create a polarization field and acquire an additional $-\hbar\lambda$ energy for each electron (positron)

Even for very high energy densities the number of pairs, the polarization energy and the single-particle energies are extremely small

Comparing the first eq. of motion with the classical wave equation $\partial^2\mathbf{A}/\partial t^2 - c^2\Delta\mathbf{A} = 4\pi c\mathbf{j}$

The density of the polarization curent

$$j(\mathbf{k}) = \frac{ecb}{V}\sum_{\mathbf{p}}\beta^*_{\mathbf{p}}\beta_{\mathbf{p}+\mathbf{k}} = \frac{4ecb}{V^2}\int d\mathbf{r}\,|B(\mathbf{r})|^2\,e^{-i\mathbf{k}\mathbf{r}} \simeq$$

$$\simeq \frac{ecb}{V}N = \frac{1}{\sqrt{35}}(6\pi^2)^{2/3}\frac{ec}{V}\left(\frac{\lambda_c}{d}\right)^2 N\ ,\ k < 1/d\ ,$$

extremely small

Introduce explicitly an external field

$A^{ext}_{\mathbf{k}_0} = A^{ext}_{-\mathbf{k}_0} = A_0 = 2a_0 \cos \omega_0 t$

Eq. of motion

$$i\dot{\beta}_{\mathbf{p}} = \Omega\beta_{\mathbf{p}} - \sum_{\mathbf{k}} g_k \beta_{\mathbf{p}-\mathbf{k}} A_{\mathbf{k}} - g_0(\beta_{\mathbf{p}-\mathbf{k}_0} + \beta_{\mathbf{p}+\mathbf{k}_0})A_0$$

where $g_0 = 2ecb\sqrt{2\pi/V\hbar\omega_0}$

Solution

$$\beta(\mathbf{r}) = B(\mathbf{r})e^{-i\Omega t + i\lambda t + i\varphi(\mathbf{r},t)} \quad ,$$

where

$$\varphi(\mathbf{r},t) = \frac{4g_0 a_0}{\omega_0} \sin \omega_0 t \cos \mathbf{k}_0 \mathbf{r}$$

This phase implies a localized energy

$$\delta\varepsilon(\mathbf{r},t) = -4\hbar g_0 a_0 \cos\omega_0 t \cos\mathbf{k}_0\mathbf{r} = -2\hbar g_0 a_0 \left[\cos(\mathbf{k}_0\mathbf{r} - \omega_0 t) + \cos(\mathbf{k}_0\mathbf{r} + \omega_0 t)\right]$$

Stationary wave driven by the external field

Note that, in contrast with the polarization energy $-\hbar\lambda$, which is quadratic in the coupling coefficient b, the energy caused by the external field is linear in b, as expected

Estimate the mean value of this energy by making use of the external field energy $W_0 = 2\hbar\omega_0 |a_0|^2$

$$\overline{\delta\varepsilon} = \frac{4}{\sqrt{35}}(6\pi^2)^{2/3}\frac{c}{\omega_0 d}\left(\frac{\lambda_c}{d}\right)^2\sqrt{\frac{e^2 W_0}{d}}$$

Even for reasonably high energy densities, is still a very low energy

The energy of the external field is distributed over the energy of the polarization field (which is very low) and the energy of the pairs

Note that the above results are sensitive to decreasing d (except for the number of pairs)

So, we can enhance the relevant values by the focalization of the energy in very small volumes

However, for usually available energies this enhancement is still insufficient for getting any appreciable result

Refractive index

The external field induces a polarization field which is stationary (the vector potential does not depend on time)

As a consequence of the stationary dynamics of the electrons and positrons

Therefore, the polarization electric field is vanishing, and we are left only with a static magnetic field (magnetization)

Under the action of an external field **the vacuum gets magnetized**

The vector potential of the polarization field $A_0^{pol} = g_0 N/2\omega_0$

This polarization field depends on the strength A_0 of the external field through the field energy W_0 which generates the number of pairs N

Consequently, we can define a static magnetic susceptibility of the polarized vacuum

$$\mu = 1 + \frac{eb}{4mc\omega_0} H_0 = 1 + \frac{(6\pi^2)^{2/3}}{4\sqrt{35}} \left(\frac{\lambda_c}{d}\right)^2 \frac{eH_0}{mc\omega_0} \ ,$$

Note the linear dependence on the strength H_0 of the external magnetic field

The vacuum polarized under the action of an external field, acquires a (very small, static) magnetic susceptibility, and, consequently, a refractive index $n = \sqrt{\mu}$ (greater than unity)

Note above the ratio of the magnetic energy (Bohr magneton in the magnetic field H_0) to the energy quanta $\hbar\omega_0$ of the external field

Conclusions

The vacuum gets polarized with electron-positron pairs under the action of an external classical field of electromagnetic radiation

The polarization field is static, *i.e.* the electric field is vanishing and the vacuum sustains only a static magnetic field

The corresponding magnetic permeability gives the refractive index of the vacuum (extremely small)

The electron-positron pairs are condensed on low-momenta states and exhibit a quasi-stationary dynamics

They acquire a single-particle energy, which is quasi-localized in space as a stationary wave driven by the monochromatic external field

The number of pairs are determined by the external energy, while the single-particle energies and the energy of the polarization field depend on the energy density of the external field

All these numerical results are extremely small, even for reasonably high external energies and energy densities

An important role in the magnitude of these effects is played by the Compton wavelength of the electron, which is very small in comparison with the extent of the spatial region over which we can concentrate the energy of the external field

The results presented here have been derived by treating the electron-positron and photon dynamics by means of classical fields

A procedure justified by the polarization process, which implies continuous creation and annihilation of electron-positron pairs under the action of a classical field of radiation

Resembling a plasma of electron-positron pairs

The coupled non-linear equations of motion have been solved for these fields, and the solution led to the results described above

Do we need a NEW THEORY? For the NEW PHYSICS?

YES, we do:

The GREAT, GOOD, CLASSICAL PHYSICS!

Another Meaning of It All

We get confined pairs, as many as high the available (laser) energy

They appear and recombine continuously. How can we see them experimentally?

Extract them by a static electric field E_s, for instance. How strong would this field be?

$$eE_s c\tau \sim mc^2 \ ,$$

where $\tau \sim \hbar/mc^2 \sim 10^{-21}s$ is their lifetime

$$eE_s c \cdot \frac{\hbar}{mc^2} \sim mc^2 \Longrightarrow E_s \sim \frac{m^2c^3}{e\hbar}$$

Whats this? This is:

Schwinger's limit