On the phase diagram of the Quantum Chromodynamics

M Apostol

Institute for Atomic Physics, Institute for Physics and Nuclear Engineering, Magurele-Bucharest

Dubna, Moscow, Autumn 2013

Quantum Chromodynamics

$$L = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \overline{\psi}_{f\alpha} \left[i\gamma^\mu (\partial_\mu \delta^{\alpha\beta} - igt^{\alpha\beta}_a A^a_\mu) - m_f \delta^{\alpha\beta} \right] \psi_{f\beta}$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^a_{bc} A^b_\mu A^c_\nu$$

$$[t_a, t_b] = i f^c_{ab} t_c$$

$a = 1...8$ gluon label; $G^a_{\mu\nu}$- gluon fields, g - coupling constant, f^a_{bc} - structure factor SU(3); eight 3×3 matrices t_a $(\alpha, \beta = 1, 2, 3)$ - SU(3) generators; $\psi_{f\alpha}$ - quark fields (bispinors), flavour $f = 1...6$ and color $\alpha = 1, 2, 3$, mass m_f; γ^μ - Dirac matrices

Description

- Analogy Quantum Electrodynamics; Yang-Mills fields and SU(3); Perturbation theory: quark (de-) confinement; mesons, hadrons (nucleons); not free, independent observables

- Lattice-gauge theory calculations; Qualitative results

- Jet production in high-energy collisions

- Various simplifications: only u and d quarks (the lightest), chiral symmetry; Broken symmetries, phase transitions

- Methods borrowed from condensed matter physics

Central concept in QCD: **broken symmetry** (chirality, Higgs; hadrons, weak bosons, etc)

Borrowed from Condensed Matter (superconductivity, superfluidity, ferromagnetism, etc)

Quark-gluon plasma-hadron transition

-Nucleus-Nucleus high-energy collisions $1TeV$/nucleon (hadr bind $1GeV$)

-Liberation of quarks and gluons: plasma

-Ignition threshold $100-150MeV$ $(125-180MeV)$

-Plasma expands, gets cool and hadronizes

-First-order transition (van der Waals):$(p+an^2)(1-bn)=nT$

-$p=0$ (or $p=const\cdot n^2$): $T=An-Bn^2$-critical curve ($n>n_c=A/2B$, $T_c=A^2/4B$)

Order parameter: second order phase transition (chirality?)

Lambda point of He (Feynmann, Onsager)

Hadronization Theory (Roum. Reps. Phys. **59** 249 (2007); Mod. Phys. Lett. B**21** 893 (2007))

-Ultrarelativistic gas of quarks

-In equilibrium with gluons (vanishing chemical potential, indefinite number)

$$E = VT^4/(\hbar c)^3 \ , \ N = VT^3/(\hbar c)^3 \ , \ T = \hbar c n^{1/3}$$

$$p = E/3 \ , \ S \simeq 4E/3T \simeq N$$

$E/N_n = 1TeV$ ($N_n = 100$ nucleons)$\Longrightarrow T_0 = 1GeV$, $N_0 = 10^3 N_n$ ($n_0 = 125fm^{-3}$, quarks) ($a = 2fm$ nucleon radius, $R_0 = aN_n^{1/3}$; $V = R^3$)

Comment

Set the nuclear quark density n in

$$T = \hbar c n^{1/3}$$

$n^{1/3} = 1/a$, or $n^{1/3} = 3/a$, $a = 2fm$

Get **ignition (threshold) temperature**

$$T = 100 - 150 MeV$$

(or $125 - 180 MeV$ for $1 = 1.5 fm$)

Expansion, cooling (goes to equilibrium!)

$$R = R_0(1 + ct/R_0)\ ,\ V = V_0(1 + ct/R_0)^3$$

$$T = T_0(1 + ct/R_0)^{-3/4}\ ,\ N = N_0(1 + ct/R_0)^{3/4}$$

Density decreases:

$$n = N/V = n_0(1 + ct/R_0)^{-9/4} = n_0(T/T_0)^3$$

Main equation: $T = \hbar c n^{1/3}$

Comment: Compare with hydrodynamical model of particle production

Fermi/Landau-Belenkij (1950-1956)

Adiabatic expansion

Condensation (transition)

$$T_t \simeq f^{1/2} T_q (T_q/T_m)^{1/2}$$

T_q - quark characteristic temperature

T_m- hadron characteristic temperature$\simeq m_0 c^2$

$m_0 = 4MeV$ - mean quark mass

f- fraction which hadronizes - outer shell$\simeq 1/N_0^{1/3}$ (saturation of nuclear forces; $f = 2 \times 10^{-2}$)

At transition $T_t = T_q$ and

$$T_t = f^{-1} T_m \ , \ \hbar c n_t^{1/3} \simeq f^{-1} m_0 c^2$$

Transition density

$$n_t = f^{-3} (m_0 c/\hbar)^3 \simeq 1 fm^{-3}$$

(Compton wavelength of m_0!)

Transition temperature

$$T_t = f^{-1} (m_0 c^2) \simeq 200 MeV$$

Critical curve, Critical point (universal)

$$T = An - Bn^2 \ , \ n_c = A/2B \ , \ T_c = A^2/4B$$

$$T_t = An_t - Bn_t^2 \ , \ n_t = 1fm^{-3} \ , \ T_t = 200MeV$$

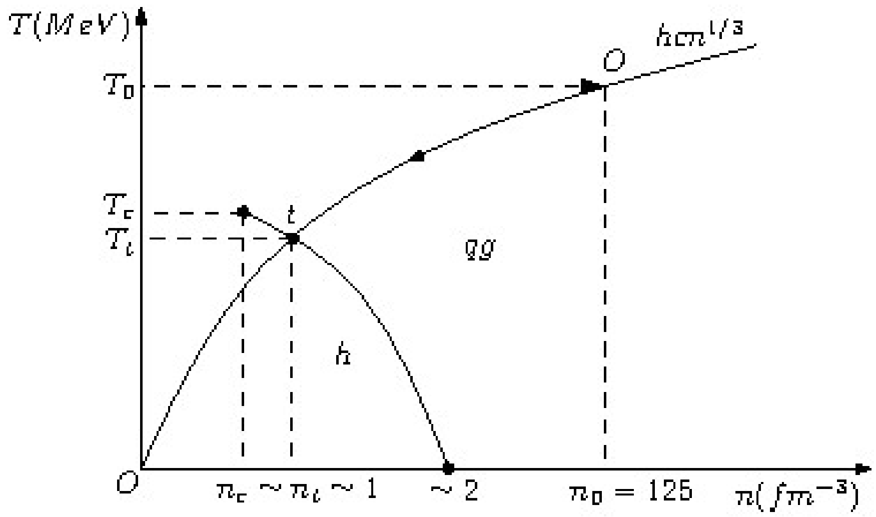
$\Longrightarrow 200 = A - B$

$n_t = 1fm^{-3}$ close to the saturation density of the Nuclear Forces (3q per $(1.5)^3 = 3.4fm^3$)

Tentatively set $n_c = A/2B = n_t = 1fm^{-3}$, $A = 2B$

$\Longrightarrow A = 400$, $B = 200$ (MeV and fm)

Hadronization Process



Hadronization of the quark-gluon plasma. Phase diagram temperature T vs quark density n. Note the hadronization curve $T = \hbar c n^{1/3}$.

Brief historical sketch:

-Gross, Politzer, Wilczek: confinement, deconfinement (1973)

-Cabibbo, Parisi (1975)-liberation of quarks

-Shuryak (1978)-QGPlasma

-Bjorken (1983): high-energy collisions

-van Hove (1984)-hadronization

-Hagedorn (1965): critical temperature of hadrons

-Fermi/Landau-Belenkij (1950-1956): hydrodynamic model part prod

-Weiner: "Surprises from QGP:when was the QGP seen?"