A few Notes on Quantal Gravity

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Gravity is Weak

Strong force 1, Electrom force 1/137, Weak force 10^{-5}

Gravitation 10^{-36}

Quanta is small: $\hbar$

Quantal Gravity ?

Big Bang

Universe $T \sim 20 \cdot 10^9$years, $R = cT$, $M \sim 10^{50}$Kg; Expanding

Planck scale: $Gm^2/r = mc^2 = \hbar c/r$

Big Bang atom: strong gravity, quantal effects; quasi-classical, however

Quantization

1 Canonical: fcts→opers, P brackets→commuts; compatibility with class motion in the qclass limit $\hbar \to 0$

2 Relativity:

- Lorentz transfs are compatible with quantal delocalization

- Time and space; time distinct in quantal motion!

- Different route: rel eqs of motion, rel Lagrangians and Hamiltonians, distinct from the class ones

- New concepts: second quantization, antiparticles, fields

-Finite c: limits on $\mathbf{r}$, $\mathbf{p}$, t, etc; not incorporated; divergencies and renormalizability

- Limited success in quantiz rel fields

3 Quantizing Gravity:

- Covariance: on curved space local transfs - contrary to quantal delocalization

-High non-linearities

Gravity has been quantized (?)

-deWitt, 1967; canonical; lack of covariance

-deWitt, 1968; S-matrix; asymptotic free waves

-non-renormalizability

-Strings (Witten), loops (Smolin), etc

The Scheme of Lifetime

Einstein Equations

$$R_{ik} = (8\pi G/c^4)(T_{ik} - \tfrac{1}{2}g_{ik}T)$$

$$F^{ik}_{;k} = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^k}(\sqrt{-g}F^{ik}) = -\frac{4\pi}{c}j^i = -\frac{4\pi}{c}\sum\frac{e_a c}{\sqrt{-g}}\delta(r-r_a)\frac{dx^i}{dx^0}$$ (Maxwell eqs)

$g^{ik}p_i p_k - m^2c^2 = 0$ (Newton eq; Ham-Jac; cons of energy)

$p_i = mcu_i$, $u_i = dx_i/ds$

- R ($(\partial g/\partial x)^2$, $g(\partial^2 g/\partial x^2)$, $g^2(\partial g/\partial x)^2$) ; T (u^2, F^2; g)

-Newton and Maxwell eqs contained, through conservation laws

Resolution Scheme

-Ten Einstein eqs for (six) g and (three) velocities u.

- Fields through Maxwell eqs

Gravitons

Empty space

$g = g_0 + h,\ h \ll 1 (g_0$ Galilean)

Linearization $R(h) \simeq R^{(1)}(h) = (1/2)\{\partial^2 h/c^2\partial t^2 - \Delta h\}$

Wave equation

$$\partial^2 h/c^2\partial t^2 - \Delta h = 0$$

Plane waves, $\omega = ck$, quanta $\varepsilon = \hbar\omega$ gravitons; transverse polarization, bosons, etc

Self-interacting gravitons

Empty space, $g = g_0 + h$, $R(h) = R^{(1)}(h) + R^{(2)}(h)$;
$R^{(1)}(h) = (1/2)\{\partial^2 h/c^2\partial t^2 - \Delta h\}$ linear

$$R_{ik}^{(1)}(h) = -R_{ik}^{(2)}(h) = (8\pi G/c^4)\{-\frac{c^4}{8\pi G}R_{ik}^{(2)}(h)\} = (8\pi G/c^4)[t_{ik}(h) - g_{ik}(h)t(h)/2]$$

t_{ik} gravitational energy-momentum (pseudo-)tensor

$$\partial^2 h_{ik}/c^2\partial t^2 - \Delta h_{ik} = (16\pi G/c^4)[t_{ik}(h) - g_{ik}(h)t(h)/2]$$

Graviton-induced lifetime

Formally $\partial^2 h/c^2\partial t^2 - \Delta h = \varepsilon(16\pi G/c^4)t(h)$, $\varepsilon \to 1$

Divergent series $h = h_0 + \varepsilon h_1 + ...$

Graviton has a limited reality, existence; lifetime

$$R \sim \frac{1}{d^2} \sim \frac{G\varepsilon}{c^4 l^3}$$

-For $d = l = \lambda$ and $\varepsilon = \hbar c/\lambda$ one gets the Planck's length by $1 = G\hbar/c^3\lambda^2$

-Any energy E is affected through graviton interaction by

$$\delta E \sim \frac{G\varepsilon_g}{c^4 \lambda_g} E$$

-Lifetime

$$\tau = \hbar/\delta E \sim \frac{\hbar c^4 \lambda_g}{G\varepsilon_g E}$$

-Graviton-graviton lft $\tau_{gg} \sim (\hbar c^4 \lambda_g / G\varepsilon_g^2)$, graviton-matter lft $\tau_{gm} = \tau_{mg} \sim (\hbar c^4 \lambda_g / G\varepsilon_g E)$, graviton-photon lft $\tau_{gph} = \tau_{phg} \sim (\hbar c^4 \lambda_g / G\varepsilon_g \varepsilon_{ph})$

-Order of magnitude; $\varepsilon = E = 1\text{MeV}$, $\tau \sim 10^{36}\lambda_g$

-Extremely long (weak gravitation); Universe age $T \sim 10^{18}s$

Kinetic Equation for Gravitons

General scheme of transport

$$\frac{\partial}{\partial t}(\Delta N/S) = c\frac{\partial}{\partial x}(\Delta N/d^3)c\tau \text{ (flux, flow, density)}$$

Graviton conductivity (Λmean free path)

$$K = (\Delta N/St)/(\Delta N/d) = c^2\tau/d^3 = c\Lambda/d^3$$

Transport velocity

$$v = KS = c^2\tau/d = c\Lambda/d$$

The Velocity of the Universe. Hubble constant

$v = c\Lambda/d = Hd$, $H = c\Lambda/d^2 \leq c/d$, any d

Limit of ballistic transport $\Lambda = d = R$

$$H = c/R = 1/T \sim 10^{-18} s^{-1}$$

Another kinetic equation

Graviton destruction at the borders of the Universe

$$dN_g/dt = -CN_m \ , \ C \sim 1/\tau$$

Matter creation at the borders of the Universe $dN_m/dt = CN_m$; $N_m \sim e^{Ct} \sim R^3$ (limited variation)

$$dR/dt \sim \frac{C}{3}R$$

Hubble constant

$$H \sim C \sim 1/\tau \sim c/R \sim 1/T$$