M Apostol

The Many-Body Theory

Its logic along the years

Dedicated to the memory of O Dumitrescu

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The Beginning

Non-relativistic Quantum Theory of identical interacting particles

Many-particle wavefunction $\psi(x_1, x_2, ...)$

Birth of QM (Born, Heisenberg, Jordan, Schrodinger)

Permutations; Pauli's principle

Note the Entanglement

The Hartree-Fock

Great advance ~1930

Mean-field: Bohr, Hartree

Exchange: purely Q; Fock, Dirac; Heisenberg's magnetism

"We got the equations of matter" Dirac ~1930

Solve them!

Pople 1940...; Kohn&Sham 1960...; molecules, nanostructures, ...

The Many-Body

~1950; Field Theory; Feynman's propagators, diagrams, etc; RPA

Nothing new! One-particle picture with perturbations

Many-body problems solved without any many-body theory (superconductivity, superfluidity, nucleons, electron liquid, Fermi liquid)

The book of Abrikosov, Gorkov, Dzyaloshinski ~1960

The book of Fetter&Walecka ~1970

Maturation and stagnation

Good titles for Research Programs:

"The One, Few and Many-Body Theory"

Non-integrability

Celestial Mechanics; Statistical Mechanics; Poincare

$$H = \sum_i m v_i^2/2 + (1/2) \sum_{ij}{}' V_{ij}$$

$$m d\mathbf{v}_i/dt = -\sum_j{}' \partial V_{ij}/\partial \mathbf{x}_i$$

Conservation laws; momentum, energy, angular momentum

Use $dV_{ij}/dt = (\partial V_{ij}/\partial \mathbf{x}_i)\mathbf{v}_i + (\partial V_{ij}/\partial \mathbf{x}_j)\mathbf{v}_j$

Get

$$\frac{d}{dt}(mv_i^2/2 + {\sum_j}' V_{ij}) = {\sum_j}' (\partial V_{ij}/\partial \mathbf{x}_j)\mathbf{v}_j$$

or

$$dE_i = d(mv_i^2/2 + {\sum_j}' V_{ij}) = {\sum_j}' (\partial V_{ij}/\partial \mathbf{x}_j)d\mathbf{x}_j$$

Non-integrable: $(\partial^2 E_i/\partial x_i \partial x_j)$

Many-Particle Forces

$$\delta x_i = g_{ij} dx_j$$

$$\delta x_i^{(1)} = g_{ij} dx_j, \;\; \delta x_i^{(2)} = g_{ij} g_{jk} dx_k, \ldots$$

$$\delta U^{(1)} = \textstyle\sum'_{ijk} (\partial V_{jk} / \partial x_j) g_{ji} dx_i,$$

$$\delta U^{(2)} = \textstyle\sum'_{ijkl} (\partial V_{jk} / \partial x_j) g_{jl} g_{li} dx_i, \ldots$$

$$F_i^{(1)} = -\sum_{jk}{}' (\partial V_{jk} / \partial x_j) g_{ji}, \;\; F_i^{(2)} = -\sum_{jkl}{}' (\partial V_{jk} / \partial x_j) g_{jl} g_{li}, \ldots$$

The Meaning

Average over j above; the mean field

$$dE_i = d(mv_i^2/2 + \left\langle \sum_j{}' V_{ij} \right\rangle) = d(mv_i^2/2 + h_i) = 0$$

Any departure: spoils the mean field picture; non-iteratively

The basic point: phenomenological parameters g_{ij}, for a limited one-particle approx

Quasi-particles and their lifetime

Hartree-Fock is such a theory

Composite particles like superconducting pairs: similar situation

Four-fermion condensate: alpha decay, nuclear cohesion energy, ...

(J Bulboaca, F Carstoiu, M Horoi, M Apostol, ...; under the leadership of O Dumitrescu)