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Research Studies on Non-Linear Effects in Seismic Risk Assessing and Mitigation

- Earthquake Statistics, Amplification Factors, Non-linear Elasticity -

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(Summary)

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SUMMARY

1. A model of seismic focus.[1] It is widely agreed that the seismic energy E released in an earthquake is related to the earthquake's magnitude M by the Gutenberg-Richter-type relationship

$$\ln E = a + bM \ . \tag{1}$$

Statistical analysis of moderate and strong earthquakes ($5.8 < M < 7$), which are probably most prone to represent a statistical ensemble, indicates values $a \simeq 10$ and $b \simeq 3.5$ (in decimal logarithms $a \simeq 4.4$ and $b \simeq 1.5$) for energy measured in J. (The error in seismic energy may be up to a factor of 10). These numerical values may be adopted for the present purpose, although the considerations made herein do not depend critically on such numerical values. Parameter a in (1) indicates the existence of a threshold energy $E_0 = e^a$ ($E_0 \simeq 4.4 \cdot 10^4$J), so that equation (1) can be recast as $E/E_0 = e^{bM}$.

It is customary to assign a region of characteristic length R to the seismic energy E, through $E \sim R^3$, and, similarly, a characteristic threshold length R_0 can be associated to the threshold energy $E_0 \sim R_0^3$, leading to

$$\ln(R/R_0) = \beta M \ , \tag{2}$$

where $\beta = b/3 = 1.17$. The two characteristic lengths R and R_0 have a double meaning, at least: on one side, they may be associated with the central core of the critical focal zone where the seismic energy accumulates, and, on the other, R may correspond to the characteristic length of the seismic region disrupted by the earthquake, R_0 being in this case a scale length. The empirical evidence in the latter case seems to support an equation of the type (2).

It is assumed that the characteristic lengths R and R_0correspond to a localized critical focal region where the seismic energy builds up by mechanical tension. It is also reasonable to assume that the process of accumulating energy in the seismic focus exhibits a uniform velocity $\mathbf{v}$, so that the accumulation of the seismic energy in focus obeys the continuity equation

$$\partial E/\partial t = -\mathbf{v} gradE \ , \tag{3}$$

where t denotes the accumulation time. Further on, the same value v of the velocity may be assumed along all three spatial coordinates, and the spatial variation of energy along each coordinate is represented as $(E + E_0)/(R + R_0)$. By such assumptions equation (3) becomes

$$dE/dt = (1/r)v\frac{E + E_0}{R + R_0} \ , \tag{4}$$

where $r = 1/3$. The factor $1/r = 3$ in front of (4) arises, therefore, from pure geometric reasons. Since for other, more special, geometries of the critical focal zone this factor may differ from 3, notation r is preferred in the interest of the generality of the treatment. For the present purpose the value of this parameter is taken as $r = 1/3$. Equation (4) leads also to consider the accumulation time $t = R/v$ as well as the threshold time $t_0 = R_0/v$, so it becomes

$$dE/dt = (1/r)\frac{E + E_0}{t + t_0} \ . \tag{5}$$

The solution of (5) is obtained straightforwardly as

$$1 + t/t_0 = (1 + E/E_0)^r \ . \tag{6}$$

For large values of energy E ($E \gg E_0$) solution (6) reads $t/t_0 \simeq (E/E_0)^r = R/R_0$, or

$$t \simeq t_0(E/E_0)^r = t_0 e^{\beta M} \ , \tag{7}$$

where the Gutenberg-Richter law (1) is used, and $\beta = br = b/3 = 1.17$. Equations (6) and (7) are the basic equations of the present model of seismic focus. According to equation (5), such a model looks like a growth model, with a typical power-law as given by (6).

2. Earthquake Statistical Distributions.[2] Let N_0 be the number of earthquakes during a long time T, characterized by the average threshold time $t_0 = T/N_0$, where N_0 is very large. The cutoff parameter t_0 may be viewed as the seismicity rate. Similarly, the frequency of N earthquakes characterized by time t can be written as $N/N_0 = 1/(1+t/t_0)$. Hence, it follows straightforwardly the temporal probability distribution

$$P(t)dt = -d(\frac{1}{1 + t/t_0}) = \frac{1}{(1 + t/t_0)^2}dt/t_0 \ , \tag{8}$$

or, making use of (6), the probability distribution in energy

$$P(E)dE = \frac{r}{(1 + E/E_0)^{1+r}}dE/E_0 \ . \tag{9}$$

Similar power-law distributions in energy have been derived recently by employing Tsalis entropy for the fragmentation of a dynamical fault-planes model. Such distributions are sometimes called Omori-type distributions, where r is an Omori parameter.

Making use of the energy distribution (9) and the Gutenberg-Richter law (1) the magnitude distribution

$$P(M)dM = \beta e^{-\beta M}dM \tag{10}$$

is obtained straightforwardly, for large energies $E \gg E_0$. The number ΔN of seisms with magnitude between M and $M + \Delta M$ is given by $\Delta N/N_0\Delta M = P(M)$, or

$$\lg(\Delta N/T) = A - BM \ , \tag{11}$$

where $A = \lg(\beta\Delta M/t_0)$ and $B = \beta/2.3$. Such a linear relationship has been checked for a large amount of earthquakes, and $A \simeq 4.6$ and $B \simeq 0.6$ were obtained, for instance, for $5.8 < M < 7.3$ (and $\Delta M = 0.1$). These values may be adopted here for the present purpose, though the numerical values of such quantities do not affect the results presented herein. Making use of the value for the parameter$B$, it is obtained $\beta \simeq 1.38$, in fair agreement with the value $\beta = 1.17$ given here. Similarly, a global rate of seismicity $1/t_0 \sim 10^{5.5}$ per year is obtained from the value of the parameter A, which is consistent with estimations of cca $10^5 - 10^6$earthquakes per year, on the average. There are appreciable deviations from the Gutenberg-Richter linear relationship (11) for extreme values of the magnitude. For low values of M such deviations are consistent with the exact relationship $P(M) = be^{bM}/(1+e^{bM})^{1+r}$ derived from the distribution given by (9) and the Gutenberg-Richter law, but for large values of the magnitude these deviations may indicate that either large seismic events are not statistical events, or the deviations may be ascribed to a magnitude saturation phenomenon.

It is also convenient to introduce the so-called recurrence law, or the excedence rate, which gives the number $N_>$of earthquakes with magnitude higher than M. The corresponding probability is readily obtained from (10) as $P_> = e^{-\beta M}$, so the excedence rate reads

$$\ln(N_>/T) = -\ln t_0 - \beta M \ . \tag{12}$$

This relationship is currently employed for analyzing the earthquake statistical distributions in magnitude. A recent analysis seems to indicate a certain universality in the value of the β slope ($B = \beta/2.3 \simeq 0.6$).

It is worth noting that equation (7) may be viewed as providing the mean recurrence time $t_r = t_0 e^{\beta M}$ for the occurrence of earthquakes of magnitude M (energy $E \gg E_0$). In fact, the mean recurrence time of earthquakes with magnitude in the range M to $M + \Delta M$ is of interest. According to (10) the rate of such earthquakes is given by $\Delta N/T = (\beta \Delta M/t_0)e^{-\beta M}$, so the mean recurrence time can be obtained as

$$t_r = (t_0/\beta \Delta M)e^{\beta M} \ . \tag{13}$$

If the seismicity rate t_0 is known, this equation may be used to predict the mean recurrence times. However, it must be noted that the accuracy of such predictions is, in fact, very low. Indeed, imposing a mean recurrence time t_r, the temporal distribution $(1/t_r)e^{-t/t_r}$ is obtained immediately from the maximum of the entropy. The deviation in the recurrence time defined as $(\bar{t^2})^{1/2} - \bar{t}$ is $(\sqrt{2} - 1)t_r$ for such distributions, which amounts to cca 41% of the mean recurrence time t_r. It is a very large deviation to be of practical use.

3. Accompanying seismic activity.[**3**] The description of the seismic activity accompanying a major seismic shock, both as foreshocks and aftershocks, is relegated to forthcoming publications. A generalized Omori's law is shown to arise by a self-replication process underwent by a generating probability distribution. The latter is a self-generating distribution and thus described by an exponential law, which leads to the original Omori's law. Time dependence of the released energy in accompanying seismic activity is given, as wel as generating exponential distributions in time, magnitude and inverse of energy, and the corresponding Omori's laws. The average deviation in magnitude in accompanying seismic activity is associated to Bath law. The seismic activity is, in general, much more complex than the simplified distinction between regular earthquakes, characterized by a mean recurrence time, and accompanying seisms, described by Omori's law. A more appropriate tool of describing the correlations between seisms is provided by the pair distribution, which is shown to obey a universal power law-exponential law, as derived from scaling arguments. The analysis of this type of distribution, as well as its application to Vrancea region is left to forthcoming publications.

4. Critical-point theory of foreshock regime.[4] Making use of the temporal distribution

$$dP = h(\tau)d\tau = \frac{1-m}{\tau_0}(\tau/\tau_0)^{-m}d\tau \tag{14}$$

assumed by the critical-point theory for the accompanying seismic activity, where the critical expoennt $m \neq 1$ (thus accounting for possible deviations from Omori's classical law), the time-dependence of the energy

$$\ln(E/E_0) \cong -\frac{1-m}{r}\ln(\tau/\tau_0) \tag{15}$$

is obtained, as well as of the magnitude

$$M = -\frac{1-m}{\beta}\ln(\tau/\tau_0) \ , \tag{16}$$

bu using (9) and the Gutenberg-Richter relationship. Time $\tau > 0$ is measured with respect to the main seismic shock, and τ_0 is a characteristic time, as required by this theory. However, the time-energy dependence for accomanying seismic activity is distinct from the one predicted by the focus model described above, which applies to main, regular earthquakes only. It is worth noting that $(1-m)/r = 2$ in (15), such that $m = 1/3$ for $r = 1/3$, providing the rate of released energy obeys $E \sim -1/\tau^2$, as suggested by empirical data (in agreement with theoretical results regarding the accompanying seismic activity).

5. Amplification factors.[5] For a linear harmonic oscillator of mass m, frequency ω_0 and friction coefficient α, such as the damping coefficient is $\lambda = \alpha/2m\omega_0$, subjected to an external periodic force of amplitide f_0, the amplification factor of the displacement x_{max} is given by

$$F_d = |x|_{max}/d_{max} \cong \frac{1}{4\lambda}(1 - e^{-\lambda(2k+1)\pi/2}) \ , \tag{17}$$

at resonance, where $d_{max} = 2f_0/m\omega^2$ and k denotes any integer. For small values of the damping coefficient λ the amplification factor may attain considerably higher-than-unity values. Indeed, for $\lambda(2k+1)\pi/2 \ll 1$ we get

$$F_d \cong (2k+1)\pi/8 \tag{18}$$

from (17). Typical values for λ allows the integer k go up to $k = 1, 2, 3, 4$, where the amplification factor reaches the values 1.18, 1.96, 2.75 and 3.53, respectively, for times $t = (2k+1)T/4$, where T is the period of the oscillations. For higher values of the damping ($\lambda > 0.25$, for instance) the amplification factor is less than unity.

A similar analysis holds for the velocity, whose amplification factor is given by

$$F_v = |\dot{x}|_{max}/v_{max} \cong \frac{1}{2\lambda}(1 - e^{-\lambda k\pi}) \ , \tag{19}$$

at resonance, where $v_{max} = f_0/m\omega_0$. For small values of the damping coefficient the amplification factor is given by

$$F_v \cong k\pi/2 \ , \tag{20}$$

and it may attain higher values than the amplification factor for displacement (up to 2π for instance, corresponding to $k = 4$). Similarly, the amplification factor for acceleration is given by

$$F_a = \left|\ddot{x}\right|_{max} / ac_{max} = \frac{1}{2}\left|\omega t \sin\omega t - 2\cos\omega t\right|_{max} \ ., \tag{21}$$

where $ac_{max} = f_0/m$. Its maximum values are (slightly less than) $F_a \cong (2k+1)\pi/4$.

This theory is extended to external shocks of very short duration, leading to amplification factors for displacement

$$F_d = 1/\sqrt{2\pi e} \tag{22}$$

(where $e \simeq 2.72$), amplification factors $F_v = 1$ for velocity and $F_a = 2.28$ for acceleration. As one can see, the amplification is less than unity for displacement, equal to unity for velocity and higher than unity for acceleration for shocks of very short duration

6. Elasticity of a uniaxial solid.[6] The wave propagation in elastic bodies, and the associated phenomena, such as the attenuation of the elastic motion, or the energy diffusion, under circumstances of anisotropy, complex structure (for instance granular or fragmentary structure), inhomogeneities, anharmonicities, etc, may be grouped under the generic term of non-linear elasticity.

Bodies with complex structures may exhibit anisotropies at macroscopic scale, which affects profoundly their elastic properties. Wave propagation in such a continuous body with axial anisotropy involves new elastic modes, associated with distinct elastic motions in the plane transversal to the anisotropy axis and along the anisotropy axis, like dilatations and compressions, shear modes, and a new mode which is termed a pinch mode. The dispersion of the elastic waves in such a body is appreciable, produced by the non-linearities induced by the anisotropy in the eigenfrequencies. The method employed here may be extended to other bodies exhibiting other types of anisotropies, or limited symmetries and dimensions, special geometries, the non-linearities playing an important role in all these situations. Such situations may be relevant for elastic discontinuities in Earth's inner zone (boundary of the inner core, cca 5000km depth), where a new manganese-iron-silicate crystalline phase was recently discovered, akin to the layered perovskite structures, only of much higher anistropy.

According to its symmetry, the elastic energy density of a uniaxial solid is governed by five elastic moduli,

$$\varepsilon_{el} = \lambda u_{ii}^2 + \mu u_{ij}^2 + \tau u_{i3}^2 + \sigma u_{33}^2 + \nu u_{33} u_{ii} \ , \tag{23}$$

where u_{ij} is the in-plane deformation tensor $(i, j = 1, 2)$, u_{i3} is the deformation vector defined by the anisotropy axis (label 3), and $u_{33}$is a scalar.

This elastic energy gives rise to higly-dispersive waves, with highly non-linear contributions to dispersion relations quartic in wavevectors,

$$\omega_1^2(\mathbf{q}) = \frac{1}{2\rho}\left(2\mu q_\perp^2 + \tau q_3^2\right) \ , \tag{24}$$

and

$$\begin{gathered}\omega_{2,3}^2(\mathbf{q}) = \tfrac{1}{4\rho}\{[4(\lambda+\mu)+\tau]\, q_\perp^2 + (\tau+4\sigma)\, q_3^2 \pm \\ \pm\left[\left[[4(\lambda+\mu)-\tau]\, q_\perp^2 + (\tau-4\sigma)\, q_3^2\right]^2 + 4(\tau+2\nu)^2 q_3^2 q_\perp^2\right]^{\frac{1}{2}})\} \ ,\end{gathered} \tag{25}$$

originating in multiple mode-couplings. The notations above refer to the wavevector component q_3 along the anisotropy axis, and to the wavevector projection $\mathbf{q}_\perp$ in the basal plane. The waves in such a solid body exhibit also combined polarizations.

7. Non-linear diffusion.[7, 8] The diffusion phenomena in complex structures exhibit non-linearities that may lead to self-organized spatio-temporal patterns. A generalized model of statistical fluid is applied to non-linear diffusion performed by microscopic collisions in a non-equlibrium kinetic gas, leading to both non-stochastic part of the Kardar-Parisi-Zhang equation and to a new diffusion equation. The former is applicable to crystal growth on solid substrates, or to water vapours ascending in atmosphere, while the latter exhibit more complex self-organized spatial patterns. These equations are solved for two dimensions, and extension to one- and three-dimensions is indicated. In plane, the radially-symmetric solutions of these equations do not conserve the diffusion, and exhibit singularities either at origin or at finite distances, as well as wavefronts propagating slower and slower in time, having the form of disks or rings. They are suggestive of the atmospheric clouds paterns, or wreaths of smoke and gaseous emanation of chimneys and smokestakes. Similarly, such patterns are associated to energy diffusion in complex structures, as, for instance, the wave localization in heterogeneous bodies, and the local effects associated to such localization. Such a localization phenomenon has been identified recently for seismic waves propagating on the Earth's surface, and suggestions have been made for extracting such type of information from the end sequence of the seismograms (coda).

The non-linear diffusion equations described herein read

$$\frac{1}{S}\frac{\partial n}{\partial t} = \triangle n \pm A(gradn)^2 \ , \tag{26}$$

where S is the diffusion coefficient and A is a non-linear coupling constant. The sign plus correponds to the non-stocahstic part of the Kardar-Parisi-Zhang equation, while the sign minus corresponds to a new equation of non-linear diffusion. The asymptotics of the solution of the former equation in two dimensions read

$$\begin{gathered} n \sim \tfrac{1}{A}\ln\left|\ln\left(\xi^2/4S\right)\right| \ , \ \ \xi^2/4S \ll 1 \ , \\ n \sim \tfrac{1}{\xi^2/4S}\cdot e^{-\xi^2/4S} \ , \ \ \xi^2/4S \gg 1 \ , \end{gathered} \tag{27}$$

where $\xi = r/\sqrt{t}$. They define a diffusion front at $\xi^2/4S \sim 1$, which means $r \sim 2\sqrt{St}$, which propagates with velocity $dr/dt \sim 1/\sqrt{t} \longrightarrow 0$ for $t \longrightarrow \infty$, covering an area which increases linear in time, as the total number of diffusing particles does. This solution has a disk-like form.

For the latter equation in (26), the solution has a logarithmic discontinuity at a finite distance ξ_0, where it looks like $n \sim -\ln\left|(\xi-\xi_0)/2\sqrt{S}\right|$. This singularity defines two fronts of diffusion, one of each side of the singularity, propagating in opposite directions, slower and slower in time, and covering an ever increasing area proportional to the total amount of diffusing particles. This solution has a ring-like form.

It may be conceivable that seismic waves localized (like particles) on Earth's surface, as a consequence of heterogeneous structures, transport seismic energy by such a non-linear mechanism of diffusion, exhibiting self-organized spatial patterns like disks or rings.

8. Quasi-classical approximation.[9] For propagating distances much longer than wavelengths the waves may be approximated by plane waves, or, equivalently, by geometric rays similar to the optical rays. This is the current approximation in the theory of propagating seismic waves. The rays may suffer refraction, reflexion, difraction, interference, etc, and in addition, they may undergo a weak dispersion, arising from the slight change in the properties of the propagating structure. This weak dispersion, which alters slightly the properties of the plane wave is treated by a standard method known as the WKB (or WKBJ) approximation. The wave is given in this case by

$$\psi(x) = \frac{C}{\sqrt{k(x)}} e^{\pm i \int_{x_0}^{x} k \cdot dx} \quad , \tag{28}$$

where $k(x)$ is the (variable) wavevector (and $\lambda(x) = 1/k(x)$ is the wavelength) C and x_0 are constants of integration. The approximation is valid for $|d\lambda/dx| \ll 1$.

Distance may be divided in infinitesimal slices $x_{n-1} < x < x_n, \ n = 0, 1, ...N+1$, and the wave may be approximated in each of them by a superposition

$$\psi_n(x) = A_n e^{ik_n x} + B_n e^{-ik_n x} \ , \ x_{n-1} < x < x_n, \ n = 0, 1, ...N+1 \tag{29}$$

of transmitted and reflected waves, where $A_0 = 1, \ B_0 = R, \ A_{N+1} = T, \ B_{N+1} = 0$, x_{-1} and x_{N+1} being arbitrary. The continuity of the wave leads to a matricial equation, whose solution coincides with (28).

The present treatment can be extended in at least three directions. First, one may account for an incidence different from normal, including this way refraction on an inhomogeneous structure. Then, higher-order contributions can be included in the matricial equation, leading thus to corrections to the rays theory of wave propagation. And, finally, defects, more or less localized, can be included in this treatment, enlarging thus appreciably the capabilities of treating the wave propagation in complex structures.

9. A non-linear equation of elastic waves.[10] Apart from statistical theories, another topic much debated in seismology is the understanding of the non-linear effects associated with the propagation of the seismic energy, and with the seismic waves in general. This is an issue in non-linear elasticity, and an instance of an exact solution to a non-linear wave equation is briefly presented here, as well as its relation to the quasi-plane waves. The intriguing issue in this connection is that, the non-linearities being present, exact solutions, as the one presented below are unphysical, and still the empirical observations are compatible with a limited type of quasi-linear behaviour in the propagation of the seismic energy and the associated effects. It is shown below that indeed, there are local amplification factors in the non-linear effects of the propagation of the seismic energy, which, however, still allow for a quasi-linear regime.

The first non-linear correction to the wave equation comes from the cubic anharmonicities, which lead to an elastic energy

$$E = \int d\mathbf{r}(\frac{\lambda}{2}u_{ii}^2 + \mu u_{ij}^2 + \frac{1}{3}Au_{ij}u_{jk}u_{ki} + Bu_{ij}^2 u_{kk} + \frac{1}{3}Cu_{ii}^3) \ , \tag{30}$$

for an isotropic elastic body, where λ and μ are the usual Lame coefficients, A, B, C are constant coefficients, and $u_{ij} = (1/2)(\partial u_i/\partial x_j + \partial u_j/\partial x_i + \partial u_k/\partial x_i \cdot \partial u_k/\partial x_j)$ is the cartesian (finite-) strain tensor. It is assumed that the coefficients in (30) are such as the stability conditions are satisfied. First, a transverse displacement, say, $u_2(x_1)$ is not affected by the cubic anharmonicities above, so that the corresponding linear wave equation is left unchanged (the transverse waves propagate with velocity $v_t = \sqrt{\mu/\rho}$, where ρ is the density of the body).

A longitudinal displacement $u_1(x_1) = u(x)$ is, however, subjected to the non-linear equation $\partial^2 u/\partial t^2 = (\partial^2 u/\partial x^2)(v_l^2 + v^2 \partial u/\partial x)$, where $v_l = \sqrt{(\lambda + 2\mu)/\rho}$ is the velocity of the longitudinal waves, and $v^2 = [3(\lambda + 2\mu) + 2(A + 3B + c)]/\rho$ is a characteristic square velocity. Leaving aside again the stability conditions, and denoting $U = \partial u/\partial x + v_l^2/v^2$, this non-linear equation becomes

$$\partial^2 U/\partial t^2 = (v^2/2)\partial^2 U^2/\partial x^2 \ . \tag{31}$$

This equation is the continuum limit of the Fermi-Pasta-Ulam equation. Its solution, and solutions of other, similar, equations have been analyzed recently by making use of the Lie algebra of the equation symmetry group and the prolongation technique. The solution $U(t,x) = g(t)f(x)$ of equation (31) can be obtained by elementary quadratures. The time dependence is given by

$$g(t) = |s|\,[\sqrt{3}\frac{1 - cn(\sqrt{\sqrt{3}\,|s|}\,|\omega t|)}{1 + cn(\sqrt{\sqrt{3}\,|s|}\,|\omega t|)} - 1]sgn(\omega^2) \ , \tag{32}$$

where $s = -g(0)$ ($\dot{g}(0) = 0$), ω is a constant of integration and cn is the Jacobi elliptic cosine-amplitude. Function $g(t)$given by (32) is a periodic function with period $\sqrt{\sqrt{3}\,|s|}\,|\omega t| = 4K$, where K is the complete elliptic integral $F(\pi/2, k)$ (~ 4) for $k^2 = (2 + \sqrt{3})/4$. It has singularities at $\sqrt{\sqrt{3}\,|s|}\,|\omega t| = 4K(n + 1/2)$, where n is an integer. These singularities make the solution of eq. (31) unphysical. The spatial dependence $f(x)$ is given by the implicit equation $\sqrt{(|f/h|)^3 - 1}\,F(1/2, 1/3, 3/2; 1 - (|f/h|)^3) = 3\,|\omega x/v|\,/2\sqrt{|h|}$, where F is the Gauss hypergeometric function and $h = f(0)$ is another constant of integration ($f'(0) = 0$). Function f goes like $f \sim |h|\,sgn(\omega/v)^2 + (\omega/2v)^2 x^2$ for $x \sim 0$, and $f \sim (\omega/2v)^2 x^2$ at infinite ($x \to \pm\infty$). It is worth noting that $f(x)$ is boundless for spatial boundaries placed at infinite, which adds to the unphysical character of the solution. The general solution of the non-linear equation for the longitudinal strain $u(t,x)$ reads then

$$u(t,x) = g(t-t_0)\int_0^x dx f(x-x_0) - (v_l/v)^2 x + c \ , \tag{33}$$

where the origin of time t_0 and the origin of space x_0 are introduced, and c is another constant of integration. The nature of this solution is worth discussing. First, it is worth noting that the displacement given by (33) implies large strain (and stress) values at the boundary of the spatial region, which is consistent with the accumulation model of the critical focal zone employed herein. Second, these large strain and stress values may lead in time to ruptures at the boundaries of the focal zone (or at the boundaries of the critical seismic region), as a consequence of the boundless increase of the time dependence (which is singular at certain times, as noted above). These ruptures may propagate, with a non-uniform velocity, which represents a distinct mechanism of dissipation of the seismic energy in the critical zone affected by non-linearities. It is not restricted to cubic anharmonicities, higher-order non-linear contributions to the wave equation leading to a similar behaviour. Third, it is worth noting that the total energy conserves, but it is non-uniformly distributed, such that ruptures may appear in time at the boundaries of the spatial region. The energy flow at the boundaries increases also boundlesly in time. The process looks rather like a vibration than a wave propagation. All these features make the solution unphysical. Exact, unphysical solutions of non-linear type described above are therefore more appropriate for the critical focal zone and for the seismic region disrupted by the earthquakes.

After all this seismic energy is dissipated in ruptures and damage of the elastic body, the non-linear contributions to the wave equation may be viewed as perturbation to the plane wave solutions of the linear equation. Indeed, introducing the perturbation parameter $\varepsilon = (v/v_l)^2$ the equation for the longitudinal displacement may be written as $\ddot{u} - v_l^2 u'' = \varepsilon v_l^2 u' u''$, whose solution reads

$$\begin{gathered} u = a\cos(\omega t - kx) + \tfrac{1}{16}\varepsilon a^2 k^2 (x + v_l t)\cos[2(\omega t - kx)] + \\ + \tfrac{1}{128}\varepsilon^2 a^3 k^4 (x+v_l t)^2 [\cos[3(\omega t - kx)] - \cos(\omega t - kx)] + ... \end{gathered} \tag{34}$$

where a is the amplitude, $\omega = v_l k$ is the frequency and k is the wavevector of the elementary plane wave. The solution given by (34) is, actually, a triple expansion in powers of the perturbation parameter ε, the ratio ak of the amplitude to the wavelength, and the ratio lk of a characteristic length $l = x + v_l t$ to the wavelength. The solution (34) is actually an asymptotic series, and it has a limited validity over finite distances and times, providing that the amplitude is much smaller than the wavelength. Such a wave may be viewed as a quasi-plane wave, *i.e.* a plane wave distorted by higher-order harmonics of limited validity in space and time. It is worth noting the amplification factor F of the order of $F \simeq 1 + \varepsilon alk^2/16$ (in displacement) brought by the non-linear effects to such quasi-plane waves, amplification which is well-documented in the analysis of the local seismic effects due to non-linearities. An estimation of the distribution of the seismic energy originating in a localized focal zone shows that the long wavelengths and small amplitudes are favoured, the ratio ak being of the order of $10^{-2} - 10^{-4}$. Therefore, one may use the quasi-plane waves pictures up to distances l very large in comparison with the wavelengths.

Another worth noting non-linear phenomenon appears in the non-linear coupling between a longitudinal displacement and a transverse one, propagating in the same direction. Beside higher-order harmonics and amplification factors, there may appear resonances at certain frequencies, due to the combined-frequency phenomenon, as, for instance, at the transverse wave frequency $\omega_2 = (\omega_1/2)(1 + v_t/v_l)$, where $\omega_1$is the frequency of the longitudinal wave. Such resonances depend on the ratio v_t/v_l of the waves velocities. Another non-linear coupling arises, for instance, from longitudinal displacements of the type $u_1(x_1)\,,\, u_2(x_2)\,,\, u_3(x_3)$, which might be relevant for the dynamics of the accumulation model of the critical focal zone. There seems not to be a simple treatment of such coupled non-linear equations.

10. Conclusions. There seems to be at least three basic features pertaining to the science of the earthquakes, according to the present image of this science. First, the energy of the earthquakes is distributed over a huge scale, according to the semi-empirical Gutenberg-Richter law (1), relating the seismic energy to magnitude M. Second, the seismic energy originates in a rather restricted critical focal zone, of a characteristic linear size given by (2); at the same time, equation (2) refers also to an epicentral length scale characteristic of the seismic region disrupted by the earthquake. Third, the large variety of the earthquakes in energy, magnitude, number, space and time suggests a statistical approach, as based on their various distributions. Such a statistical approach is also suggested by the distribution in magnitudes of the differential number of earthquakes (equation (11)), by a similar distribution of the earthquakes with magnitudes exceeding a given value (excedence, or recurrence law given by equation (12)), by the Omori temporal distribution of the aftershocks (which goes like $t^{-\gamma}$, where $\gamma = 1^{+}$), and, in general, by Omori-type power laws, where a positive Omori parameter r appears, like in (9), by the average aftershock magnitude (Bath's law, this aftershock magnitude being 1.2 less than the magnitude of the main shock), and by the time Poisson-like distribution of the recurrence times. All these laws are semi-empirical, having a limited validity. Such a limitation comes mainly from the fact that very small seisms, or very great earthquakes, by their own nature, do not reliably belong to a statistical ensemble. It is also woth noting an intriguing issue much debated today in seismology, regarding the effects of the non-linearities on the propagation of the seismic energy, and the corresponding estimation of such local effects, especially in studies of seismic risk and hazard.

An attempt of a systematic understanding of such basic features in seismology is made here, by introducing an accumulation, or growth, model for the concentration of the seismic energy in the critical focal zone. This model relates the accumulation time to the seimic energy (equation (3)), and introduces a characteristic parameter r, whose value $r = 1/3$ is derived on geometrical grounds. It turns out that this parameter r is an Omori parameter. Indeed, the second main theoretical point made here is the interpretation of the accumulation time as the average recurrence time of the earthquakes with corresponding energy (and magnitude), as given by the accumulation model. On this basis, the temporal distribution (8) of the earthquake average recurrence times is derived, the

Omori distribution in energy (equation (9)) and the exponential distribution in magnitudes with the exponent $\beta = br = 3.5/3 = 1.17$ (equations (10) and (11)). The derivation of Omori's law and Bath law for accompanying seismic activity are relegated to forthcoming publications. The differential distribution of the earthquakes in magnitudes (11), as well as the excedence rate (recurrence law) (12) are derived from the exponential distribution in magnitudes with $\beta = 1.17$, in agreement with empirical observations. The time Poisson-like distribution of earthquakes is also derived for a fixed mean recurence time (equation of type $(1/t_r)\exp(-t/t_r)$), and the differential average recurrence time is given for earthquakes with magnitude in the range M to $M + \Delta M$ (equation (13)). The errors in estimating both the magnitude and time distributions are discussed, and the errors asssociated with the seismicity rate $1/t_0$ are shown to be critical for the statistical prediction of long succession times of the great earthquakes (t_0 being also a threshold time introduced by the accumulation model). An application of these results is made to the great earthquakes in the seismic region Vrancea, Romania, in the past 200years. The universal pair-correlation distribution for earhtquakes and its application to Vrancea region are left to forthcoming publications.

It is shown, by analyzing the cubic anharmonic corrections to the elastic waves equation corresponding to longitudinal displacements, that the non-linearities have a disruptive effect on the critical focal zone, or the epicentral region greatly affected by the earthquake. The exact solution of this equation (equations (32) and (33)) has an unphysical character, exhibiting time singularities and a boundless increase at the boundaries of the spatial region. Such an unphysical behaviour is also specific to higher-order non-linearities. Consequently, ruptures may appear at the boundaries of the critical zone, which may propagate in the whole body of the region. However, the propagating seismic energy is distributed mainly on long wavelengths and small amplitudes, such that for small values of the ratio of the amplitude to the wavelength the linear picture of quasi-plane waves is still valid, in a perturbational picture, for limited distances and times (as controlled by the ratio of a characteristic length l to the wavelength, according to equation (34)). As a consequence of the non-linearirites the quasi-plane waves are distorted by higher-order harmonics, and exhibit local amplification factors in displacement, velocity and acceleration, as documented by empirical evidence. The non-linearities may lead, in this perturbational approach, to other effects, as resonances, combined-frequency phenomenon, or non-linear coupling between various kinds of elastic waves, which enriches considerably the linear phenomenology of waves propagation.

The remaining of critical-point theory, amplification factors, elastic waves in a uniaxial solid, non-linear diffusion and a new derivation of the rays theory pertain to various generalizations and extensions into new methods for treating seismic phenomenology discussed partly in Refs.[11]-[17]

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