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On a possible coupling between ortho- and parapositronium

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Abstract

It is shown that the quantum beats observed in the 3γ-annihilation cross-section of orthopositronium in magnetic field may induce similar oscillations, with the same frequency, but much smaller in amplitude, in the 2γ-annihilation cross-section of parapositronium (511keV gamma radiation), as a consequence of the coupling between the positronium hyperfine energy levels brought about by local magnetic interactions in matter. Though the induced oscillations in the parapositronium 2γ-annihilation occur over the orthopositronium lifetime, rather severe experimental restrictions are involved in observing them, due to their small amplitude. These induced quantum beats, as well as the coupling between ortho- and parapositronium, could be used as a new method for probing local magnetic interactions, which was termed "positronium spin resonance" (V. G. Baryshevsky et al, Phys. Lett. **A136** 428 (1989)), by analogy with "muon spin resonance". In addition, the local magnetic interactions may enhance the rate constant of the orthopositronium 3γ-decay, while their component constant in time may produce a small shift in the frequency of these quantum beats.

Some time ago, there has been shown[1, 2] that the 3γ-annihilation rate of (triplet spin-state) orthopositronium exhibits quantum beats in moderate magnetic fields. The existence of these quantum beats have been confirmed recently.[3, 4] The frequency of these oscillations is given by $\Omega = x^2\Delta W/4\hbar$, where $x = 4\mu H/\Delta W$ and $\Delta W \simeq 8.3 \cdot 10^{-4}$eV is the hyperfine energy splitting between the triplet and singlet (parapositronium) spin-states, H is the magnetic field and μ is the Bohr magneton. For an external magnetic field $H = 500$Gs (corresponding to the parameter value $x = 0.014$) the beat frequency is $\Omega = 6.5 \cdot 10^7 s^{-1}$, sufficiently high to be seen as $6 - 7$ cycles during the orthopositronium lifetime $\gamma_t^{-1} \simeq 10^{-7}$s, and sufficiently low to lie well inside the usual (γ triple-coincidence) detector resolution $10^9 s^{-1}$.

For the most favourable geometric arrangement of the experimental setup,[2] the orthopositronium 3γ-annihilation rate (summed over the polarizations of the gamma quanta) in magnetic field reads

$$\gamma_t(t) = \gamma_t[1 + (p/4)\sin\Omega t] \tag{1}$$

where p is the positrons polarization. It corresponds to the magnetic field H aligned along the z-axis, the polarization $\mathbf{p}$ aligned along the x-axis, and the plane containing the three gamma counters (at 120°to each other) tilted by $\pi/4$ with respect to the z-axis. The contributions to $\gamma_t(t)$ given by (1) are kept to the lowest order in the magnetic field H.

The kinetic equation $\partial N_t/\partial t = -\gamma_t(t)N_t$ for the triplet population N_t is easily solved for the damping rate $\gamma_t(t)$ given by (1), leading to

$$N_t = N_{0t}e^{-\gamma_t t}[1 - (\gamma_p p/2\Omega)\sin^2(\Omega t/2)] \tag{2}$$

for $\gamma_p p/2\Omega \ll 1$, where N_{0t} is the initial population. Under typical experimental conditions ($H \sim 500$Gs, $p \simeq 0.25$) the relative amplitude $\gamma_p p/2\Omega$ of these oscillations is about 1%.

The origin of these quantum beats resides in the Zeeman splitting of the hyperfine energy levels of orthopositronium brought about by the external magnetic field H. The hyperfine spin states of positronium are denoted by $\chi_{0,1,2,3}$, where $\chi_0$corresponds to the (singlet) parapositronium state with energy $W_0$and $\chi_{1,2,3}$ correspond to the degenerate (triplet) orthopositronium states with energy W_1. In magnetic field the singlet state $\chi_0$and the triplet state $\chi_1$mix up according to $\widetilde{\chi}_0 = (1 - x^2/8)\chi_0 + (x/2)\chi_1$, $\widetilde{\chi}_1 = (1 - x^2/8)\chi_1 - (x/2)\chi_0$ (for small values of the parameter x), and the energy levels become $E_0 = W_0 + x^2\Delta W/4$, $E_1 = W_1 - x^2\Delta W/4$, where $\Delta W = W_1 - W_0$. It is the Zeeman splitting $E_{2,3} - E_1 = x^2\Delta W/4 = \hbar\Omega$ in the triplet energy levels which causes the quantum beats with frequency Ω.

Under these circumstances, the question raises whether similar quantum beats could be induced in the 2γ-annihilation rate of (singlet spin-state) parapositronium (511keV gamma radiation), whose cross-section is by a fine-structure constant factor higher than the 3γ-annihilation rate of the orthopositronium. The quantum beats in the positronium gamma annihilation, as well as the ortho-parapositronium coupling, would turn then into a new method of probing the local magnetic interactions in matter, which may be called the "positronium spin resonance",[1, 2] by analogy with the "muon spin resonance".

Local magnetic interactions in matter may bring about transitions between ortho- and parapositronium states. Let g denotes a generical transition rate for such processes. They can be associated to the magnetic interactions that are responsible for the linewidth of the paramagnetic resonance, like, for instance, dipolar magnetic interactions. During collisions with atoms in matter such interaction acquires a time dependence with a broad range of frequencies. The pickoff annihilation process for orthopositronium in matter is well documented. During this process, the positron in the orthopositronium (whose wavefunction overlaps with the neighbouring electrons wavefunctions) picks up a neighbouring electron with antiparallel spin, and decay into two quanta of gamma radiation. Obviously, this is an ortho- to parapositronium transition (the reverse process exists also, though the parapositronium lifetime is very short). The magnetic interactions in matter can be characterized by a local magnetic field h which, usually, as shown by the linewidth of the paramagnetic resonance, varies in the range from 1Gs to hundred Gs. The corresponding transition rates vary between $10^7 - 10^8 s^{-1}$ and $10^9 s^{-1}$ (as estimated from the magnetic energy μh divided by $\hbar$). The transition rate g can be associated to such values. In order to preserve the quantum beats the transition rate g must be smaller than the frequency Ω corresponding to the Zeeman splitting $x^2\Delta W/4$ (which in turn is limited by the detector resolution). For an external magnetic field $H = 500$Gs the Zeeman splitting (and quantum beats) frequency is $\Omega = 6.5 \cdot 10^7 s^{-1}$, and for an external magnetic field $H = 1.5$kGs this frequency reaches the detection limit $\Omega = 10^9 s^{-1}$. It follows that g must lie within this range, or be much smaller than $10^7 s^{-1}$, as for weak magnetic samples. Making use of μh as a measure for the magnetic energy, and representing the transition rate by $g \sim \mu h/\hbar$, the inequality $g \ll x^2\Delta W/4\hbar$ implies $h \ll xH$. For higher values of g, the external magnetic field would need to be enhanced, in order to keep distinct the Zeeman splitting and the quantum beats, but the detection limit is overpassed in this case. In addition, the amplitude of the beats decreases with increasing their frequency, which also imposes severe restrictions in identifying experimentally the oscillations. In any case, the following inequalities are satisfied, $g, \gamma_t \ll \Omega \ll \gamma_s$, where $\gamma_s \simeq 10^{10} s^{-1}$ is the parapositronium 2γ-annihilation rate, and perhaps $g \ll \gamma_t$ in some cases. These inequalities make the induced oscillations in the parapositronium 2γ-annihilation a very small effect.

A spin-flip transition rate between ortho- and parapositronium has been discussed recently in connection to the quenching rate of positronium in Xe, or Kr, as caused by the spin-orbit interaction.[5]

Positronium spin-flip, as caused by electron scattering at low energies, has also been estimated.[6] Spin-flip scattering of positronium on Li,[7] as well as on matter atoms,[8, 9] has also been studied. The cross-section estimates for these processes are consistent with a transition rate in the range indicated above. Long time ago, Telegdi et al[10] suggested an upper limit $g \simeq 10^8 s^{-1}$ for a presumable conversion rate of ortho- and parapositronium, at least in some amorphous insulators.

Assuming a transition rate g between ortho- and parapositronium states in matter, the kinetic equations

$$\begin{aligned} \partial N_t/\partial t &= -\gamma_t(t)N_t - gN_t + gN_s \ , \\ \partial N_s/\partial t &= -\gamma_s N_s - gN_s + gN_t \ , \end{aligned} \tag{3}$$

can be written for positronium triplet population N_t and singlet population N_s, where $\gamma_t(t)$ is given by (1). The solution of these equations is readily written as

$$\begin{aligned} N_t &\simeq N_{0t}e^{-(\gamma_t+g)t}[1-(\gamma_p p/2\Omega)\sin^2(\Omega t/2)] \ , \\ N_s &\simeq N_{0s}e^{-\gamma_s t} + N_{0t}(g/\gamma_s)e^{-(\gamma_t+g)t} \ [1-(\gamma_t p/2\Omega)\sin^2(\Omega t/2)] \ , \end{aligned} \tag{4}$$

for $g, \gamma_t \ll \Omega \ll \gamma_s$. One can see from (4) that the triplet damping rate is renormalized from γ_t to $\gamma_t + g$, and the same quantum beats appear in the 2γ-annihilation rate (cross-section) of parapositronium, as a consequence of the coupling brought about by the transition rate g. These beats are induced by the quantum beats in the orthopositronium 3γ-annihilation rate, via the g-coupling, and their amplitude is smaller by a factor g/γ_sthan the amplitude of the 3γ-anihilation oscillations. For weak magnetic interactions ($g \sim 10^7 s^{-1}$ at most) this ratio is $\sim 10^{-3}$ or less, while for stronger magnetic interactions (the limiting value $g \sim 10^9 s^{-1}$) this ratio is about 1/10. It is worth noting that the induced oscillations in the parapositronium 2γ-annihilation rate occurr during the lifetime γ_t^{-1} lifetime of orthopositronium, as shown in (4), similarly with the original beats in the orthopositronium 3γ-annihilation.

In conclusion, quantum beats may be induced in the parapositronium 2γ-annihi-

lation rate by similar quantum beats exhibited by the orthopositronium 3γ-annihilation rate in external magnetic field, with a much smaller amplitude (by a few orders of magnitude), as a consequence of a possible coupling between ortho- and parapositronium states brought about by local magnetic interactions in matter. Providing they may be brought into the reach of experimental detection conditions, these induced oscillations may provide a new probe for local magnetic interactions, which was termed "positronium spin rotation",[1] by analogy with "muon spin rotation". In addition, it is easy to see that the component constant in time of the local magnetic interactions gives rise to a shift $\delta\Omega/\Omega = 2\delta x/x = 2h/H$ in frequency, whose maximum (limiting) value is $2x$. For $H = 500$Gs and $x = 0.014$ this value is $\sim 3\%$.

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